Dominance and Rationalizability

σ_i weakly (strongly) dominates σ'_i if

$$u_i(\sigma_i, s_{-i}) \geq (>) u_i(\sigma'_i, s_{-i})$$ with at least one strict

Prisoner's Dilemma Game

	R	L
U	2,2	0,3
D	3,0	1,1

a unique dominant strategy equilibrium (D,L)

this is Paretodominated? Does it occur?

Public Goods Experiment

Players randomly matched in pairs

May donate or keep a token

The token has a fixed commonly known public value of 15

It has a randomly drawn private value uniform on 10-20

V=private gain/public gain

So if the private value is 20 and you donate you lose 5, the other player gets 15; V= -1/3

If the private value is 10 and you donate you get 5 the other player gets 15; V=+1/3

Data from Levine/Palfrey, experiments conducted with caltech undergraduates

Based on Palfrey and Prisbey

V	donating a token
0.3	100%
0.2	92%
0.1	100%
0	83%
-0.1	55%
-0.2	13%
-0.3	20%

Nash Equilibrium

Definition

players can anticipate on another's strategies

σ is a *Nash equilibrium* profile if for each $i \in 1, \ldots N$

$u_i(\sigma) = \max_{\sigma'_i} u_i(\sigma'_i, \sigma_{-i})$

Mixed Strategies: The Kitty Genovese Problem

Description of the problem

Model:

n people all identical

benefit if someone calls the police is x

cost of calling the police is 1

Assumption: $x > 1$

Look for symmetric mixed strategy equilibrium where *p* is probability of each person calling the police

p is the symmetric equilibrium probability for each player to call the police

each player *i* must be indifferent between calling the police or not

if *i* calls the police, gets $x-1$ for sure.

If *i* doesn't, gets 0 with probability $(1-p)^{n-1}$, gets x with probability $1-(1-p)^{n-1}$

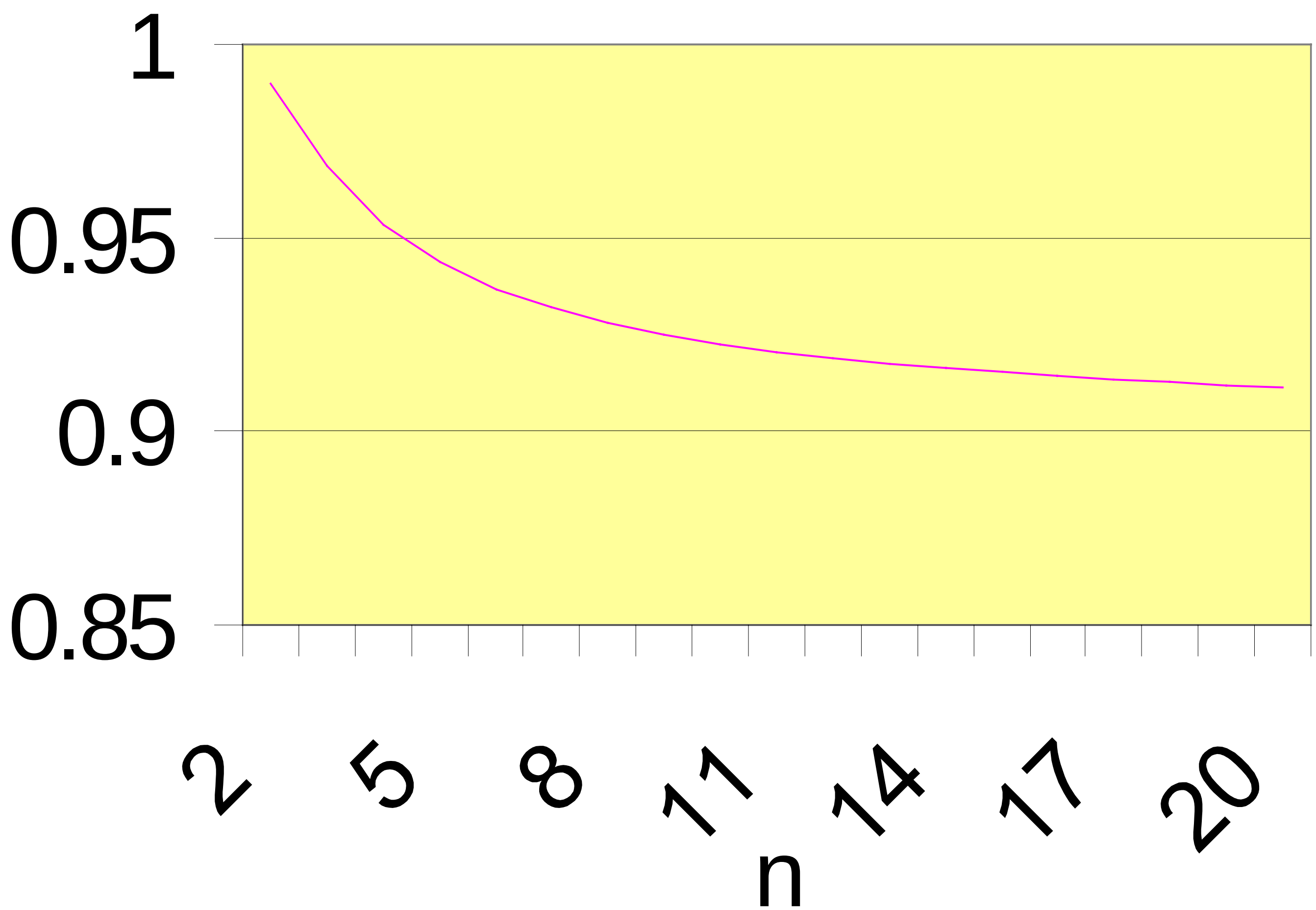
so indifference when $x-1 = x\big(1-(1-p)^{n-1}\big)$

solve for $p = 1-(1/x)^{1/(n-1)}$

probability police is called

$$1 - (1-p)^n = 1 - \left(\frac{1}{x}\right)^{\frac{n}{n-1}}$$

probability police are called



x=10

Coordination Games

	L	R
U	1,1	0,0
D	0,0	1,1

three equilibria (U,L) (D,R) (.5U,.5R)

too many equilibria?? introspection possible?

the rush hour traffic game – introspection clearly impossible, yet we seem to observe Nash equilibrium

equilibrium through learning?

Coordination Experiments

Van Huyck, Battalio and Beil [1990]

Actions $A = \{1, 2, \ldots \overline{e}\}$

Utility $u(a_i, a_{-i}) = b_0 \min(a_j) - b a_i$ where $b_0 > b > 0$

Everyone doing a' the same thing is always a Nash equilibrium

$a' = \overline{e}$ is efficient

the bigger is a' the more efficient, but the “riskier”

a model of “riskier” some probability of one player playing $a' = 1$

story of the stag-hunt game

$\overline{e} = 7$, 14-16 players

treatments: A $b_0 = 2b$

B $b = 0$

In final period treatment A:

77 subjects playing $a_i = 1$

30 subjects playing something else

minimum was always 1

In final period treatment B:

87 subjects playing $a_i = 7$

0 playing something else

with two players $a_i = 7$ was more common

1/2 Dominance

Coordination Game

	L (p_2)	R
U (p_1)	2,2	-10,0
D	0,-10	1,1

risk dominance:

indifference between U,D

$$2p_2 - 10(1-p_2) = (1-p_2)$$
$$13p_2 = 11, p_2 = 11/13$$

if U,R opponent must play equilibrium w/ 11/13

if D,L opponent must play equilibrium w/ 2/13

½ dominance: if each player puts weight of at least ½ on equilibrium strategy, then it is optimal for everyone to keep playing equilibrium

(same as risk dominance in 2x2 games)

Correlated Equilibrium

Chicken

6,6	2,7
7,2	0,0

three Nash equilibria (2,7), (7,2) and mixed equilibrium w/ probabilities (2/3,1/3) and payoffs

(4 2/3, 4 2/3)

6,6	2,7
7,2	0,0

correlated strategy

1/3	1/3
1/3	0

is a correlated equilibrium giving utility (5,5)

What is public randomization?

Approximate Equilibria and Near Equilibria

- exact: $u_i(s_i | \sigma_{-i}) \geq u_i(s_i' | \sigma_{-i})$

  approximate: $u_i(s_i | \sigma_{-i}) + \varepsilon \geq u_i(s_i' | \sigma_{-i})$

- Approximate equilibrium can be very different from exact equilibrium

Radner's work on finite repeated PD

gang of four on reputation

upper and lower hemi-continuity

A small portion of the population playing "non-optimally" may significantly change the incentives for other players causing a large shift in equilibrium behavior.

Quantal Response Equilibrium

(McKelvey and Palfrey)

propensity to play a strategy

$$p_i(s_i) = \exp(\lambda_i u_i(s_i, \sigma_{-i}))$$

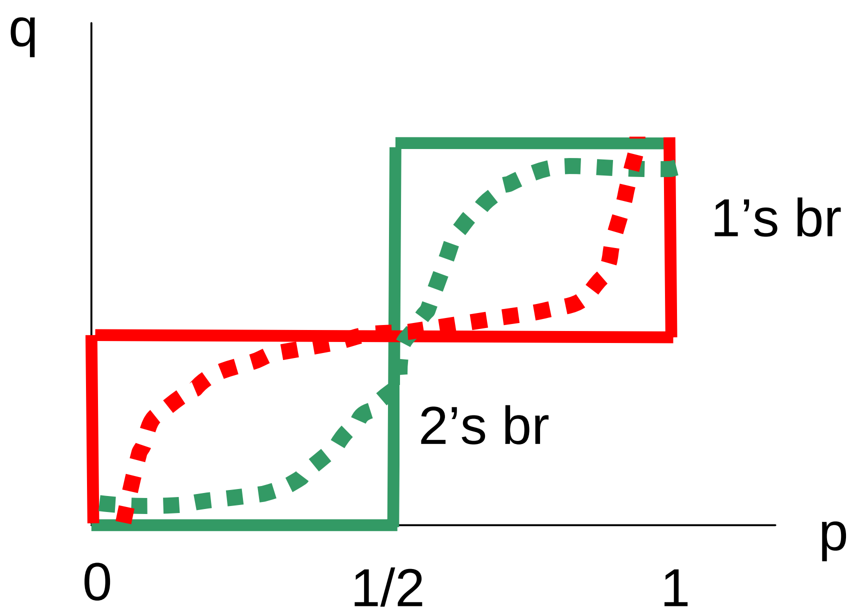
$$\sigma_i(s_i) = p_i(s_i) / \sum_{s_i'} p_i(s_i')$$

as $\lambda_i \to \infty$ approaches best response

as $\lambda_i \to 0$ approaches uniform distribution

Smoothed Best Response Correspondence Example

	L ($\sigma_2(L) = q$)	R
U ($\sigma_1(U) = p$)	1,1	0,0
D	0,0	1,1



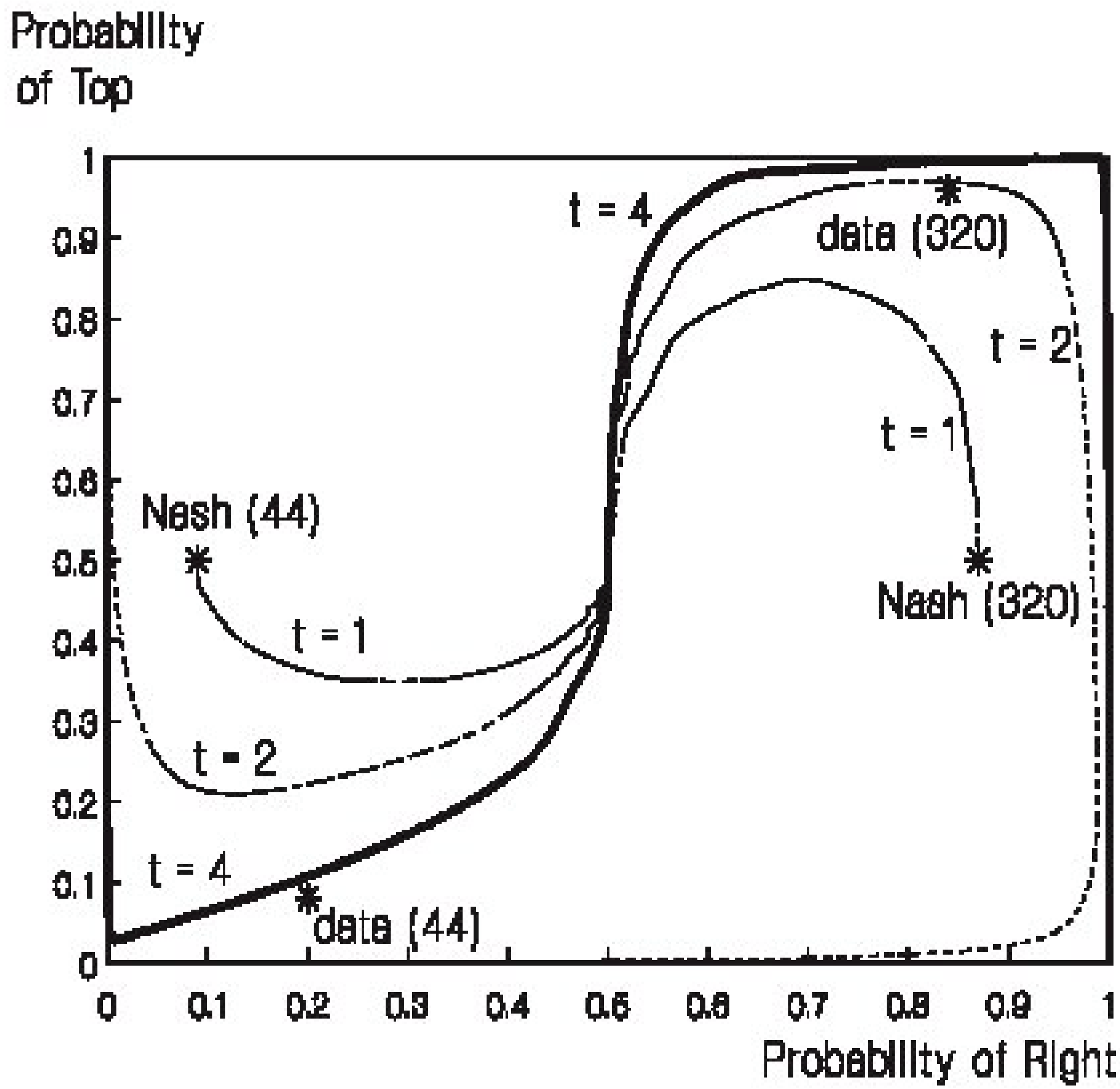
Goeree and Holt: Matching Pennies

Symmetric

	50% (48%)	50% (52%)
50% (48%)	80,40	40,80
50% (52%)	40,80	80,40

	12.5% (16%)	87.5% (84%)
50% (96%)	320,40	40,80
50% (4%)	40,80	80,40

	(80%)	(20%)
50% (8%)	44,40	40,80
50% (92%)	40,80	80,40


Probability
of Top
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of Right
t = 4
data (320)
t = 2
t = 1
Nash (44)
Nash (320)
t = 1
t = 2
t = 4
data (44)

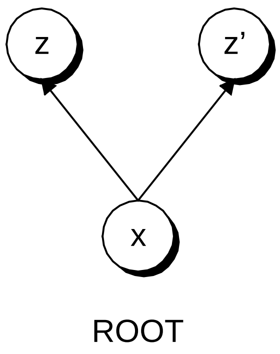
Extensive Form Games

Definition of Extensive Form Game

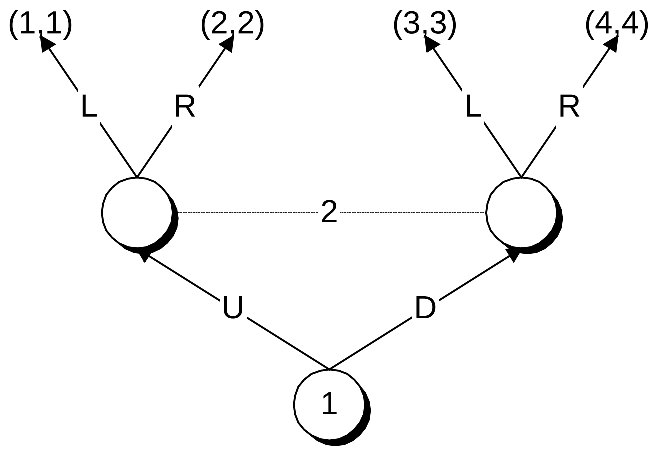
a finite game tree X with nodes $x \in X$

nodes are partially ordered and have a single root (minimal element)

terminal nodes are $z \in Z$ (maximal elements)



Example: a simple simultaneous move game



Behavior Strategies

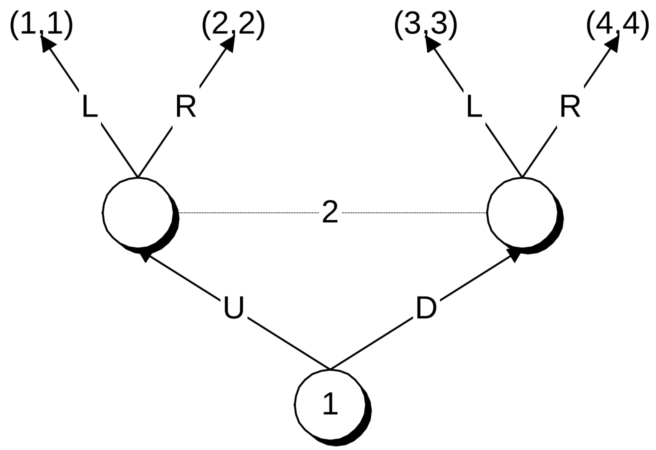
a *pure strategy* is a map from information sets to feasible actions $s_i(h_i) \in A(h_i)$

a *behavior strategy* is a map from information sets to probability distributions over feasible actions $\pi_i(h_i) \in P(A(h_i))$

Nature's move is a behavior strategy for Nature and is a fixed part of the description of the game

We may now define $u_i(\pi)$

normal form are the payoffs $u_i(s)$ derived from the game tree

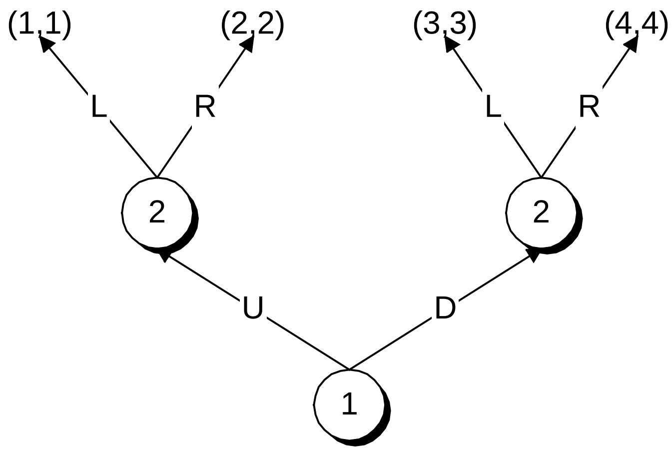


	L	R
U	1,1	2,2
D	3,3	4,4

Kuhn's Theorem:

every mixed strategy gives rise to a unique behavior strategy

The converse is NOT true



1 plays .5 U

behavior: 2 plays .5L at U; .5L at R

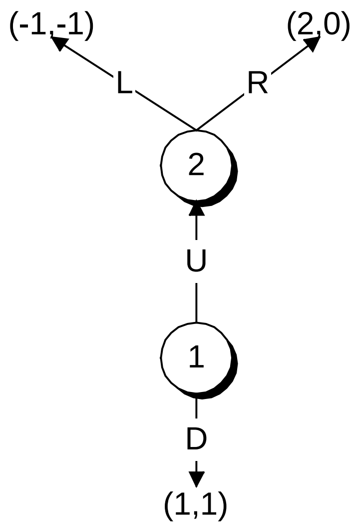
mixed: 2 plays .5(LL),.5(RR)

2 plays .25(LL),.25(RL),.25(LR),.25(RR)

however: if two mixed strategies give rise to the same behavior strategy, they are *equivalent*, that is they yield the same payoff vector for each opponents profile $u(\sigma_i, s_{-i}) = u(\sigma'_i, s_{-i})$

Trembling Hand Perfection

Selten Game



	L	R
U	-1,-1	2,0
D	1,1	1,1

subgame perfect

equilibria:

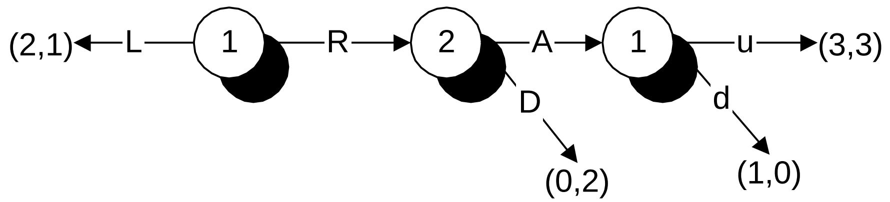
UR is subgame perfect

D and .5 or more L is Nash but not subgame perfect

can also solve by weak dominance

or by trembling hand perfection

Example of Trembling Hand not Subgame Perfect



	A	D	
Lu=Ld	2,1	2,1	(n-2)/n
Ru	3,3	0,2	1/n
Fd	1,0	0,2	1/n
	1/n	(n-1)/2	

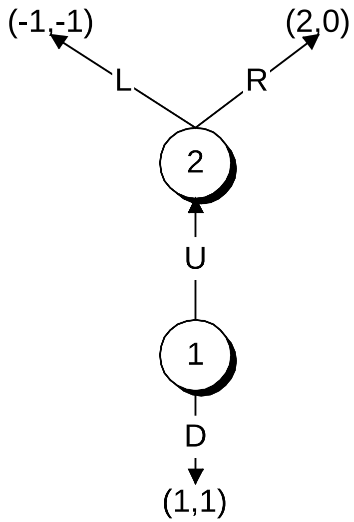
Here Ld,D is trembling hand perfect but not subgame perfect *definition of the agent normal form*

each information set is treated as a different player, e.g. 1a, 1b if player 1 has two information sets; players 1a and 1b have the same payoffs as player 1

extensive form trembling hand perfection is trembling hand perfection in the agent normal form

what is sequentiality??

Robustness – The Selten Game



genericity in normal form

	L	R
U	-1,-1	2**,0**
D	1**,1*($\pm\varepsilon$)	1,1

Self Confirming Equilibrium

$s_i \in S_i$ pure strategies for *i;* $\sigma_i \in \Sigma_i$ mixed

H_i information sets for *i*

$\bar{H}(\sigma)$ reached with positive probability under σ

$\pi_i \in \Pi_i$ behavior strategies

$\hat{\pi}(h_i | \sigma_i)$ map from mixed to behavior strategies

$\hat{\rho}(\pi)$, $\hat{\rho}(\sigma) \equiv \hat{\rho}(\hat{\pi}(\sigma))$ distribution over terminal nodes

μ_i a probability measure on Π_{-i}

$u_i(s_i|\mu_i)$ preferences

$$\Pi_{-i}(\sigma_{-i}|J) \equiv \{\pi_{-i}|\pi_i(h_i) = \hat{\pi}(h_i|\sigma_i), \forall h_i \in H_{-i} \cap J\}$$

Notions of Equilibrium

Nash equilibrium

a mixed profile σ such that for each $s_i \in \mathrm{supp}(\sigma_i)$ there exist beliefs μ_i such that

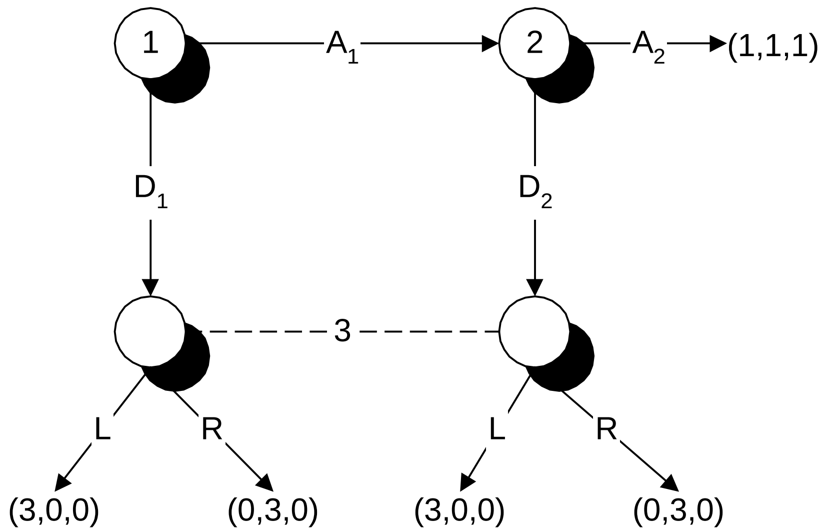
- s_i maximizes $u_i(\cdot|\mu_i)$
- $\mu_i(\Pi_{-i}(\sigma_{-i}|H)) = 1$

Unitary Self-Confirming Equilibrium

- $\mu_i(\Pi_{-i}(\sigma_{-i}|\overline{H}(\sigma))) = 1$

(=Nash with two players)

Fudenberg-Kreps Example



A_1, A_2 is self-confirming, but not Nash

any strategy for 3 makes it optimal for either 1 or 2 to play down

but in self-confirming, 1 can believe 3 plays R; 2 that he plays L

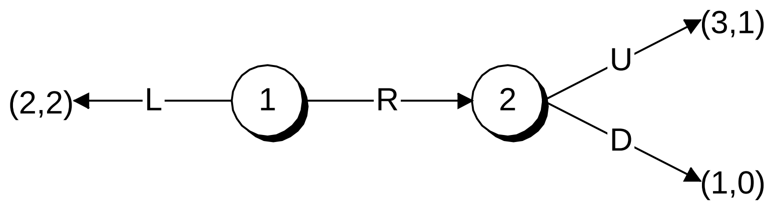
Heterogeneous Self-Confirming equilibrium

- $\mu_i(\Pi_{-i}(\sigma_{-i}|\overline{H}(s_i,\sigma))) = 1$

Can summarize by means of “observation function”

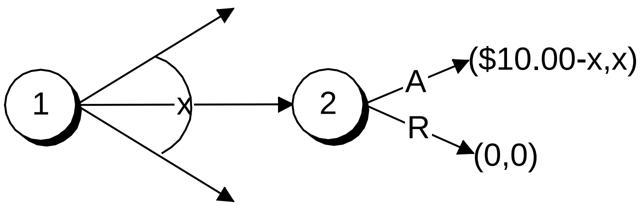
$$J(s_i,\sigma) = H, \overline{H}(\sigma), \overline{H}(s_i,\sigma)$$

Public Randomization



Remark: In games with perfect information, the set of heterogeneous self-confirming equilibrium payoffs (and the probability distributions over outcomes) are convex

Ultimatum Bargaining Results



Raw US Data for Ultimatum

x	Offers	Rejection Probability
$2.00	1	100%
$3.25	2	50%
$4.00	7	14%
$4.25	1	0%
$4.50	2	100%
$4.75	1	0%
$5.00	13	0%
	27	

US $10.00 stake games, round 10

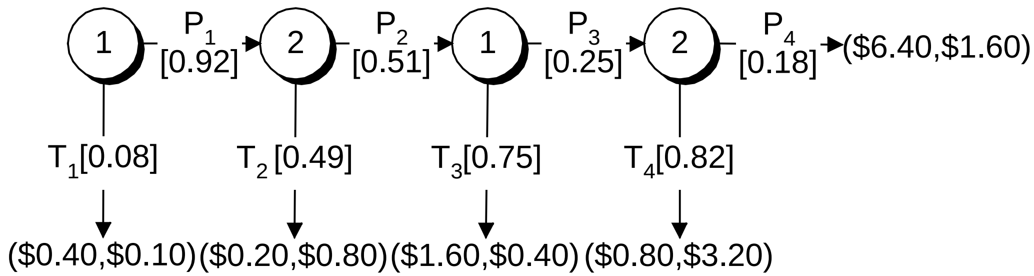
Trials	Rnd	Cntry Stake	Case	Expected Loss			Max Gain	Ratio
				Pl 1	Pl 2	Both		
27	10	US	H	$0.00	$0.67	$0.34	$10.00	3.4%
27	10	US	U	$1.30	$0.67	$0.99	$10.00	9.9%
10	10	USx3	H	$0.00	$1.28	$0.64	$30.00	2.1%
10	10	USx3	U	$6.45	$1.28	$3.86	$30.00	12.9%
30	10	Yugo	H	$0.00	$0.99	$0.50	$10?	5.0%
30	10	Yugo	U	$1.57	$0.99	$1.28	$10?	12.8%
29	10	Jpn	H	$0.00	$0.53	$0.27	$10?	2.7%
29	10	Jpn	U	$1.85	$0.53	$1.19	$10?	11.9%
30	10	Isrl	H	$0.00	$0.38	$0.19	$10?	1.9%
30	10	Isrl	U	$3.16	$0.38	$1.77	$10?	17.7%
	WC		H			$5.00	$10.00	50.0%

Rnds=Rounds, WC=Worst Case, H=Heterogeneous, U=Unitary

Comments on Ultimatum

- every offer by player 1 is a best response to beliefs that all other offers will be rejected so player 1's heterogeneous losses are always zero.
- big player 1 losses in the unitary case
- player 2 losses all knowing losses from rejected offers; magnitudes indicate that subgame perfection does quite badly
- as in centipede, tripling the stakes increases the size of losses a bit less than proportionally (losses roughly double).

Centipede Game: Palfrey and McKelvey



Numbers in square brackets correspond to the observed conditional probabilities of play corresponding to rounds 6-10, stakes 1x below.

This game has a unique self-confirming equilibrium; in it player 1 with probability 1 plays T_1

Summary of Experimental Results

Trials / Rnd	Rnds	Stake	Case	Expected Loss			Max Gain	Ratio
				Pl 1	Pl 2	Both		
29*	6-10	1x	H	$0.00	$0.03	$0.02	$4.00	0.4%
29*	6-10	1x	U	$0.26	$0.17	$0.22	$4.00	5.4%
	WC	1x	H			$0.80	$4.00	20.0%
29	1-10	1x	H	$0.00	$0.08	$0.04	$4.00	1.0%
10	1-10	4x	H	$0.00	$0.28	$0.14	$16.00	0.9%

Rnds=Rounds, WC=Worst Case, H=Heterogeneous, U=Unitary

*The data on which from which this case is computed is reported above.

Comments on Experimental Results

- heterogeneous loss per player is small; because payoffs are doubling in each stage, equilibrium is very sensitive to a small number of player 2's giving money away at the end of the game.
- unknowing losses far greater than knowing losses
- quadrupling the stakes very nearly causes $\bar{\varepsilon}$ to quadruple
- theory has substantial predictive power: see WC
- losses conditional on reaching the final stage are quite large--inconsistent with subgame perfection. McKelvey and Palfrey estimated an incomplete information model where some "types" of player 2 liked to pass in the final stage. This cannot explain many players dropping out early so their estimated model fits poorly.