1. Principal Agent

a)

	0	1
Pay	0,0	1,1
Not Pay	0,0 [unique pure Nash]	5,-1

b) 1 can mix up to 50% on pay, 2 chooses 0
c) pay is weakly dominated for 1
d) pure is (pay,1); mixed is (50-50,1)
e) minmax for player 1 is 0; minmax for player 2 is 0
f) worst equilibrium is 0; best equilibrium is 1 provided

$$(1-\delta)4-\delta 1 \le 0$$

$$4/5 \le \delta$$

g) many answers: most obvious – play pay, 1 unless there has been a deviation, then switch to 0
h) convex hull of 0,0; 1,1; 5,-1 above the point 0,0

2. Auto Repair

a) normal form

	Repair	Not
Repair	$p-c$, $\theta v - p$	0, πv
Not	p, $\pi v - p$	0, πv [unique pure Nash]

b) some mixed Nash but all yield payoff 0; minmax for 1 is 0; pure precommitment is repair $p-c$; mixed precommitment mix so that 2 is indifferent

$$\alpha\theta v + (1-\alpha)\pi v = \pi v + p$$

$$\alpha = \frac{p}{(\theta-\pi)v}$$

so payoff is $p-\alpha c$.
c) worst is obviously 0
d)

$$v = (1-\delta)(p-c) + \delta(\theta v + (1-\theta)w(n))$$

$$v = (1-\delta)p + \delta(\pi v + (1-\pi)w(n))$$

$$v = (1-\delta)(p-c) + \delta(\theta v + (1-\theta)w(n))$$

$$\frac{(1-\delta\pi)}{(1-\pi)}v - \frac{(1-\delta)}{(1-\pi)}p = \delta w(n)$$

$$v = (1-\delta)(p-c) + \delta\theta v + (1-\theta)\left[\frac{(1-\delta\pi)}{(1-\pi)}v - \frac{(1-\delta)}{(1-\pi)}p\right]$$

$$v\left[1 - \frac{1-\theta+\delta\theta-\delta\pi}{(1-\pi)}\right] = (1-\delta)(p-c) - (1-\theta)\frac{(1-\delta)}{(1-\pi)}p$$

$$v(\theta-\pi) = (1-\pi)(p-c) - (1-\theta)p = (\theta-\pi)p - (1-\pi)c$$

$$v = p - \frac{1-\pi}{\theta-\pi}c$$

$$\begin{aligned} w(n) &= \frac{(1-\delta\pi)}{\delta(1-\pi)}v - \frac{(1-\delta)}{\delta(1-\pi)}p \\ &= \frac{(1-\delta\pi)}{\delta(1-\pi)}\left[p - \frac{1-\pi}{\theta-\pi}c\right] - \frac{(1-\delta)}{\delta(1-\pi)}p \\ &= p - \frac{(1-\delta\pi)}{\delta(\theta-\pi)}c \geq 0 \end{aligned}$$

$$\delta \geq \frac{c}{p(\theta-\pi)+\pi c} < 1$$

$$(\theta-\pi)p - c > -\pi c$$

3. Auction

a) dominant strategy to announce truthfully; ¾ chance one person has low value and get revenue $8; ¼ chance both have high value and get revenue $12, so expected revenue is 9.
b) this is a 2x2 symmetric game; strategy is what to bid when type is $12

calculate payoff to $12 type only, since only that type has a choice

both bid $12
½ opponent bids 12 and is 12: you win ½ and get 0; lose ½ and get 6, expected value 3
½ opponent bids 8 and is 8; you win always and get 4, expected value 4

you bid 8, he bids 12
½ opponent bids 12 and is 12; you lose and get 6; expected value 6
½ opponent bids 8 and is 8; you win ½ and get 4; lose ½ and get 4; expected value 4

you bid 12 he bids 8
you always win and get 4

both bid $8
¼ chance win versus 12, getting 4
¼ chance lose versus 12, getting 6
¼ chance win versus 8, getting 4

¼ chance lose versus 8, getting 4

	\$12	\$8
\$12	3.5,3.5	4,5
\$8	5,4	4.25, 4.25

So bidding 8 is dominant strategy equilibrium, and expected revenue to the seller is \$8.

4) Mechanism Design

$$(1/3)[u(H,H;H)+u(L,H;L)+u(H,L;H)]$$
$$+(1/3)[t(H,H)+t(L,H)+t(H,L)]=$$
$$(x-1)/3+(1/3)[t(H,H)+t(L,H)+t(H,L)]\geq$$
$$(1/3)[u(H,H;H)+u(L,H;H)+u(H,L;H)]$$
$$+(1/3)[t(H,H)+t(H,H)+t(H,L)]=$$
$$x/3+(1/3)[2t(H,H)+t(H,L)]$$

$$(x-1)/3+(1/3)[t(H,H)+t(L,H)+t(H,L)]\geq$$
$$x/3+(1/3)[2t(H,H)+t(H,L)]$$

$$t(L,H)-t(H,H)\geq 1$$

$t(H,H)=0 \Rightarrow t(L,H)=1$ and then choosing $t(H,L)=-1$ give the other constraint these constraints can be interpreted as “budget balance” when there are two players.