Econ 504
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Econ 504
First Midterm

1. *Chain Store Paradox*

   Consider a repeated entry game between a long-lived player and a sequence of n short-lived players, each of whom plays once. In period t, the long-lived player and short-lived player t simultaneously choose how much output to produce (long-lived) and whether or not to enter the market (short-lived). The long-lived player may produce *low* output or *high* output. If there is no entry, the short-lived player gets nothing and the long-lived player gets 6 for *low* output and 5 for *high* output. If there is entry and the long-lived player produces *low* output, they both get 1. If there is entry and the long-lived player produces *high* output they both get -1. The long-lived player's payoff is the average payoff from all n games that he plays.

   (a) If the game is played once (a single short-lived player; $n = 1$), show the normal form.

	out	in
lo	6,0	1,1
hi	5,0	-1,-1

   (3 points)

   (b) What is the unique subgame perfect equilibrium when there are n short-lived players?

   In the final period the unique Nash equilibrium is (lo,in), so in the next to last period it must again be (lo,in), and so forth (3 points)

   (c) Consider the following strategy profile. The long-lived player begins by playing *high* and continues playing *high* as long as he himself has always played *high* and the short-lived players have always played *do not enter*; otherwise, the long-lived player plays *low*. Short-lived player $t = 1$ plays *do not enter*. Short-lived player $t > 1$ plays *do not enter* provided every previous short-lived player has played *do not enter* and the long-lived player has always played *high*. Otherwise, the short-lived player plays *enter*. What is the smallest value of ϵ for which this an ϵ-equilibrium?

   The only suboptimal play is for the long-lived player to choose high output in the last period. This costs 1. There are t periods, so the average cost per period is $1/t$, which is the smallest ϵ for which this is an ϵ-equilibrium. (4 points)

2. *Grab-a-Dollar*

Consider the three move game of “grab-a-dollar.” Player one may either take or pass. If he takes, he gets 1, player two gets 0. If he passes, player two gets to move, and may either take or pass. If he takes, he gets 2 and player one gets 0. If he passes, player one gets a final move. If player one takes he gets 4 and player 2 nothing; if he passes player two gets 8 and player 1 gets nothing.

(a) Write down the extensive and normal form of this game. (2 points)

```
1–P–>2–P–>1–P–>0,8
|    |    |
T    T    T
|    |    |
1,0  0,2  4,0
```

REDUCED normal form

	T	P
T at 1	1,0	1,0
T at 3	0,2	4,0
Pass	0,2	0,8

(b) Find the subgame perfect equilibrium of this game. (2 points)

T, T, T - that is optimal to take in the last round; so optimal to take in the next to last round, etc.

(c) Find a Nash equilibrium of this game in which one player strictly mixes. (3 points)

Player 1 must Take at 1 in any Nash equilibrium. Pass is strictly dominated, so never played. If 1 plays T at 3 with any positive probability, then 2 must play T, meaning the 1 must play T at 1. So indeed, Player 1 must Take at 1. But player 2 can mix between T and P provided that π the probability of T is not so small that player 1 want to T at 3. So $(1-\pi)4 \leq 1$ or $\pi \geq 3/4$

(d) Prove that in every self-confirming equilibrium of this game, with probability 1 player one grabs immediately. (3 points)

As above: If play reaches the third node with positive probability, Player 1 must Take, and Player 2 sees that, so must Take. If player reaches the second node with positive probability, Player 2 must Take, and Player 1 must know that. So Player 1 has to Take.

3. State whether the following are true or false. If false, provide a counter example (the simpler the better). If true, provide a proof.

(a) If a strategy is not a best response to any opposing *pure* strategy profile, then it is strictly dominated.

No:

	L	R
U	10,0	0,0
M	0,0	10,0
D	6,0	6,0

D is a not a best response to any pure strategy but it is not strictly dominated. This can be shown by direct calculation but note also that B is a best response to 50:50 L/R. (3 points)

(b) If a Nash equilibrium is trembling hand perfect, then it does not put positive probability on any weakly dominated strategy. (4 points)

True: a weakly dominated strategy will no longer be a best response when the other player strictly trembles.

(c) If a Nash equilibrium is subgame perfect, then it does not put positive probability on any weakly dominated strategy.

False: in a simultaneous move game Nash and subgame perfect are the same thing; so a Nash equilibrium in a game that plays a weakly dominated strategy is also subgame perfect. (3 points)

4. *Gridlock.*

A two-player game of perfect information is played on a two-dimensional grid. There are n rows and m columns. The players alternate choosing a grid point (row and column). Each choice blocks all grid points above (or equal) and to the left (or equal). For example, if you play row 2, column 2 (2,2), nobody afterwards can choose (2,2), (1,2), (2,1), or (1,1). The player who makes the last move loses and gets a payoff of -1, while the other player gets +1. Prove that player one has a winning strategy. *Hint*: whatever you do, don't try to compute the winning strategy.

(10 points) The game does not end in a draw, so one player has a winning strategy. If it is player 2, then for any move player 1 makes, 2 has a winning countermove. In particular, if player 1 movies top left, 2 had a winning countermove - that necessarily covers top left. So Player 1 could have made 2's move and won the game, proving that 1 and not 2 must be the winner.