1a. Efficiency requires the object go to the high value person; incentive compatibility leads to payments that are independent of the players own announcement - it is easy enough to work out that they are the other players value, hence this is equivalent to a second price auction. (10 pts)

b. The key is to observe that in the first price auction both types have to have the same upper support of their bidding functions. If not, the player with higher support could lower his bid slightly while still having probability one of winning. So this means that a first bidder with value a bit less than 2 will lose to a second bidder with value 1, so the auction is ex post in efficient - it may not go to the highest value individual. (15 points)

c. revenue equivalence is not going to hold here with ex post inefficiency in the first but not second price auction and the probability winning the object different in the first and second price auction. (5 points)

Question 2

player 1	player 2	%
0	4	60
4	0	20
0	2	20

If player 1 contributes 0, player 2's contribution determines the level x of public good. Player 2 chooses his contribution according to:

$$\max_{x \in \{0,1,\ldots,8\}} 205x - 5x^2 - 164x$$

So, when player 1 contributes 0, player 2's best response is to contribute 4. Payoffs to each player are $(740, 84)$. Note that 740 is the highest payoff that player 1 can get given player 2's best response. So this is the unique subgame perfect equilibrium, which was played in 60% of the matches. (10 points)

If player 2 threatens player 1 to contribute 0 regardless of player 1's action, then player 1 will solve the maximization problem above and contribute 4. Payoffs to each player are $(84, 740)$. This is a Nash equilibrium, but it's not subgame perfect. (10 points)

The third case is clearly not a Nash equilibrium. Given that player 1 contributes 0, player 2 could be better off if he contributed 4 instead of 2. Payoffs to each player are $(390, 62)$

Heterogeneous self confirming equilibrium losses:

	player 1	player 2
$(0, 4)$	0	0
$(4, 0)$	0	0
$(0, 2)$	0	22

In each of the first two cases, both players are optimizing with respect to their beliefs about their opponents' play, so known losses are zero. In the third case, player 2 is not best responding to player 1's action. By contributing 2 instead of 4, player 2 is losing 22. (10 points)

Question 3

Let player 1 be the hunter and player 2 the gatherer.

	$H2$	$G2$
$H1$	$0,0$	$2,2$
$G1$	$1,1$	$0,0$

This game has two strict Nash equilibria, $(H1, G2)$ and $(G1, H2)$, and one mixed, $((\frac{1}{3}H1, \frac{2}{3}G1), (\frac{2}{3}H2, \frac{1}{3}G2))$

Consider all probability distributions over outcomes:

	$H2$	$G2$
$H1$	a	b
$G1$	c	d

where $a + b + c + d = 1$, $a, b, c, d \geq 0$, such that:

player 1 has no incentives to deviate from the recommendation:

$$\frac{a}{a+b}(0) + \frac{b}{a+b}(2) \geq \frac{a}{a+b}(1) + \frac{b}{a+b}(0) \Rightarrow a \leq 2b \qquad (i)$$
$$\frac{c}{c+d}(1) + \frac{d}{c+d}(0) \geq \frac{c}{c+d}(0) + \frac{d}{c+d}(2) \Rightarrow 2d \leq c \qquad (ii)$$

and player 2 has no incentives to deviate from the recommendation:

$$\frac{a}{a+c}(0) + \frac{c}{a+c}(1) \geq \frac{a}{a+c}(2) + \frac{c}{a+c}(0) \Rightarrow 2a \leq c \qquad (iii)$$
$$\frac{b}{b+d}(2) + \frac{d}{b+d}(0) \geq \frac{b}{b+d}(0) + \frac{d}{b+d}(1) \Rightarrow d \leq 2b \qquad (iv)$$

The set of correlated equilibria is given by all probability distributions over outcomes that satisfy $(i), (ii), (iii)$ and (iv). (20 points)

Correlated equilibria that are also Nash equilibria:
$b = 1$ gives $(H1, G2)$.
This is also the only Pareto efficient correlated equilibrium.
$c = 1$ gives $(H2, G1)$
$a = \frac{2}{9}, b = \frac{1}{9}, c = \frac{4}{9}, d = \frac{2}{9}$ gives $((\frac{1}{3}H1, \frac{2}{3}G1), (\frac{2}{3}H2, \frac{1}{3}G2))$ (10 points)