Signalling And Screening

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Johannes Hörner

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Signalling refers to any activity by a party whose purpose is to influence the perception and thereby the actions of other parties. This presupposes that one market participant holds private information that for some reason cannot be verifiably disclosed, and which affects the other participants' incentives. Excellent surveys include Kreps and Sobel (1994) and Riley (2001). The classic example of market signalling is due to Spence (1973, 1974). Consider a labor market in which firms know less than workers about their innate productivity. Under certain conditions, some workers may wish to signal their ability to potential employers, and do so by choosing a level of education that distinguish themselves from workers with lower productivity.

1. Mathematical Formulation: We consider here the simplest game-theoretic version of Spence's model. A worker's productivity, or *type*, is either θ_H or θ_L, with $\theta_H > \theta_L > 0$. Productivity is private information. Firms share a common prior p, with $p = \Pr\{\theta = \theta_H\} \in (0,1)$. Before entering the job market, the worker chooses a costly education level $e \geq 0$. (The reader is referred to Krishna and Morgan, this volume, for the case of costless signals.) Workers maximize $U(w, e; \theta) = w - c(e; \theta)$, where w is their wage and $c(e; \theta)$ is the cost of education. Assume that $c(0; \theta) = 0$, $c_e(e; \theta) > 0$, $c_{ee}(e; \theta) > 0$, $c_\theta(e; \theta) < 0$ for all $e > 0$, where $c_e(\cdot; \theta)$ and $c_\theta(e; \cdot)$ denote the derivatives of the cost with respect to education and types, respectively, and

$c_{ee}(\cdot;\theta)$ is the second derivative of cost with respect to education. The key assumption made in the literature is that on the cross-derivative: $c_{e\theta}(e;\theta) < 0$. That is, the marginal cost is lower for a high-productivity worker. This *single-crossing* condition ensures that the indifference curves of a high and a low-productivity worker cross at most once, with the indifference curve of the high-productivity worker having a smaller slope where they do. See Figure 1.

To focus on signalling, assume that education does not affect productivity. If a firm assigns probability $p(e)$ to the high-productivity worker conditional on education e, the worker's expected productivity is $(1-p(e))\,\theta_L + p(e)\,\theta_H$. If the worker accepts wage w, the firm's profit is then $(1-p(e))\,\theta_L + p(e)\,\theta_H - w$.

The Basic Signalling Game: There is one worker and two firms. In the first stage, the worker chooses education. Education is observable. In the second stage, firms compete through wages in Bertrand-competition fashion. Following usual arguments, the wage $w(e)$ offered and accepted equals the expected productivity of the worker, given the observed education. A (Perfect Bayesian) Equilibrium specifies an education function $e^* : \{\theta_L, \theta_H\} \to \mathbb{R}_+$, and a belief function $p^* : \mathbb{R}_+ \to [0,1]$, giving respectively the education chosen by each type, and the probability assigned to a high-productivity worker conditional on each possible education level, so that the worker's choice is optimal given the wage determined by p^*, and the belief function is derived from this choice using Bayes' rule whenever possible.

Either the worker chooses distinct education levels depending on his productivity, or he does not. An equilibrium is *separating* if $e^*(\theta_L) \neq e^*(\theta_H)$, and *pooling* otherwise. In the first case, education perfectly reveals productivity, so that the worker's wage equals his productivity: $w(e^*(\theta_i)) = \theta_i$, for $i = L, H$. In the second case, education reveals no information, and the wage is equal to his expected productivity given the prior p: $w(e^*(\theta_i)) = (1-p)\,\theta_L + p\theta_H =: E(\theta)$, for $i = L, H$.

Observe that, in any separating equilibrium, the low-productivity worker gets the lowest possible wage. Since education is costly, this implies that he chooses no education: $e^*(\theta_L) = 0$.

The high-productivity worker, on the other hand, gets the highest possible wage. Therefore, the corresponding education level $e^*(\theta_H)$ must be high enough to deter the low-productivity worker from pretending he has high productivity. That is, it must be that $e^*(\theta_H) \geq e'$, where e' solves $\theta_L - c(0;\theta_L) = \theta_H - c(e';\theta_L)$. At the same time, the education level $e^*(\theta_H)$ cannot be too high, since the high-productivity worker must choose it. That it, it must be that $e^*(\theta_H) \leq e''$, where e'' solves $\theta_L - c(0;\theta_H) = \theta_H - c(e'';\theta_H)$. Single-crossing implies that the interval $[e', e'']$ is non-empty. Indeed, since the indifference curves $\{(e,w) : U(w,e;\theta_H) = U(\theta_L, 0;\theta_H)\}$ and $\{(e,w) : U(w,e;\theta_L) = U(\theta_L, 0;\theta_L)\}$ cross at $(0,\theta_L)$, the point (e'',θ_H) which lies along the first one must be to the right of the point (e',θ_H) which lies along the second. See Figure 2. Because a high-productivity worker is more willing to trade off an increase in education to induce an increase in wage, it is possible to find a suitable education level that is worth acquiring if and only if the worker's productivity is high.

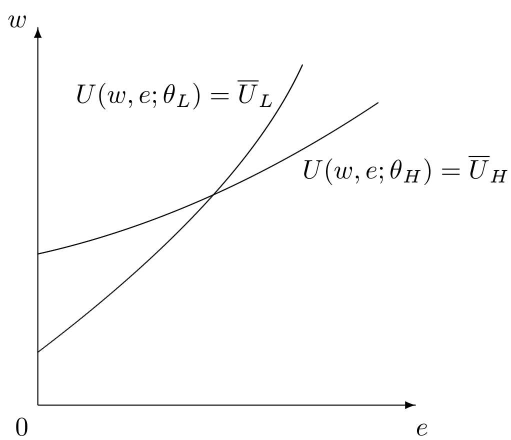

Figure 1

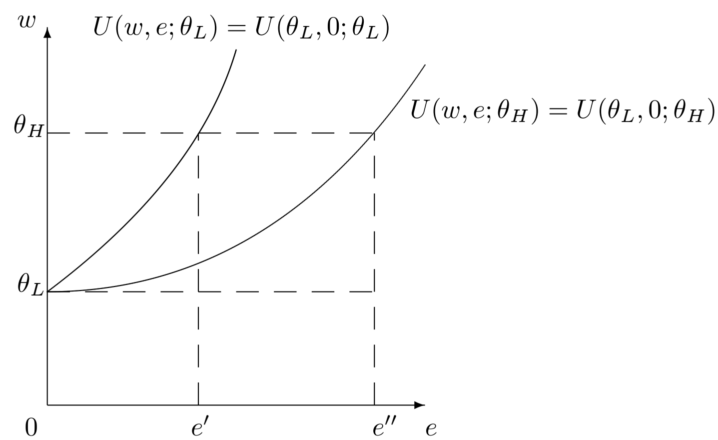

Figure 2

Each education level $e^* \in [e', e'']$ can be supported in equilibrium, for suitable beliefs. For instance, by setting $p^*(e) = 0$ if $e < e^*$, and $p^*(e) = 1$ otherwise, a high-productivity worker optimally chooses $e^*(\theta_H) = e^*$. Observe that these equilibria are Pareto-ranked. The best equilibrium outcome, involving $e^* = e'$, is known as the *Riley outcome* (Riley, 1979).

Compared to the case in which signalling is not available, the low-productivity worker is worse off. Without signalling, the worker would not acquire any education, independently of his type and he would receive the wage $E(\theta)$. Here instead, the low-productivity worker earns only θ_L. Surprisingly, the high-productivity worker may also be worse off. As no education is interpreted as evidence of low productivity, the outcome without signalling is not available to him any longer. In a separating equilibrium, his utility is at most $\theta_H - c(e'; \theta_H)$. Without signalling opportunities, his utility is $E(\theta) - c(0; \theta_H)$. While $E(\theta)$ tends to θ_H as p tends to one, e' is independent of p. Therefore, if p is large enough, the high-productivity is worse off. See Figure 3.

There is also a continuum of pooling outcomes. Let $\hat{e}$ solve $U(\theta_L, 0; \theta_L) = U(E(\theta), \hat{e}; \theta_L)$. Every level of education $e^* \in [0, \hat{e}]$ can be supported by the beliefs $p^*(e) = 0$ if $e < e^*$, $p^*(e) = p$ otherwise. Since education is costly, we need only check that the worker prefers $e(\theta_i) = e^*$ $(i = L, H)$ to no education at all. This is true for the low-productivity worker by definition of $\hat{e}$, and follows from single-crossing for the high-productivity worker. Here as well, the outcomes are Pareto-ranked, with the best equilibrium outcome, sometimes referred to as the *Hellwig outcome*, specifying $e^* = 0$ (Hellwig, 1986). In addition to these separating and pooling outcomes, there also exists a continuum of equilibria in mixed strategies.

The Basic Screening Game: While it is standard in the literature to call signalling models those in which the informed party moves first, they are closely related to screening models, in which the uninformed parties take the lead. Classic references include Rothschild and Stiglitz (1976) and Wilson (1977) in the context of insurance markets. In these models, the two firms simultaneously announce a menu of pairs (e, w). Given these contracts, the worker chooses which contract to accept, if any. We sketch here the main results of this model. An equilibrium is *separating* if the worker accepts distinct contracts depending on his type, and *pooling* otherwise.

Observe that, in equilibrium, firms must just break even. Otherwise, if the worker of type $i = L, H$ accepts contract (e_i, w_i), a contract $(e_i, w_i + \varepsilon)$ for small $\varepsilon > 0$ would attract both types of worker, and the firm earning less than half the aggregate profits would gain by offering it.

Also, there can be no pooling equilibrium. Because a pooling contract (e^*, w^*) would have to break even, a firm whose rival offered this contract would gain by offering a contract (e, w) specifying a higher wage and education level, accepted only by the high-productivity worker. See Figure 4.

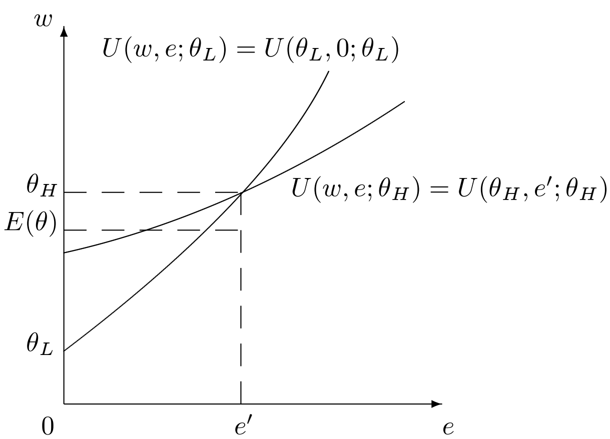

Figure 3

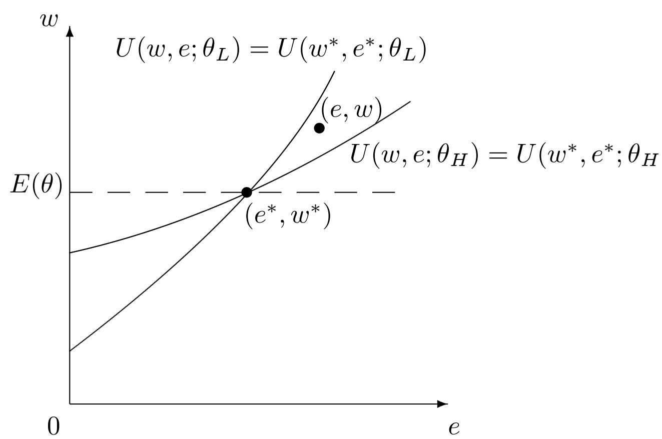

Figure 4

Finally, in any separating equilibrium, wages paid must equal the worker's productivity. In particular, the contract accepted by the low-productivity worker specifies education $e_L = 0$ and wage $w_L = \theta_L$. Indeed, if $(e_L, w_L) \neq (0, \theta_L)$, then the firm whose rival offered (e_L, w_L) would gain by offering a contract with either a slightly lower wage, or a slightly lower education, independently of the worker's type accepting it. Similarly, if the wage accepted by the high-productivity worker fell short of θ_H, a firm whose rival offered the contract accepted by the low-productivity worker would gain by offering a contract specifying a slightly higher wage and education than those specified by the contract accepted by the high-productivity worker. Since $w_i = \theta_i$, $i = L, H$, and $e_L = 0$, it follows that e_H solves $\theta_H - c(e_H; \theta_L) = \theta_L - c(0; \theta_L)$, that is, $e_H = e'$: if instead the low-productivity worker preferred his contract to (e_H, w_H), at least one firm would gain by offering a contract specifying a wage and education just below w_H and e_H.

However, such an equilibrium need not exist for large p. If $E(\theta) - c(0; \theta_H) > \theta_H - c(e'; \theta_H)$, the contract $(0, E(\theta) - \varepsilon)$ for small $\varepsilon > 0$ attracts both types of workers and makes profits. Thus, if the cost of sorting outweighs the gain, no equilibrium exists. As emphasized by Riley (2001), existence requires a strengthening of single-crossing, as marginal cost must be sufficiently lower for a high-productivity worker, given p. This is the same condition as earlier, under which the high-productivity worker prefers signalling to be unavailable. While equilibria in mixed strategies exist (Dasgupta and Maskin, 1986), they have not been characterized.

2. Extensions and Refinements. To a large extent, the theoretical literature on signalling has focused on selecting among the equilibria, while the literature on screening has addressed the nonexistence issue.

The early literature takes the view that the screening model ignores the dynamic adjustments between firms. To account for these, Wilson (1977) and Riley (1979) define equilibria differently. A set of contracts is a Wilson equilibrium if no firm has a profitable deviation that remains profitable once existing contracts that lose money after the deviation are withdrawn, while it is a Riley, or reactive equilibrium, if no firm has a profitable deviation that remains profitable once new contracts that make money after the deviation are added. Under either definition, equilibria exist. Wilson equilibria involve some pooling, while the unique Riley equilibrium is separating. Hellwig (1987) offers a game-theoretic treatment of Wilson, by modelling a second stage in which firms may withdraw any contract offered previously. In the two-type case, the Hellwig outcome can be supported as an equilibrium.

Formal game-theoretic treatments of signalling appear in the 1980s. Many refinements have been applied to and inspired by the basic signalling game, shedding new light on the somewhat ad hoc selection procedures used previously. For a survey on refinements, see Govindan and Wilson, this volume. While sequential equilibrium (Kreps and Wilson, 1982) does not reduce the multiplicity of equilibria, the *Intuitive Criterion* (Cho and Kreps, 1987) selects the Riley outcome in the basic signalling model. This result, as striking as it is, has several limitations. First, uniqueness does not obtain with more types. Second, the Riley outcome is not necessarily persuasive when the probability of the high-productivity worker is nearly one. As long as this probability is less than one, the high-productivity worker acquires education $e' > 0$, independently of p. But if he is known to be of high productivity, we should expect him not to acquire education, as it serves no signalling purpose. Third, the motivation behind the Intuitive Criterion also underlies the more stringent *Perfect Sequential Equilibrium* (Grossman and Perry, 1986). Yet such an equilibrium fails to exist in the situation described earlier, in which the basic screening game has no equilibrium. An alternative is offered by the concept of *Undefeated Equilibrium* (Mailath, Okuno-Fujiwara and Postlewaite, 1993), which selects the Riley outcome when it is also a Perfect Sequential Equilibrium outcome, and the Hellwig outcome otherwise.

In settings with more types, stronger refinements are needed to select the Riley outcome. These include Banks and Sobel's (1987) *Divinity* and *Universal Divinity*, Cho and Kreps' (1987) criterion *D1*, and Kohlberg and Mertens' (1986) *stability* concepts. See also Cho and Sobel (1990). It is worth pointing out that, with a continuum of types and under weak assumptions, the separating outcome is unique in the signalling model (Mailath, 1987), while no equilibrium exists in the screening model (Riley, 2001).

The single-crossing condition has been generalized to multidimensional signals by Engers (1987) for the case of screening and by Cho and Sobel (1990) and Ramey (1996) for the case of signalling. Quinzii and Rochet (1985) considers multidimensional types. Little is known about equilibria when single-crossing fails, as may occur in applications.

Maskin and Tirole (1992) enlarge the set of contracts. In the screening version, firms offer contracts that let the worker choose ex post among a set of pairs (e, w). In the signalling version, the worker offers such a contract to the firm. Under weak assumptions, the set of equilibrium outcomes coincide. In particular, only the Riley outcome obtains when the basic screening model has an equilibrium.

Acquiring education takes time, and there is no reason to expect firms to wait until graduation before drawing inferences. In Nöldeke and van Damme (1990) and Swinkels (1999), firms make offers before workers complete their education. Offers are public in Nöldeke and van Damme, and private in Swinkels. As the time between offers gets small, only the Riley outcome satisfies Kohlberg and Mertens' never a weak best response criterion when offers are public, while only the Hellwig outcome is a sequential equilibrium outcome when offers are private.

Following Spence's early suggestion, Nöldeke and Samuelson (1997) extend the basic signalling model by considering a dynamic model in which agents adjust their beliefs and actions to past market outcomes, and introduce perturbations into the process. The dynamic process admits at most two recurrent sets, closely related to the Riley and Hellwig outcomes. Several known refinements reappear in their characterization.

3. APPLICATION. Signalling has found many applications besides education, insurance and labor. Whenever possible, the reader is referred to surveys.

INDUSTRIAL ORGANIZATION: Signalling helps explain limit (or predatory) pricing. Milgrom and Roberts (1982) shows that low price may signal an incumbent's low cost. In Milgrom and Roberts (1986), advertising is a signal that a firm's experience good is of high quality. Bagwell and Riordan (1991) show that introductory pricing can serve the same purpose. In Gal-Or (1989), warranties signal product durability. See Tirole (1988) for a general survey, and Bagwell (2001) for a survey specific to advertising and pricing.

FINANCE: Myers and Majluf (1984) show that stock issues may signal the firm's value, so that the choice of financing (equity or debt) affects a firm's investment policy when the firm is better informed about returns than investors. In Bhattacharya (1979), dividends signal cash flows, as managers are better informed about those than investors. In Leland and Pyle (1977), the owner's stake signals the firm's underlying quality in an initial public offering, provided the owner is risk-averse. These early applications have been extended in several directions, and their predictions empirically tested. See Allen and Morris (2001).

Political Science: Signalling has been applied to electoral competition. Banks (1990a) shows how campaign platforms signal the candidates' future actions if elected, while Banks (1990b) argues that agenda-setting signals the bureaucrat's private information about the "reversion level" if the proposal is turned down. See Banks (1991) for a survey. Lohman (1993) shows how individuals engage in costly political actions to signal their private information, if politicians are responsive to turnout. Prat (2002) considers a signalling game in which an interest group has private information about candidates, based on which they can offer contributions that are used as campaign advertising.

Social Norms: Bernheim (1994) develops a theory of social conformity based on signalling. Agents are motivated both by private tastes and status. When society censures extreme preferences, consumers with centrist preferences may choose to pool, while extremists refuse to conform. Fang (2001) provides a model of social culture rich enough to endogenously generate the single-crossing condition supporting the separating equilibrium. Austen-Smith and Fryer (2005) study signalling when workers have both a social and an economic type, and education affects both the wage and their perception by their peers. Peer pressure may induce educational underinvestment by accepted types.

Biology: Following Zahavi's (1975) "handicap principle", asserting that animal signals are reliable because they are costly, a large literature on signalling has emerged in biology. Grafen (1990) provides a game-theoretic treatment. See Maynard Smith and Harper (2003) for a survey.

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