Econ 506A (2008)
Topics in Advanced Theory I
GAME THEORY

Haluk Ergin & David Levine

Bargaining with Complete Information

The Rubinstein-Stahl Model

Bargaining Theory

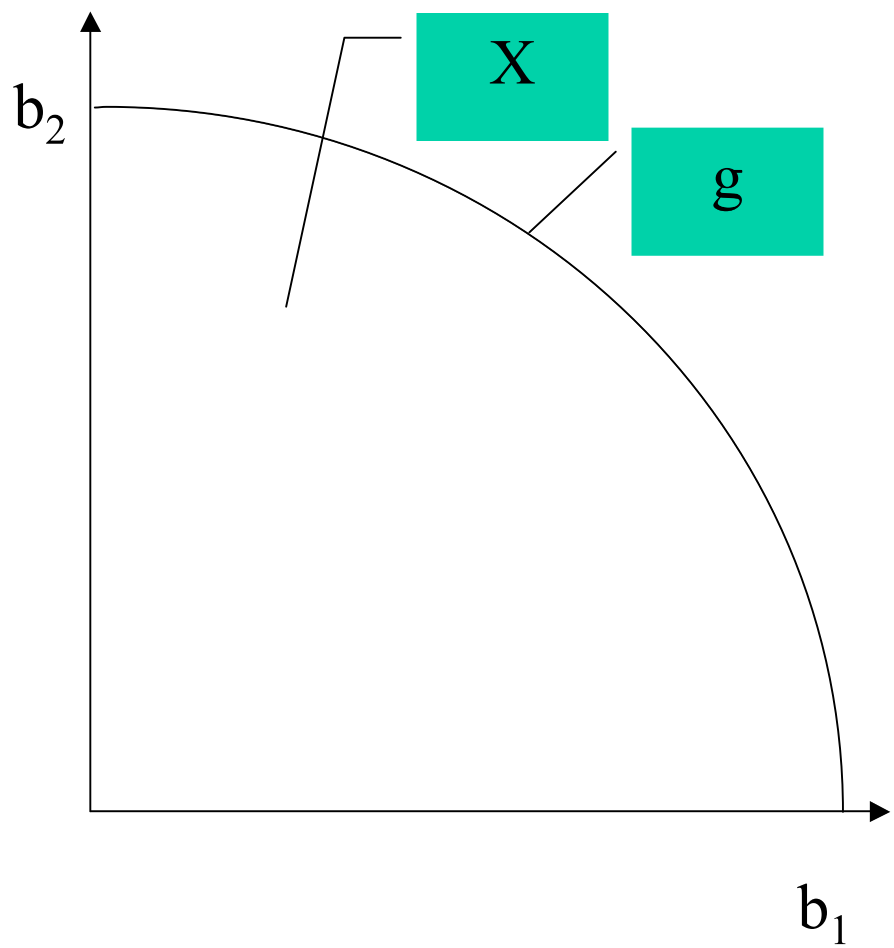
Slides: Courtesy of Paul Milgrom and Muhamet Yildiz

Bargaining Theory

- Cooperative (Axiomatic)
  - Edgeworth
  - Nash Bargaining
  - Variations of Nash
  - Shapley Value

- Non-cooperative
  - **Rubinstein-Stahl (*)** (complete info)
  - Asymmetric info
    - Rubinstein, Admati-Perry, Crampton, Gul, Sonenchein, Wilson; Abreu and Gul
  - Non-common priors
    - Posner, Bazerman, Yildiz

Rubinstein-Stahl Model



- $N = \{1,2\}$
- X = feasible expected-utility pairs ($x,y \in X$)
- $U_i(x,t) = \delta_i^t x_i$
- $D = (0,0) \in X$ disagreement payoffs
- g is concave, continuous, and strictly decreasing

Timeline

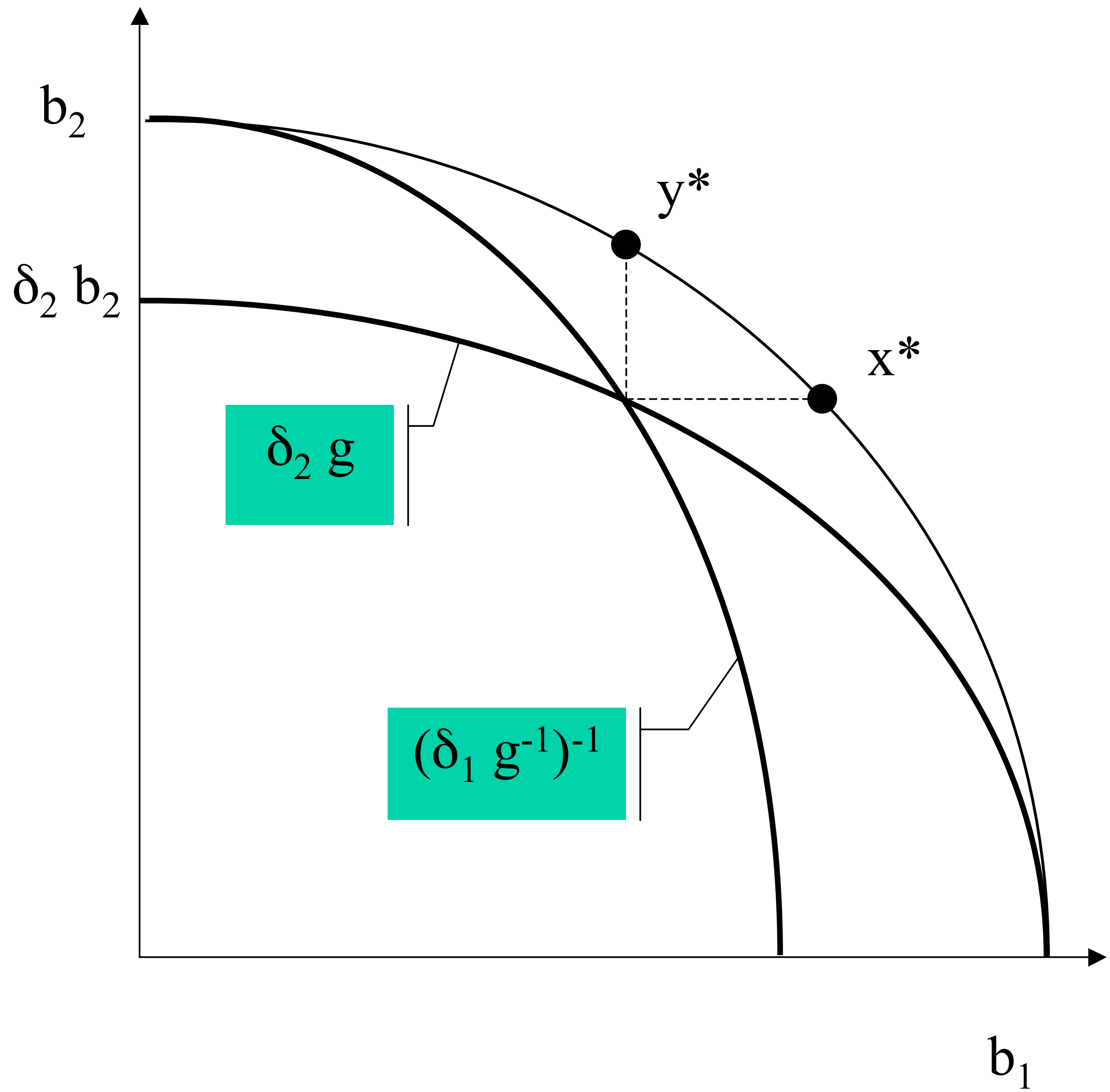
T = {0,1,…, t,…}

At each t, if t is even,

- Player 1 offers some x
- Player 2 Accepts or Rejects the offer
- If the offer is Accepted, the game ends yielding x
- Otherwise, we proceed to t + 1

if t is odd,

- Player 2 offers some y
- Player 1 Accepts or Rejects the offer
- If the offer is Accepted, the game ends yielding y
- Otherwise, we proceed to t + 1



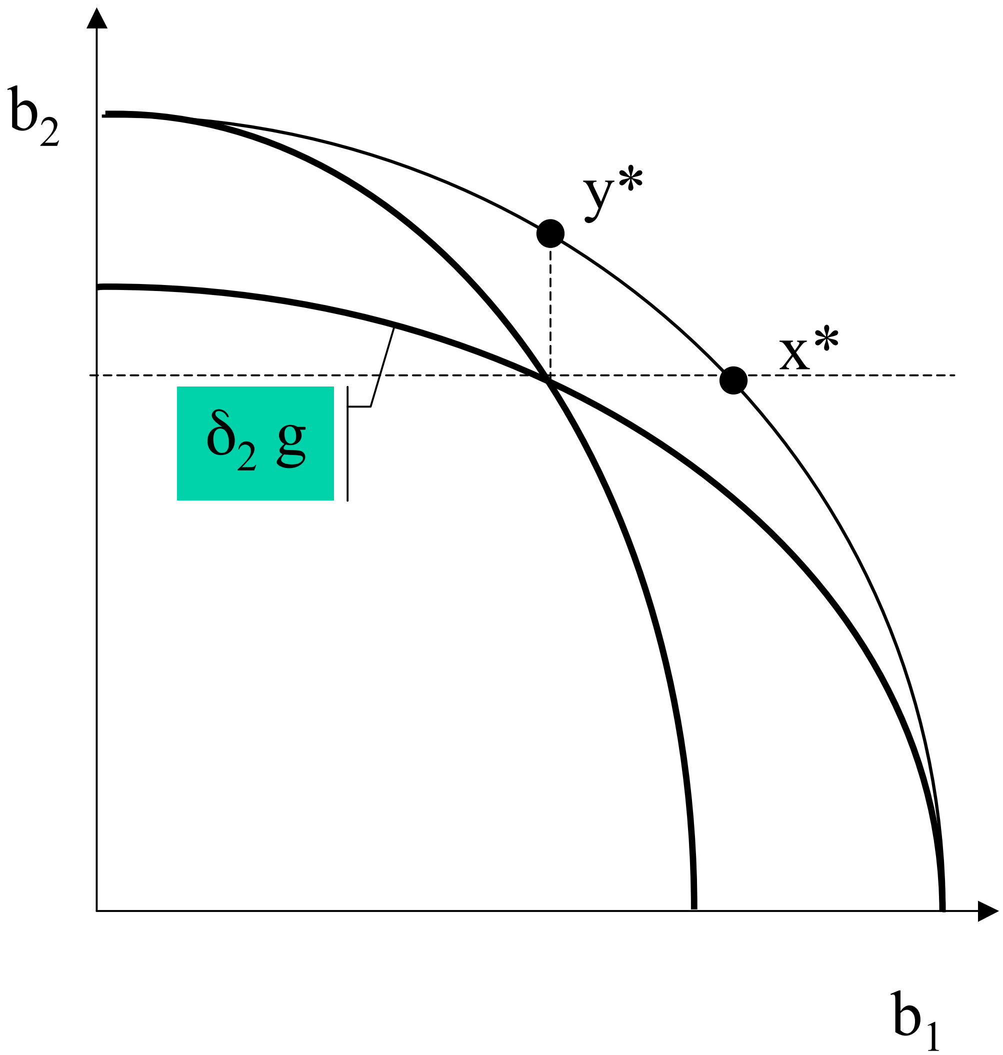
b_2
$\delta_2 b_2$
y^*
x^*
$\delta_2 g$
$(\delta_1 g^{-1})^{-1}$
b_1

Theorem [OR 122.1]

The following is the unique subgame-perfect equilibrium:

- player 1 always offers x*;
- player 2 accepts an offer x iff $x_2 \geq x^*_2$;
- player 2 always offers y*;
- player 1 accepts an offer y iff $y_1 \geq y^*_1$;

Proof (it is a SPE)



Use single deviation principle:

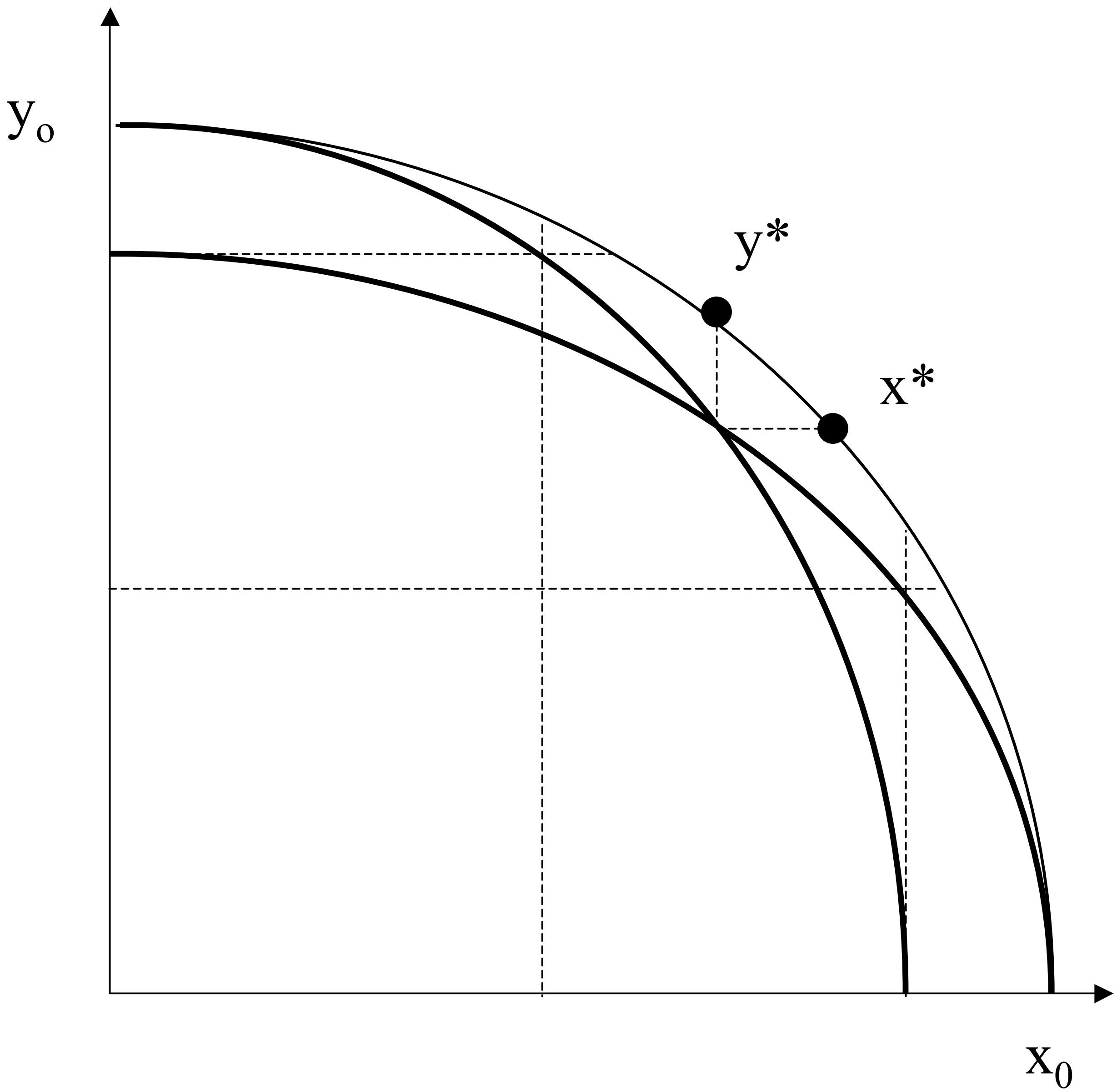
1. If player 2 rejects an offer x at t, she will get y_2* at t+1. Hence, Accept iff $x_2 \geq \delta_2 y^*_2 = x^*_2$ is optimal at t.
2. At t, it is optimal for 1 to offer

$x^* = \text{argmax } \{x_1 | x_2 \geq x^*_2\}$.

“Extensive-form rationalizability” [FT 4.6]

Definition: In a multistage game with observable actions, action a_i^t is conditionally dominated at stage t given history h^{t-1} iff, in the subgame starting at h^{t-1}, every strategy for player i that assigns positive probability to a_i^t is strictly dominated.

Theorem: In any perfect-information game, every subgame-perfect equilibrium survives iterated elimination of conditionally dominated strategies.


y_o
y*
x*
x_0

Comparative statics

- Player *i*'s equilibrium share (both x^*_i and y^*_i) is increasing in his patience δ_i and decreasing in his opponent's patience δ_j.
- As the discount rates converge to 1, x* and y* become arbitrarily close. However limiting equilibrium payoffs are not well defined, since the limit depends on the relative rates of convergence of the discount rates.
- If the contracts that the two players can offer were different, then the limiting payoffs could have been well defined. (Yildiz, 2003)

Example: Split the pie

Two players bargain on division of a pie of size 1.

Feasible set: (z_1,z_2) s.t. $z_1 \geq 0$, $z_2 \geq 0$, $1 \geq z_1+z_2$: $g(z_1)=1-z_1$.

Unique SPE strategies, solve for $z_1= \delta_1(1-z_2)$ and $z_2= \delta_2(1-z_1)$:

$z_1=\delta_1(1-\delta_2)/(1-\delta_1\delta_2)$ and $z_2=\delta_2(1-\delta_1)/(1-\delta_1\delta_2)$

$g(x_1^*)=z_2 \Rightarrow x_1^*=(1-\delta_2)/(1-\delta_1\delta_2)$, $x_2^*=z_2=\delta_2(1-\delta_1)/(1-\delta_1\delta_2)$;

$y_2^*= g(z_1) \Rightarrow y_2^*= (1-\delta_1)/(1-\delta_1\delta_2)$, $y_1^*=z_1=\delta_1(1-\delta_2)/(1-\delta_1\delta_2)$,

If $\delta=\delta_1=\delta_2$ then $x_1^*= y_2^*=1/[1+\delta]$.

Some Complete Information Extensions

- **Different procedures about who makes an offer and when (deterministic or random):**

When discount rates are same and close to one, SPE payoff of player *i* is proportional to the average frequency with which she makes offers.

- **More than two players:**

Uniqueness of SPE is lost with for all known reasonable procedures, unless if one restricts to Markov strategies.