THE POLITICAL ECONOMY OF DEMOCRACY

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There are reasons to think that a fourth wave of democratization is coming. There are now more democracies on earth than ever before. Since 1991, no fewer than 40 governments have undertaken the transition to democracy. All these newly democratizing nations and redemocratizing nations, as well as the efforts to create suprastate constitutions, especially that of the European Union, have made more relevant and necessary than ever to understand legislative procedures and alternative political constitutions. The historical formal split into the distinct studies of political sciences biased the way economists and political scientists approached many questions and placed artificial constraints on the study of many important social issues. Thus, the importance of a unified study of political economy that explores the frontiers of the interaction between politics and economics has become nowadays an unavoidable necessity. The characterization of political economy as a synthesis of fields will provide sparks and an exciting research agenda for enlightening our understanding of democracies.

The workshop on “The Political Economy of Democracy”, held in Barcelona on 5-7 June 2008 under the sponsorship of the BBVA Foundation, brought together intellectual leaders from economics and political science to obtain a balanced understanding of common topics of analysis, such as pre-electoral maneuvering, elections, coalition building and governance, within a single comprehensive framework. Particular attention was devoted to fields of active development such as endogenous entry of candidates, politicians' and voters' behavior, negotiations and agreements, and political regimes.

In the current political economy literature, citizen-candidate models provide a framework to address the issue of endogenous entry of candidates. Since the seminal papers of Osborne and Slivinski (1996) and Besley and Coate (1997), citizen-candidate models have typically suffered from multiple equilibria and thus, they lack clean empirical predictions. Grosser and Palfrey study a citizen-candidate model with private information about the candidates' preferred policies. Introducing private information has the advantage of provid-
Many Enemies, Much Honor? 
On the Competitiveness of Elections 
in Proportional Representation Systems

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4.1. Introduction

About one third of all countries and more than thirty percent of all established democracies use a proportional representation (PR) electoral system. In its purest form, a proportional electoral system maps the share of votes obtained by each party in the election into an equal share of seats in the legislature. Since the seminal work of Duverger (1954), PR has been held responsible (at least partially) for the proliferation of political parties in PR democracies. More recently, the political debate shifted its focus to the relation between the number of legislative parties and the quality of the political environment in terms of competence, or corruption of elected politicians. While the existing literature contains numerous studies supporting Duverger’s hypothesis, the connection between the number of legislative parties and their investment in quality has been overlooked both theoretically and empirically. In this paper, we build on Iaryczower and Mattozzi (2008a) to develop a simple theoretical framework in which the quality and the number of candidates running for office are endogenous equilibrium outcomes.

and provide conditions under which elections in PR would result in a positive association between the quality and number of candidates running for office.

The essential features of the model are the following. Potential candidates are horizontally differentiated according to a policy position they represent. In particular, they are endowed with a policy position they can champion in government if they choose to run for office and get elected. With their given policy positions, candidates who choose to run for office then compete along a bounded vertical dimension, which we represent as costly activities (investment of money, time or effort) that increase voters' perception of the quality of a candidate's platform. Politicians derive utility exclusively from rents they can appropriate while in office. We assume that there is a large finite number of risk averse and fully rational voters.

The mapping of votes' shares into seats' shares is given by the electoral system. In this paper we consider the case of a perfect PR system, where vote shares are transformed into seat shares one to one. Regarding the mapping from seats to the distribution of rents, we assume that candidates participate in the distribution of rents proportionally to the share of seats obtained in the election (see for example Lizzetti and Persico (2001)). As for policy outcomes, we adopt the simplifying assumption that the policy outcome is given by a lottery between the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly). This assumption captures in a stylized fashion the additional uncertainty faced by voters that is introduced by the process of post-election bargaining in PR. An electoral equilibrium is a Subgame Perfect Nash Equilibrium in pure strategies of the game of electoral competition, i.e., a strategy profile such that (i) voters cannot obtain a preferred policy outcome by voting for a different candidate in any voting game (on and off the equilibrium path), (ii) the location and quality decisions of other candidates, and given voters' strategy, candidates cannot increase their expected rents by modifying their investment in quality, (iii) candidates running for office collect non-negative rents, and (iv) candidates not running for office prefer not to enter: they would make negative rents in an equilibrium of the continuation game.

We start our analysis by focusing on electoral equilibria with two candidates running for office, and we construct an equilibrium where candidates obtain no rents. In electoral equilibria in which two candidates run for office without choosing maximal quality, candidates invest more in quality the less differentiated they are in the policy space and, given differentiation, the weaker is voters' ideological focus (Stokes 1963). In an equilibrium with no rents, however, a heightened responsiveness of voters to candidates' quality must result in a larger ideological differentiation between candidates running for office, without (directly) influencing the equilibrium investment in quality. We then extend the analysis to symmetric electoral equilibria with three or more candidates running for office. Within symmetric equilibria, a larger number of candidates leads to less differentiation in the ideological dimension, and thus to candidates being closer substitutes for each other. For the same reason as in two-candidate equilibria then, in symmetric equilibria candidates invest more in quality the larger the number of candidates running for office.

Within the class of symmetric electoral equilibria, we also explore how changes in the "supply side" parameters of the model can affect the number and quality of candidates running for office. In particular, we show that changes in the fixed cost of running for office, or shifts in the cost function of quality induce a positive correlation between the equilibrium number of candidates running for office and their quality. We also explore the role of "demand-side" factors, such as the responsiveness of voters to quality differentiation among candidates. We show that in symmetric equilibria in which no candidate obtains positive rents, a less ideologically focused electorate leads to more differentiation in the policy positions represented in the election, and to a smaller number of candidates. If instead candidates obtain positive rents in equilibrium, the impact of demand-side factors can be absorbed by the expected level of rents without affecting the number of candidates running for office.

Finally, we also show that the positive relation between quality and number of parties extends to the case of limited asymmetry among equilibrium candidates. In particular, we show that in this case there is a positive equilibrium relation between the quality of candidates and the effective number of parties.

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3. Austen-Smith and Banks (1988), Baron and Diermeier (2001), and Penson, Roland, and Tabellini (2005) study strategic voting induced by the process of government and coalition formation among elected representatives in PR for a given number of parties (three for Austen-Smith and Banks (1988), Baron and Diermeier (2001), four for Penson, Roland, and Tabellini (2005)). Iacocca and Marzoli (2008b) explore alternative specifications of the policy function mapping elected representatives into policy outcomes.
of voting for candidate \( k \), and \( \sigma = (\sigma_1(\cdot), \ldots, \sigma_n(\cdot)) \) denotes a voting strategy profile. An electoral equilibrium is a Subgame Perfect Nash Equilibrium of the game of electoral competition; i.e., a set of candidates running for office \( k^* \), policy positions \( x^* \), quality choices \( \theta^* \), and a voting profile \( \sigma^* \) such that:

(i) \( \theta^* \) is optimal for \( k \) given \( \{k^*, x^*, \theta^*(k^*, x^*, \theta^*(x^*)) \} \); i.e., \( \theta^* \) is a (pure Nash) equilibrium of the continuation game \( \Gamma_{k^*} \);

(ii) if \( k \in K \), then \( \Pi_k(k^*, x^*, \theta^*, \sigma^*(k^*, x^*, \theta^*(k^*, x^*)) \geq 0 \) (no exit condition);

(iii) if \( k \notin K \), then \( \Pi_k(k^*, \cup_k x^*, \theta^*, \sigma^*(k^*, \cup_k x^*, \theta^*(k^*, x^*)) \leq 0 \), in an equilibrium of the continuation game (non-profitable entry).

An outcome of the game is a set of candidates running for office \( k \), policy positions \( x \), and quality choices \( \theta \). A polity is a triplet \( (\alpha, \varepsilon, F) \in \mathcal{P}_k^*: \) We say that Proportional Representation admits an electoral equilibrium with outcome \( (K, x^*_k, \theta^*_k) \) if there exist a set of polities \( F \subseteq \mathcal{P}_k^* \) with positive measure such that if a polity \( \rho \in F \) then there exists an electoral equilibrium with outcome \( (K, x^*_k, \theta^*_k) \).

### 4.3. Results

We start by characterizing the properties of electoral equilibria with two candidates running for office. First note that, in the absence of investment in quality, equilibrium imposes only relatively weak constraints on the composition of the field of candidates. In particular, the equilibrium requirement of non-negative rents for candidates running for office implies a lower bound on ideological differentiation, while the no-entry condition imposes an upper bound on ideological differentiation. Consider next two candidates 1 and 2 representing policy positions \( x_1 = \Delta_1 \) and \( x_2 = x_0 + \Delta_2 \) with quality \( \theta_1 \) and \( \theta_2 \), and let \( \hat{x}_i \in R \) denote the (unique) value of \( x \) such that \( u(\theta_1, x_1; x) = u(\theta_2, x_i; x) \), so that \( u(\theta_1, x_1; z) > u(\theta_2, x_i; z) \) if and only if \( z > \hat{x}_2 \).

\[
\hat{x}_2 = \frac{x_0 + \Delta_2}{2} + \alpha \frac{u(\theta_1) - [u(\theta_2)]}{\Delta}.
\] (4.2)

Note next that in our model strategic voting is in fact equivalent to sincere voting on and off the equilibrium path. Since the probability that each candidate running for office is elected and implements his ideology is proportional to the share of votes received in the election, voting for a candidate who is not the most preferred one is always a strictly dominated strategy. In fact, by switching her vote to her most preferred candidate, a voter only affects the lottery's weights of ex-
actively two candidates and, with two alternatives, strategic voting and sincere voting coincide. Thus candidate 1’s vote share given \((x, 0)\) is \(m_1(\theta, x) = \min \{0, \frac{x}{2}\} \). Note that if \(\theta_1 > \theta_2(\theta, x)\), where \(m_1(\theta_1, \theta_2, x) \equiv 0\), the vote share mapping \(m_1(\theta_1, \theta_2, x)\) is differentiable and the marginal vote share is given by
\[
\frac{\partial m_1}{\partial \theta_1} = \frac{\partial v(\theta)}{\Delta}, \tag{4.3}
\]
that is, the marginal impact of quality on vote share given the identity of \(k\)’s relevant competitors is well-defined, increases with \(\alpha\), and decreases with \(\Delta\). In the next proposition we focus on equilibria in which exactly two candidates run for office.

**Proposition 4.1.** Proportional Representation admits an electoral equilibrium in which exactly two candidates run for office. In any two-candidate equilibrium, candidates choose the same quality,
\[
\theta^*_1 = \theta^*_2 = 0^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) = \alpha \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) \leq C^{-1}\left(\frac{1}{2} - F\right). \tag{4.4}
\]
Furthermore, the more responsive are voters to differences in quality between candidates (the higher is \(\alpha\)), the higher is candidates’ investment in quality and, if candidates do not capture positive rents, also the higher is the degree of ideological polarization between candidates (\(\Delta\)).

**Proof.** To prove this result, we show that if \(\tilde{\epsilon} \leq \frac{1}{4}, \tilde{\epsilon} + F > \frac{1}{2}\), and,
\[
\frac{2\tilde{\epsilon}}{\Psi^{-1}\left(\frac{1}{2} - F\right)} \leq \alpha \leq \frac{1 - 2\tilde{\epsilon}}{\Psi^{-1}\left(\frac{1}{2} - F\right)},
\]
in which two symmetrically located candidates run for office with non-maximal quality, and capture zero rents (showing that PR admits an equilibrium with two candidates collecting positive rents follows a similar logic and is therefore omitted).

Suppose that candidates 1 and 2 run for office, and that \(\max(\theta^*_1, \theta^*_2) < 1\). This implies that the FOCs must be satisfied with equality and, in particular, that \(\frac{\partial v(\theta)}{\Delta} = C(\theta)\) for \(k = 1, 2\), and hence that \(\Delta \geq \alpha \Psi(1)\). Then,
\[
\theta^*_1 = \theta^*_2 = 0^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right). \tag{4.5}
\]

Note that when \(\theta^*_2 = 0^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right), 1\)’s marginal profit is well-defined, continuous and decreasing at all points \(\theta > 0\). Since the condition for non-negative rents is part of the equilibrium definition, it follows that \(\theta^*_1 = 0^*\) is indeed a best response. Furthermore, since \(\theta^*_1 = \theta^*_2\), we have that \(\bar{\theta} = \Delta + \frac{\Delta}{2}\). Given that in equilibrium candidates must collect nonnegative rents, then it must be true that \(\Pi^*_1 = \Delta + \frac{\Delta}{2} - C(\theta^*_1) - F \geq 0\) and \(\Pi^*_2 = 1 - \Delta - \frac{\Delta}{2} - C(\theta^*_2) - F \geq 0\), or equivalently,
\[
F + C(0^*) - \frac{\Delta}{2} \leq \Delta \leq 1 - \frac{\Delta}{2} - C(0^*) - F. \tag{4.6}
\]
There exists \(\Delta \geq 0\) satisfying (5) if and only if \(\theta^* \leq C^{-1}\left(\frac{1}{2} - F\right)\) or, substituting from (4.5), if and only if \(\Delta \leq \alpha \Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right) \leq \alpha \Psi(1)\). Therefore, if \(\Delta \leq \alpha \Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right) \leq \alpha \Psi(1)\), also \(\Delta \geq \alpha \Psi(1)\). Choose then \(\Delta = \alpha \Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right) \leq \alpha \Psi(1)\). From (4.5), \(\theta^* = C^{-1}\left(\frac{1}{2} - F\right)\) or \(C(\theta^*) = \frac{1}{2} - F\). Also, since inequalities in (4.6) hold as equalities, \(\Delta = F + C(0^*) - \frac{\Delta}{2} = \frac{1}{2} - F\). Hence, \(\Pi^*_1 = \Pi^*_2 = 0\).

Consider next entry of a third candidate \(j\) with \(\xi \in (\xi_1, \xi_2)\) and assume the following continuation play: \(\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta} = 1\). The optimality conditions for \(k = 1\) and \(k = 2\) are \(\frac{\alpha}{(1 - \delta_j)\Delta} \Psi(1) \geq 1\) and \(\frac{\alpha}{\delta_j} \Psi(1) \geq 1\), where \(\tilde{\epsilon} = \frac{\xi - \xi^*}{\Delta}\). The necessary first order condition for \(j\) is \(\frac{\alpha}{(1 - \delta_j)\Delta} \Psi(1) = 1\), which is implied by the previous inequalities. These conditions are satisfied if and only if
\[
\max(\delta_j, 1 - \delta_j) \Delta \leq \alpha \Psi(1). \tag{4.7}
\]
Now suppose that \(\delta_j < \frac{1}{2}\). Then (4.7) is \((1 - \delta_j) \Delta \leq \alpha \Psi(1)\), and thus we need \(\alpha \Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right) \leq \Delta \leq \frac{\alpha}{1 - \delta_j} \Psi(1)\). This is feasible if
\[
\frac{\Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right) - \Psi(1)}{\Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right)} \leq \delta_j \leq \frac{1}{2}. \tag{4.8}
\]
Suppose instead that \(\delta_j > \frac{1}{2}\). Then (4.7) is \(\delta_j \Delta \leq \alpha \Psi(1)\), and we need \(\alpha \Psi\left(C^{-1}\left(\frac{1}{2} - F\right)\right) \leq \Delta \leq \frac{\alpha}{\delta_j} \Psi(1)\). This is feasible if:

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3 See Inzerower and Mattezi (2008) for a formal argument.
\[
\frac{1}{2} \leq \delta_j \leq \frac{\Psi(1)}{\Psi(C^{-1}\left(\frac{1}{2} - F\right))}
\]  

(4.9)

Combining (4.8) and (4.9) we obtain

\[
\frac{\Psi(C^{-1}\left(\frac{1}{2} - F\right)) - \Psi(1)}{\Psi(C^{-1}\left(\frac{1}{2} - F\right))} \leq \delta_j \leq \frac{\frac{1}{2} \Psi(1)}{\Psi(C^{-1}\left(\frac{1}{2} - F\right))}.
\]  

(4.10)

When (4.10) holds, i.e., following the entry of a centrist candidate, and \( \Delta = \alpha \Psi(C^{-1}\left(\frac{1}{2} - F\right)) \), then \( \tilde{\theta}_1 = \tilde{\theta}_2 = \tilde{\theta}_j = 1 \) is a joint best response provided that the incumbent candidates choose not to drop from the race. A sufficient condition for the latter statement to be true (when (4.10) holds) is \( \alpha \leq \frac{1 - 2\bar{\epsilon}}{\Psi(1)} \). When \( \tilde{\theta}_1 = \tilde{\theta}_2 = \tilde{\theta}_j = 1 \) we have that \( \tilde{\Pi}_j = \frac{\Delta}{2} - \bar{\epsilon} - F < 0 \), since \( \bar{\epsilon} - F > \frac{1}{2} \) and \( \Delta < 1 \). Now consider entries such that \( \delta_j > \frac{\Psi(1)}{\Psi(C^{-1}\left(\frac{1}{2} - F\right))} = \frac{\Psi(1)}{\Delta} \). In this case \( j \) enters relatively close to \( k = 1 \), and a strategy profile such that all three candidates choose maximal quality cannot be an equilibrium of the continuation game. Consider instead \( \tilde{\theta}_2 \in (0, 1) \), and \( \tilde{\theta}_1 = \tilde{\theta}_j = 1 \). The FOC for \( k = 2 \) is \( \frac{\alpha}{\delta_j} \tilde{\theta}_j \Psi(\tilde{\theta}_j) = \Psi(1) = 1 \), or equivalently \( \tilde{\theta}_2 = \frac{\Psi^{-1}\left(\frac{\alpha}{\delta_j} \tilde{\theta}_j \Psi(1)\right)}{\Psi^{-1}\left(\frac{\alpha}{\delta_j} \tilde{\theta}_j \Psi(1)\right)} \). The FOC for \( k = 1 \) is, as before, \( (1 - \delta_j) \Delta \alpha \Psi(1) \), and the FOC for \( j \) is not relevant. Therefore, we need \( \alpha \Psi(C^{-1}\left(\frac{1}{2} - F\right)) \) \( \leq \Delta < \frac{\alpha}{1 - \delta_j} \Psi(1) \), which is feasible if

\[
\frac{\Psi(C^{-1}\left(\frac{1}{2} - F\right)) - \Psi(1)}{\Psi(C^{-1}\left(\frac{1}{2} - F\right))} \geq \delta_j
\]  

(4.11)

and this always holds with \( \delta_j > \frac{\Psi(1)}{\Psi(C^{-1}\left(\frac{1}{2} - F\right))} \). We need to show now that \( \tilde{\Pi}_j = \tilde{x}_j(1, \tilde{\theta}_j) - \frac{x_j + x_j}{2} - \bar{\epsilon} - F < 0 \). If \( \tilde{x}_j \) were fixed, \( j \) would be better off by choosing

\[\frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \quad \text{and} \quad \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \quad \text{for} \quad \tilde{x}_j \text{and} \tilde{\theta}_j.\]

Note that when \( \delta_j = (\alpha) \frac{1}{2} \), we need \( \Delta = \frac{\alpha \Psi(1) (\Delta \frac{1}{2} \Psi(1))}.\) From (8) this holds for all "feasible" \( \delta_j \), if and only if \( \Delta \leq \alpha \Psi(1)(\frac{1}{2} - F). \) But then, since we also need \( \Delta \leq \alpha \Psi(1)(\frac{1}{2} - F), \) this must hold with equality. It is not surprising that, given zero profit in equilibrium, it must be the case that a unique \( \Delta \) is the one that covers all possible \( \delta_j \) in (4.10).

Note that before we were satisfying (4.8) with \( \delta_j < 1/2 \), and now we are satisfying (4.11), which is the first part of (4.8), with \( \delta_j > 1/2 \). The reason is that before we were forcing \( k \) to keep choosing maximal quality even when \( i \) was entering relatively far away from him.
equilibrium. If we consider a decrease in \( F \), the latter inequality becomes less binding and can hold for a smaller value of \( \Delta \), which implies a higher quality and a larger number of candidates running for office. A similar logic applies in the case of downward parallel or proportional shifts of the cost function. Hence, changes in the "supply side" of the political environment induce a positive correlation between the number of candidates running for office and their equilibrium quality. We summarize this conclusion informally in the following remark:

Remark 4.1. Everything else constant, reductions in the fixed cost of running for office \( F \), and/or downward (parallel or proportional) shifts in the cost function \( C(\cdot) \), increase both the number of candidates running for office and their quality.

If we focus on changes on the demand side of the political environment, however, the comparison is less clear. Consider changes in the responsiveness of voters to candidates' quality (\( \alpha \)). Increasing \( \alpha \) has the direct effect of making a given field of candidates "more aggressive" in quality competition. This has the effect of reducing the expected rents of all participants in the election. In a LSE where candidates running for office collect positive rents, the system has enough flexibility so that as voters become more responsive to the candidates' quality, quality competition can become tighter without affecting the equilibrium number of candidates. As candidates "compete away" their rents, however, increased voters' responsiveness to candidates' quality must lead to changes in the level of ideological differentiation and, eventually, in the number of candidates deciding to run for office. In fact, when candidates collect no rents in equilibrium, optimal quality depends on \( \alpha \) only indirectly, through the equilibrium level of differentiation \( \Delta \), which is increasing in \( \alpha \).

In this case it follows that a less ideologically focused electorate must lead to a smaller number of candidates running for office. The overall effect on quality, however, is ambiguous.

So far we focused on location-symmetric equilibria. Note that in the class of LSE, it follows immediately that the number of candidates (inversely related to the degree of ideological differentiation between candidates) is directly related to the level of quality competition; i.e. the larger the number of candidates, the closer substitutes candidates are to each other, and therefore the more intense quality competition is. This result generalizes with some caveats to configurations of candidates with limited asymmetry. Our first objective is to find a pro-

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1. Formally, an electoral equilibrium is location-symmetric (LSE) if the distance between any two candidates is equal to \( \Delta \), and the candidates follow \( \hat{\Delta}_k = \Delta \) and \( \hat{\Delta}_k = -\Delta \).

2. When equilibrium rents are equal to zero, it follows that \( \Delta = C(0) \) and \( \hat{\Delta}_k = \Delta \), which is increasing in \( \alpha \).

Note that we can easily extend our previous analysis and results to accommodate some limited asymmetry in location. While a full characterization of asymmetric equilibria is beyond the scope of this paper...
per way to measure the number of candidates in an asymmetric environment. Consider, for example, comparing an outcome with four-minority candidates each obtaining one percent of the vote and a fifth, one capturing the remaining ninety-six percent, with a second outcome where three candidates each obtain a third of the votes. As this example suggests, looking at the number of candidates in the context of asymmetric political configurations can be misleading, since the number of relevant candidates can be said to be larger in the latter outcome than in the former. One measure that overcomes this problem, and it is largely used in the political science literature, is the effective number of parties introduced by Laakso and Taagepera (1979). The Laakso-Taagepera effective number of parties or candidates for our purposes) is defined as $e = 1/H$, where in turn $H = \sum_{i=1}^{N} m_i^2$ is the Herfindahl index, which is commonly employed to measure concentration of industries in industrial organization. The popularity of the effective number of candidates is due to a number of attractive properties (see Encausas and Jacekmin (1980)). First, it is symmetric, or invariant to permutations of vote shares, between candidates. Second, it satisfies the transfer principle: the transfer of a part of a candidate's vote share to a candidate with a bigger vote share must not increase the effective number of candidates. For a given number of candidates, this condition implies that $e$ attains its maximum value when the candidates have equal vote shares, and its minimum value when a single candidate captures (almost) the entire electorate. Third, the value of $e$ for symmetric candidates must increase when the effective number of candidates grows from $K$ to $K+1$. In particular, the effective number of candidates (weakly) decreases when we transfer vote share from one candidate to another with a higher initial vote share. Given the definition of effective number of candidates, we can show the following result:

**Proposition 4.2.** Consider an electoral equilibrium with three candidates running for office such that $\Delta_3 > \Delta_2$. Then $\theta_2 = \theta_3^* = \theta_3^*$, with the inequalities strict if quality is non-maximal in equilibrium. Consider an alternative electoral equilibrium with $x_3 > x_2$. Then $\theta_2^* > \theta_3^* > \theta_3^*$, and $\theta_3^* < \theta_3^*$. Furthermore, if $\Psi$ is convex, then the new equilibrium has both a smaller effective number of candidates and a lower average quality of candidates. Similarly, consider a LSE with $K$ parties, and an alternative equilibrium with $x_3 > x_2$ and $x_3^* \in (x_2, x_3^*)$. Then if $\Psi$ is convex, the new equilibrium has both a smaller effective number of candidates and a lower average quality of candidates.

**Proof.** In order to prove this result, first we need to introduce some additional notation, which will prove useful to handle non-symmetric configurations of candidates. Provided that $\theta_2 = \theta_3 = (\theta_3^*, \theta_3^*)$, the $k$th vote share $m_k(\theta_3^*, \theta_3^*)$ can be expressed as

$$m_k(\theta_3^*, \theta_3^*) = \frac{\Delta_3}{2} + \alpha \left[ \frac{\nu(\theta_3^*) - \nu(\theta_3^*)}{\Delta_3^*} + \frac{\nu(\theta_3^*) - \nu(\theta_3^*)}{\Delta_3^*} \right].$$

where $\Delta_3^*$ and $\Delta_3^*$ denote the distance between the policy represented by $k$ and that of its neighbors, $\theta_m$ and $\theta_{m'}$ denote the campaign effort of $k$'s neighbors, and $\Delta_3^* = \Delta_3^* + \Delta_3^*$. Letting $\delta_2 = \Delta_3^*$, it follows that $k$'s FOC is given by

$$\theta_3^* = \Psi^{-1} \left( \frac{\delta_2 (1 - \delta_2) \Delta_3^*}{\alpha} \right).$$

for $k \in (2, K-1)$. In the case of three candidates running for office with $\Delta_3 > \Delta_2$ FOCs deliver $\delta_2 = \Psi^{-1} \left( \frac{\delta_2 (1 - \delta_2) \Delta_3^*}{\alpha} \right); \theta_3^* = \Psi^{-1} \left( \frac{\delta_2 \Delta_3^*}{\alpha} \right)$, and $\theta_3^* = \Psi^{-1} \left( \frac{\delta_2 (1 - \delta_2) \Delta_3^*}{\alpha} \right)$.

It follows immediately that $\theta_3^* > \theta_3^* \geq \theta_3^*$ since $\Psi(\cdot)$ is decreasing and $\delta_2 < \frac{1}{2}$. Next, note that since $\delta_3 (1 - \delta_3)$ is monotonically increasing in $\delta_3$ for $\delta_3 < \frac{1}{2}$, then $\delta_3 < \delta_2$ implies that $\theta_3^* > \theta_3^* > \theta_3^*$, and $\theta_3^* < \theta_3^*$. Now $\theta_3^* = \frac{1}{3} \sum \theta_3^*$, and since $\theta_3^* > \theta_3^*$ it is enough to show that $\theta_3^* > \theta_3^* > \theta_3^*$. This can be written as

$$\Psi^{-1} \left( \frac{\delta_2 \Delta_3^*}{\alpha} \right) + \Psi^{-1} \left( \frac{\Delta_3^*}{\alpha} \right) > \Psi^{-1} \left( \frac{\delta_2 \Delta_3^*}{\alpha} \right) + \Psi^{-1} \left( \frac{(1 - \delta_3) \Delta_3^*}{\alpha} \right)$$

which follows from convexity of $\Psi$. In fact, if $\Psi$ is decreasing and convex then $\Psi^{-1}$ is also convex. The last part of the proposition can be proved in a similar way.

Note that the result of Proposition 4.2 holds when $\Psi$ is convex, which is not implied by the assumptions of $\nu$ concave and $C$ convex. Convexity of $\Psi$, however, is satisfied in the case of many commonly used parametric specifications. For example, when $C(\theta) = \theta^a$, with $A > 0$ and $B > 1$, and $\nu$ belongs to the class of hyperbolic absolute risk aversion (HARA) utility functions, i.e., $\nu(\theta) = \frac{1}{d} \left( \frac{\theta^a}{1 - d} \right)$ with $a > 0$ and $d < 1$. The HARA class includes as special cases the constant absolute risk aversion (with $b = 1$ and $d \rightarrow -\infty$), constant relative risk aversion (with
b \to 0 \text{ and } a = 1 - d), \text{ logarithmic (with } b \to 0 \text{ and } d \to 0), \text{ as well as power and exponential utility functions.}

References


