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SYMPOSIUM

The pro-competitive effect of campaign limits in non-majoritarian elections

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Abstract We study a model of elections in non-majoritarian systems that captures the link between competition in policies and competition in campaign spending. We argue that the overall competitiveness of the political arena depends on both the endogenous number of parties contesting the election and the endogenous level of campaign spending. These two dimensions are linked together through their combined effect on the total equilibrium level of political rents. We illustrate the key insights of the model with an analysis of the competitive effects of campaign spending limits. We show that under some conditions spending caps can be pro-competitive, leading to an increase in the number of parties contesting the elections.

Keywords Campaign spending · Elections · Campaign limits · Political parties

JEL Classification D72 · D78 · C72

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1 Introduction

Non-majoritarian electoral systems create a link between the degree of ideological differentiation among parties competing for votes, and the incentives of new parties to enter the political arena. Parties that represent policies that are too far apart from one another invite the entry of new competitors, who seek to attract the niche of voters that do not find any of the options available to be particularly appealing. This link is well known in the literature: there is little hope of thinking about representation and ideological differentiation in non-majoritarian elections without letting the number of parties adjust freely in equilibrium.[1] This, however, is only part of the story.

While parties' ideological positions are surely important in shaping citizens' voting decisions, a paramount ingredient of modern day elections is the campaign competition between parties and the costs that are associated with it. Parties and candidates spend heavily in electoral campaigns to change voters' impressions about them, and their effort pays off: campaign spending sways votes (Green and Krasno 1988; Kenny and McBurnett 1994; Gerber 1998; Coleman and Manna 2000; Stratmann 2009), and induces would-be voters to participate in the election (Gerber and Green 2000; Rekkas 2007).

The key point here is not just that campaign competition is important in modern elections—it is—but that a partial equilibrium analysis of campaign spending is not called for either. Since voters and vote shares are more responsive to differentials in campaign spending between two parties, the more similar their policy positions are, a smaller differentiation in policies leads to more intense campaign competition, and then to higher costs of running the campaign in equilibrium. As a result, the *effective size of the cake* (i.e. total political rents) shrinks and so does the number of parties that can be supported in equilibrium.

In this paper, we introduce a model that captures the link between competition in policies and competition in campaign spending. The key insight of the paper is that the overall competitiveness of the political arena depends on both the endogenous number of parties contesting the election *and* the endogenous level of campaign competition. Our model shows that these two dimensions are linked together through their combined effect on the total equilibrium level of political rents.

We illustrate the key insights of the model through the analysis of *campaign finance limits* (or *spending caps*).[2] We show that increasing the spending cap leads (eventually) to a smaller number of parties, reducing the set of alternatives available to voters. In fact, increasing the spending cap can eliminate competition altogether. Thus, while relaxing the spending cap can increase the competitiveness of the campaign for parties that participate in the election, it can also reduce the number of parties that enter the competition and therefore reduce competition in the ideological dimension.

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[1] See Osborne and Slivinski (1996) and Iaryczower and Mattozzi (2009).

[2] As Prat (2002) puts it, "In principle one may restrict [campaign contributions] or [campaign spending], or both. The U.S. Supreme Court has ruled that limits on spending are unconstitutional because they restrict the right to free speech. In contrast, limits on spending are in place in most European countries."

Our results might suggest that if voters become more responsive to campaigning—thus inducing parties to engage in a more intense campaign competition—the equilibrium number of parties should be smaller as well. We show that this is not necessarily the case. In particular, we provide sufficient conditions for equilibrium that are determined by "supply side" factors only; i.e., the variable cost of campaigning at the spending cap or the fixed cost of entry into the electoral competition. The responsiveness of voters to campaigning plays no role in defining the number of parties that can be competing for votes in equilibrium.

This paper builds on a large literature, touching on each of the components of the model. A number of influential papers study elections in majoritarian and proportional systems. Myerson (1993) and Lizzeri and Persico (2001) focus on how the nature of electoral competition affects promises of redistribution made by candidates in the election. Austen-Smith and Banks (1988), Baron and Diermeier (2001), Austen-Smith (2000), Persson et al. (2003), Persson et al. (2006), and Baron et al. (2011) consider models of elections and legislative outcomes in proportional representation systems were rational voters anticipate the effect of their vote on the bargaining game between parties in the elected legislature. They do this in the context of a fixed number of parties (and without introducing campaign competition). Palfrey (1984, 1989), Feddersen et al. (1990), Feddersen (1992), Osborne and Slivinski (1996), Besley and Coate (1997), and Iaryczower and Mattozzi (2009) introduce entry in elections. With few exceptions (Osborne and Slivinski 1996; Iaryczower and Mattozzi 2009), these papers work in the context of plurality elections.[3]

A second strand of literature deals with the campaign competition dimension. In our paper, we formalize campaign competition as differentiation in a common value dimension. This builds on the large literature that following Stokes (1963)'s original critique to the Downsian model, incorporates competition in *valence* issues (typically within majoritarian electoral systems, and two exogenously given parties). See Groseclose (2001), Aragones and Palfrey (2002), and Bernhardt et al. (2011) for models where one party has an exogenous valence advantage. For models of endogenous valence see Carrillo and Castanheira (2008), Meirowitz (2008), Ashworth and Bueno de Mesquita (2009), Eyster and Kittsteiner (2007), Callander (2008), and Herrera et al. (2008). See also Morton and Myerson (2011).

The rest of the paper is organized as follows. Section 2 presents the model. In Sect. 3, we characterize equilibria of the model with a focus on the analysis of spending caps. We conclude in Sect. 4. Most of the proofs can be found in the Appendix.

2 The model

There are three stages in the game. In the first stage, a finite set of political parties simultaneously decide whether or not to participate in the election. In the second stage, all parties simultaneously choose a level of campaign spending. In the third stage, a finite set of voters vote.

[3] For models of differentiation and entry in industrial organization, see d'Aspremont et al. (1979), Shaked and Sutton (1982), and Perloff and Salop (1985).

The ideology space is $X \equiv \{t/T : t = 0, 1, \ldots, T\} \subset [0, 1]$, where T is a large integer. In any $x \in X$, there is a party who will perfectly represent policy x if elected.[4] Parties care about the spoils they can appropriate from being in office, and must pay a fixed cost F to participate in the election.[5] We denote the set of parties at the end of the first stage by $\mathcal{K} = \{1, \ldots, K\}$. In the second stage, all parties contesting the election simultaneously choose a level of campaign spending θ_k, which cannot exceed a *spending cap* L; i.e., $\theta_k \in [0, L]$. Parties can spend θ_k at a cost $C_v(\theta_k)$, $C_v(\cdot)$ increasing and convex. In the third stage, N fully strategic voters vote in an election, where we think as N being a large finite number. A voter i with ideal point $z^i \in X$ ranks parties according to the utility function $u(\cdot; z^i)$, which assigns to party k with characteristics (θ_k, x_k) the payoff $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$, with v increasing and concave. The parameter α captures voters' responsiveness to campaigning. Voters' ideal points are uniformly distributed in X.

Let $\theta_{\mathcal{K}} \equiv \{\theta_k\}_{k \in \mathcal{K}}$ and $x_{\mathcal{K}} \equiv \{x_k\}_{k \in \mathcal{K}}$ denote the level of campaigning and policy positions of the parties contesting the election. We assume that each party k obtains a share of the total seats in the legislature equal to her share of votes in the election, $s_k(\theta_{\mathcal{K}}, x_{\mathcal{K}})$, and that the policy outcome is the result of a *probabilistic compromise* among the parties represented in the legislature, where the likelihood of the policy represented by a party emerging as the policy outcome is increasing in the candidate's vote share or seat share in the assembly.[6] The expected share of rents appropriated by party k, denoted m_k, is proportional to his vote share in the election. For simplicity, and without any real loss of generality, we assume that $m_k(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = s_k(\theta_{\mathcal{K}}, x_{\mathcal{K}})$. Normalizing total political rents to one and letting $C(\cdot) \equiv C_v(\cdot) + F$, we can write the expected payoff of a party k contesting the election as

$$\Pi_k(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = m_k(\theta_{\mathcal{K}}, x_{\mathcal{K}}) - C(\theta_k). \tag{1}$$

A strategy for party k is a decision of whether or not to participate in the election and campaign spending $\theta_k(\mathcal{K}, x_{\mathcal{K}}) \geq 0$. A strategy for voter i is a function $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) \in \mathcal{K}$, where $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = k$ indicates the choice of voting for party k, and $\sigma = \{\sigma_1(\cdot), \ldots, \sigma_N(\cdot)\}$ denotes a voting strategy profile. An *electoral equilibrium* is a pure strategy Subgame Perfect Nash Equilibrium of the game of electoral competition, i.e., a strategy profile such that (*i*) voters cannot obtain a better policy outcome by voting for a different party in any voting game (on and off the equilibrium path), (*ii*) given the location and campaign decisions of other parties, and given voters' voting strategy, parties cannot increase their expected rents by modifying their campaign levels, (*iii*) parties contesting the election earn non-negative rents, and (*iv*) parties not contesting the election prefer not to enter: would earn negative rents in an equilibrium of the continuation game.

[4] This assumption captures the fact that commitments to any other alternative policy are not credible in the context of a static model.

[5] In the benchmark model, we assume that parties are purely office motivated, i.e., they do not care about policy outcomes per se, but only as a tool to get votes. We can show that as long as office motivation dominates policy motivation, all our results are qualitatively unchanged.

[6] For a similar approach see, e.g., Grossman and Helpman (1996) and Persico and Sahuguet (2006).

3 The institutional determinants of competition in short-run and long-run electoral equilibria

We begin our analysis focusing on a short-run horizon in which the entry of "new" parties is ruled out. To differentiate this from our full equilibrium analysis with entry, we call equilibria in this restricted setting *short-run electoral equilibria*. A short-run electoral equilibrium is a configuration of parties competing in the election (the number of parties competing in the election, together with their position in the policy space), a choice of campaign spending by each party competing in the election, and a voting strategy by voters such that (*i*) voters cannot obtain a better policy outcome by voting for a different party in any voting game (on and off the equilibrium path), (*ii*) given the location and campaign decisions of other parties, and given voters' voting strategy, parties cannot increase their expected rents by modifying their campaign levels, and (*iii*) parties contesting the election earn non-negative rents. Note that this drops requirement (*iv*) in the definition of electoral equilibrium, which required that parties not contesting the election prefer not to enter (would earn negative rents in an equilibrium of the continuation game).

We begin in Sect. 3.1 with the simplest case in which two parties compete in the election. We then move on to the multiple party case in Sect. 3.2. The main result of the section (Theorem 1) is presented in Sect. 3.3, which deals with long-run electoral equilibria.

3.1 Short-run electoral equilibrium with two parties

In this section, we establish two classes of results. We start by characterizing equilibria in which parties are unconstrained in campaigning. We then analyze the effect of raising the spending cap, taking voters' characteristics as given.

Consider two candidates 1 and 2 representing policy positions x_1 and x_2 with ideological differentiation $\Delta \equiv x_2 - x_1$, and campaign spending θ_1 and θ_2. Let $\tilde{x}_{12} \in \mathcal{R}$ denote the (unique) value of x such that $u(\theta_1, x_1; x) = u(\theta_2, x_2; x)$, so that $u(\theta_1, x_1; z^i) > u(\theta_2, x_2; z^i)$ if and only if $z^i > \tilde{x}_{12}$

$$\tilde{x}_{12} = \frac{x_1 + x_2}{2} + \alpha \frac{[v(\theta_1) - v(\theta_2)]}{\Delta}. \tag{2}$$

Note that with two parties competing in the election, there is no room for strategic voting, and thus all voters vote for their preferred alternative in equilibrium. Hence, as long as $\tilde{x}_{12} \in (0, 1)$, candidates vote share mappings are given by $m_1(\theta, x) = \tilde{x}_{12}(\theta, x)$ and $m_2(\theta, x) = 1 - \tilde{x}_{12}(\theta, x)$. We first show that when voters are sufficiently ideological—and thus relatively unresponsive to campaign spending—it is always possible to support a short-run electoral equilibrium in which parties are not constrained in campaigning. What "sufficiently ideological" means here precisely depends on the cost of campaigning evaluated at the spending cap, $C(L)$. By letting $\Psi(\cdot) \equiv v'(\cdot)/C_v'(\cdot)$, and defining the bound $b \equiv 1/\Psi(L)$ if $C(L) \leq 1/2$ and $b \equiv 1/\Psi(C^{-1}(1/2))$ otherwise, we have the following result.[7]

Proposition 1 *For any $\alpha \leq b$, there exists a short-run electoral equilibrium with two parties unconstrained in campaigning. In any such equilibrium, (i) there is a lower bound on ideological differentiation $\underline{\Delta}(\alpha)$, which is increasing in α, and (ii) campaign competition is neutral for electoral outcomes. In fact*

$$\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1}(\Delta/\alpha). \tag{3}$$

The fact that there cannot be a campaign differential between parties is a feature of all equilibria with two parties: while parties' total rents depend on their ideological positions, campaign incentives—i.e., marginal rents—depend only on their ideological differentiation. It follows that in a two-party equilibrium both parties must have the same incentives in campaigning.

Note also that even for relatively low responsiveness to campaigning (i.e., $\alpha < b$), Proposition 1 identifies a lower bound on ideological differentiation as a necessary condition for parties to be unconstrained in campaigning. This suggests that when instead voters are sufficiently responsive (α is big enough), no feasible ideological differentiation would induce a soft enough competition. As we show in the next proposition, in such an environment parties always campaign aggressively. However, this does not mean that such intense competition can always be carried out: depending on the total campaign cost at the spending cap, either parties are able to sustain a high level of competition (in which case parties are necessarily constrained by the cap) or, when they are not, the system must lose competition altogether. In other words, when voters are easily swayed by campaign spending, relaxing the spending cap can end up eliminating competition in equilibrium.

Proposition 2 *Suppose $\alpha > b$. Then if $C(L) \leq 1/2$, there exists a short-run electoral equilibrium with two parties, constrained in campaigning. If instead $C(L) > 1/2$, then in all pure strategy electoral equilibria, a single party runs unopposed, without campaigning.*

Two remarks follow. First, note that together with Proposition 1 this result implies that when the spending cap is relatively low, a two-party short-run electoral equilibrium in pure strategies can be sustained independently of the value of α. In other words, the conditions constraining the number of parties in equilibrium are entirely determined by "supply side" factors; i.e., the cost of campaigning at the spending cap, $C(L)$. Voters' responsiveness to campaign advertising only affect parties' equilibrium rents.

Second, note that in keeping with the equilibrium notion employed in the paper, the second part of Proposition 2 states that if $C(L) > 1/2$, then in all electoral equilibria in *pure strategies* a single party runs unopposed. The result, however, extends

[7] We defined the ideology space as $X \equiv \{t/T : t = 0, 1, \ldots, T\} \subset [0, 1]$ for a large integer T. In the formal analysis, we consider the limit of the discrete case as T goes to infinity and treat the policy space as an interval of $\mathcal{R}$. This simplification allows us to take derivatives of market vote shares and (as it will become evident throughout the analysis) does not sacrifice anything of importance.

to equilibria in mixed strategies as well. Since this is an interesting result in itself, we discuss this briefly below.

Proof (*Rent Dissipation in mixed-strategy equilibria*) When $C(L) > 1/2$ and $\alpha > b$, there cannot be a short-run electoral equilibrium in pure strategies with more than one party. While two-party mixed-strategy equilibria certainly exist in the campaign stage, the more relevant question for our purpose is whether in these equilibria parties will completely dissipate their variable rents (excluding the fixed cost F). If these were true parties would earn negative total rents. Therefore, a short-run electoral equilibrium with two parties does not exist. It turns out that this is exactly what happens when voters' responsiveness to campaign is high enough. To see why this is the case, notice that for sufficiently high α, the game of campaign competition can be approximated by an all-pay auction between parties. More precisely, for every $\epsilon > 0$ consider the discrete version of the campaigning game where $\theta = \{0, \epsilon, 2\epsilon, \ldots, L\}$ and, for simplicity, let $C_v(L) > 1$ and $(x_1 + x_2)/2 = 1/2$, i.e., the case in which parties ideological position are symmetric and the spending cap is never binding. Simple algebraic manipulation of Eq. (2) shows that there exists a threshold $\bar{\alpha}(\epsilon) \equiv \Delta/2\epsilon$ such that if $\alpha > \bar{\alpha}(\epsilon)$ we can write the expected variable rents of party $k = 1, 2$ as

$$\Pi_k^v = \begin{cases} 1 - C_v(\theta_k) & \text{if} \quad \theta_k > \theta_{-k} \\ \frac{1}{2} - C_v(\theta_k) & \text{if} \quad \theta_k = \theta_{-k} \\ -C_v(\theta_k) & \text{if} \quad \theta_k < \theta_{-k}. \end{cases}$$

Since the campaigning stage is a discrete symmetric all-pay auction, we can use the results of Baye et al. (1994) to conclude that there exists a symmetric mixed-strategy equilibrium in which each party puts positive probability on all pure strategies θ_k such that $\theta_k \leq C_v^{-1}(1)$. As ϵ approaches zero, the equilibrium distributions converge uniformly to the continuous uniform distribution, which is the *unique* equilibrium of a two-player all-pay auction with a continuous strategy space (Baye et al. 1996). Further, as ϵ approaches zero, expected variable rents converge to zero, and there is full rent dissipation. As a consequence, for any ϵ we can find a sufficiently large α such that the variable rents of this mixed-strategy equilibrium of the campaign stage are arbitrarily small and, in particular, smaller than the fixed cost F. Hence, restricting attention to the only class of equilibria in the campaign stage that survives when ϵ approaches zero, a two-party short-run electoral equilibrium does not exist, and all equilibria have a single party not investing in campaigning (this equilibrium exists trivially for all values of (α, L)). □

To sum up, in this section, we established two classes of results. First, we characterized equilibria in which parties are unconstrained in campaigning. We showed that for any given spending cap, if voters are sufficiently unresponsive to parties' campaign efforts (i.e., sufficiently ideological), then there exists a short-run equilibrium with two parties unconstrained in campaigning. In any such equilibrium, both parties must spend an equal amount on campaigning, and as a result, campaign competition is neutral for electoral outcomes. Campaign competition does however affect political rents. In fact we showed that political rents increase—as equilibrium campaign spending decreases—the larger is the ideological differentiation among parties in the election. Second, we considered the effect of raising the spending cap, taking voters' characteristics as given. We showed that when voters are highly responsive to campaigning, raising spending caps can have an anti-competitive effect in equilibrium. This result is due to the fact that in this situation raising the spending cap increases the cost of campaign competition and eventually leads to the impossibility of sustaining competition in equilibrium: for high enough L, all equilibria (in pure *and* mixed strategies) have a single party running in the election.

3.2 Short-run electoral equilibrium with multiple parties

In this section, we extend the previous analysis of short-run electoral equilibria to the case of $K \geq 3$ parties. Considering a multiparty environment introduces some non-trivial theoretical considerations, which we discuss and address immediately below. We then present the main results of this section in Proposition 3 and its corollaries.

The first potential complication in dealing with multiparty equilibria concerns the characterization of the vote share functions. With only two alternatives, there is no room for strategic voting, and thus in equilibrium all voters vote for their preferred party in every subgame. With more than two parties, instead, a voter might conceivably benefit from voting for a party other than her most preferred, if by doing so she reduces the likelihood of ending up with her least preferred policy outcome. Lemma 3 in the Appendix rules out this possibility and shows that in any voting subgame of any electoral equilibrium, voters vote for their preferred alternative. This result simplifies considerably the characterization of electoral equilibria, assuring uniquely determined, smooth and well-behaved vote share functions for all parties on and off the equilibrium path.

The second consideration brought by multiparty competition in non-majoritarian elections concerns the incentives to spend in campaigning and is substantially more involved. The first point to note is that for any number K of parties competing in the election, small changes in party k's campaign spending only lead to changes in the distribution of votes between k and its two "effective" competitors, one to each side of the policy spectrum. *Given the identity* of k's relevant competitors at a particular campaign spending profile, the marginal impact of k's campaign spending on vote shares is always local in nature and therefore well defined. In particular, the marginal benefit of campaign spending increases the larger is voters' responsiveness to campaign spending, the less differentiated k is on average with regard to his effective competitors, and given this, the less symmetric is k's differentiation with regard to his relevant competitors.

The complication arises because the identity of k's effective competitors will not necessarily remain fixed at different campaign spending profiles, and in particular, it will not always coincide with that of k's closest neighbors. But since closer parties in the policy space are better substitutes for each other, changes in party k's campaign spending will have a stronger impact on how voters rank k relative to its closest competitors than to more distant parties in the policy space. As a result, changes in the identity of a party's relevant competitors will lead to non-differentiabilities in the mapping from campaign spending to vote shares and discontinuities in the marginal vote share mapping.

In Proposition 3, however, we show that under some conditions, the action identified as optimal by the first-order condition will indeed be a best response. In particular, we prove that this is true for all strategy profiles with $K \geq 3$, where all parties representing *interior* positions $k = 2, \ldots, K-1$ choose equal campaign spending, i.e., $\theta_k^* = \theta^*$ for all $k = 2, \ldots, K-1$. The intuition for this result is that with symmetry, the discontinuities described above occur at levels of campaigning that are all larger than the optimal solution and thus never reached in a best response (the interested reader is referred to "Appendix B" for a more detailed discussion).

To state this formally, we define a class of electoral equilibria in which all parties contesting the election are located at the same distance to their closest neighbors. We call equilibria of this class location-symmetric (LS) electoral equilibria.

Definition 1 An electoral equilibrium is a location-symmetric (LS) electoral equilibrium if $x_{k+1} - x_k = \Delta$ for any $k < K$, and $x_1 = 1 - x_K$.

The next lemma shows that the non-differentiabilities in the mapping of campaign investment to vote shares discussed above are not relevant in a LS equilibrium.

Lemma 1 *Consider a LS equilibrium with $K \geq 3$ parties contesting the election such that $\theta_k^* < L$ for all k. Then parties' equilibrium campaign spending is given by*

$$\theta_k^* = \Psi^{-1}(\Delta/2\alpha) \quad \textit{for all} \quad k = 2, \ldots, K-1 \quad \textit{and} \quad \theta_1^* = \theta_K^* = \Psi^{-1}(\Delta/\alpha). \quad (4)$$

Notice that in any LS equilibrium, it will always be the case that $\theta_1^* = \theta_K^*$ and that $\theta_k^* = \theta^*$ for all $k = 2, \ldots, K-1$.

Building on Lemmas 1 and 3—and letting $\underline{a} \equiv C(L)/2\Psi(L)$ and $\overline{a} \equiv C(L)/\Psi(L)$—the next result extends Propositions 1 and 2 to the case of multiple parties.

Proposition 3 *Take an integer $K \geq 3$ as given. If $K \times C(L) \leq 1$, there exists a short-run LS equilibrium with K parties. Moreover, for any α, there is a short-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ such that*

1. *If $\alpha \leq \underline{a}$, then all parties are campaign unconstrained, i.e., $\theta_1^* = \Psi^{-1}(\Delta^*(\alpha)/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta^*(\alpha)/2\alpha) < L$. Further, for every party k campaign, spending is decreasing with ideological differentiation and voters' ideological focus.*
2. *If $\underline{a} \leq \alpha \leq \overline{a}$, then only parties representing interior ideological positions are constrained by campaign spending caps, i.e., $\theta_1^*(\alpha) = \Psi^{-1}(\Delta^*(\alpha)/\alpha) < \theta^*(\alpha) = L$.*
3. *If $\alpha \geq \overline{a}$, then all parties are campaign constrained, i.e., $\theta_1^*(\alpha) = \theta^*(\alpha) = L$.*

Together with the results of Sect. 3.1, Proposition 3 implies that for any integer $K \geq 2$ such that $K \times C(L) \leq 1$, there exists a short-run electoral equilibrium with K parties. This result generalizes our previous conclusions and yields two key implications.

First, note that—differently to a model in which the number of parties is given, in which limiting campaign spending can possibly increase but never reduce parties' rents—here tightening the campaign spending cap can lead to a larger number of parties in equilibrium, increasing competitive pressures as a result. By the same logic, relaxing the campaign spending cap leads (eventually) to less parties, reducing the set of alternatives available to voters, and the competitiveness of the election. In fact, as we showed in the previous section for the specific case of two parties, increasing the spending cap can possibly eliminate competition altogether. This is indeed true for any K: we can find an α large enough such that if the spending cap L grows large, the unique short-run equilibrium has a single party not investing in campaigning.[8] Second, note that the conditions constraining the number of parties in equilibrium are entirely determined by supply side factors: the cost of campaigning at the spending cap, $C(L)$. Instead the responsiveness of voters to campaigning plays no role in defining the number of parties that can be competing for votes in a short-run equilibrium.

The uncoupling of supply and demand factors in the determination of the number of parties does not mean, of course, that "demand-side" factors are irrelevant for equilibrium. Instead, the level of voters' ideological focus impacts the intensity of campaign competition between the *given* number of parties competing for votes in the election. In particular, for any given spending cap L, if voters are sufficiently ideological (α is low), parties will be unconstrained by spending caps in equilibrium. As voters' ideological focus diminishes, first centrist candidates and eventually all candidates will hit campaign constraints. What makes this possible is that in these equilibria, parties earn strictly positive rents from participating in the electoral competition. This allows the system enough flexibility so that the number of parties in the election can be independent of demand-side factors: as voters become more responsive to campaign spending, campaign competition becomes tighter and candidates "compete away" their rents. The campaign finance limits prevent complete rent dissipation. We establish this formally in the next corollary.

Corollary 1 *Take $K \geq 3$ as given and assume $K \times C(L) \leq 1$. Then for any α, there is a short-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with associated rents $(\Pi_1^*(\alpha), \Pi^*(\alpha))$ for parties representing extreme and interior ideological positions respectively, such that*

1. $\Pi_1^*(\alpha)$ *is strictly decreasing for all $\alpha < \overline{a}$, and positive and constant for all $\alpha > \overline{a}$.*
2. $\Pi^*(\alpha)$ *is strictly decreasing for all $\alpha < \underline{a}$, and positive and constant for all $\alpha > \underline{a}$.*

It is worth noticing that the mere presence of campaign limits does not prevent complete rent dissipation in equilibrium. Indeed, we can show that there exists an a' such that for all $\alpha < a'$ there exists a short-run LS equilibrium with at least three parties in which $\theta_k^*(\alpha) < L$ for all k, and all interior parties earn zero rents. This result follows from the second part of Lemma 6 in the appendix.

3.3 Electoral competition in long-run equilibria

So far we focused on short-run electoral equilibria, in which incumbent parties cannot be challenged by new parties, even if they represent poor alternatives for a large fraction of voters. In a longer time horizon, however, we expect entry to be a relevant factor in shaping electoral outcomes. In this section, we extend the analysis

[8] The logic behind this result is very similar to the case of two parties and exploits the property that in any equilibrium of an all-pay auction with any number of players and identical valuations, there is complete rent dissipation (see Theorem 1 of Baye et al. 1996).

to *long-run* electoral equilibria. A strategy profile is a long-run electoral equilibrium (or simply an electoral equilibrium) if it is a short-run electoral equilibrium, and in addition parties not participating would obtain negative rents if they chose to enter the electoral competition.

Do our previous short-run results stand in this long-run setting? Remarkably, the answer is an unqualified yes. As in a short-run equilibrium, it is still the case that increasing the spending cap reduces the maximum number of parties that can be contesting the election. In the long-run analysis, in addition, the fixed cost of entry provides a lower bound on the equilibrium number of parties.[9] Still, as before, there are always conditions for which the responsiveness of voters to campaigning plays no role in defining the number of parties that can be competing for votes in equilibrium. In particular, the uncoupling of supply and demand for the determination of the equilibrium number of parties is still valid in the long-run equilibrium analysis: the equilibrium number of parties is determined entirely by the variable cost of campaigning at the spending cap, $C_v(L)$, and the fixed cost of entry F. Finally, reducing voters' ideological focus has the effect of increasing the intensity of campaign competition and reducing parties' rents. As in Corollary 1, it is the excess rents in equilibrium which allows the uncoupling of demand and supply side in the determination of the equilibrium number of parties. These results follow from Theorem 1 below.

Theorem 1 *Take $K \geq 2$ as given. If $K \times C(L) \leq 1$ and $K \times F \geq 1/2$, there exists a long-run electoral equilibrium with K parties. Moreover, there exist thresholds $(\underline{\alpha}, \overline{\alpha})$ and, for any α, either a long-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ or a two-party electoral equilibrium $(x_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ such that*

1. *If $\alpha \leq \underline{\alpha}$, then all parties campaign unconstrained, and for every party k, campaign spending is decreasing in parties' ideological differentiation. In particular, $\theta_1^* = \Psi^{-1}(\Delta^*(\alpha)/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta^*(\alpha)/2\alpha) < L$ for $K > 2$, and $\theta^* = \Psi^{-1}(\Delta^*(\alpha)/\alpha)$ for $K = 2$.*
2. *If $\alpha \geq \overline{\alpha}$ all parties are campaign constrained.*

The main logic driving the results of Theorem 1 is best grasped by considering the special case of two parties unconstrained in campaigning. This is covered in Lemma 2 below.

Lemma 2 *Suppose that $2 \times C(L) \leq 1$ and $2 \times F \geq 1/2$. Then for any $\alpha \leq \overline{\alpha}_{2P}$ there exists a long-run two-party electoral equilibrium $(x_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ such that both parties campaign unconstrained, and campaign spending is decreasing in parties' ideological differentiation. In particular, $\theta^* = \Psi^{-1}(\Delta^*(\alpha)/\alpha) < L$.*

Proof of Lemma 2 We showed in Proposition 1 that if two parties compete for votes in the election, and (i) voters vote for their preferred party, (ii) both parties are unconstrained in campaign spending and choose

$$\theta_k^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right),$$

[9] Furthermore, the long-run analysis introduces new bounds on ideological differentiation between parties in equilibrium. We elaborate on this at the end of this section.

and (iii) parties' ideological differentiation $\Delta \geq \alpha\Psi(L)$ (with $C(L) \leq 1/2$), then there is a location of the left party x_1 such that a short-run electoral equilibrium exists. We next show that if conditions (i) and (ii) are satisfied, and parties' ideological differentiation $\Delta \in D^*$, where

$$D^* \equiv \{\Delta : \max\{2C_v(L), 1-2F, \alpha\Psi(L)\} \leq \Delta \leq \min\{1-2C_v(L), 2C(L)\}\},$$

then there is a location of the left party x_1 such that a long-run electoral equilibrium exists. First notice that for D^* to be nonempty, it is sufficient that

$$\alpha \leq \min\left\{2\frac{C(L)}{\Psi(L)}, \frac{1-2C_v(L)}{\Psi(L)}\right\} \equiv \overline{\alpha}_{2P},$$

and $C(L) = C_v(L) + F \leq 1/2$ and $F \geq 1/4$ (which in turn implies $C_v(L) \leq F$). Hence, if $2 \times C(L) \leq 1, 2 \times F \geq 1/2$, and $\alpha \leq \overline{\alpha}_{2P}$ a long-run two-party electoral equilibrium exists. We now show why these conditions are sufficient.

Let $\Delta = \alpha\Psi(L)(1+\varepsilon)$ for $\varepsilon > 0$. First we show that entry in (x_1, x_2) is not profitable. Suppose that j enters at $x_j \in (x_1, x_2)$ and consider the following continuation: $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = L$. Letting $\delta_j^r \equiv (x_2 - x_j)/\Delta$, we have that the necessary first-order conditions (FOC) for a maximum for $k = 1$ and $k = 2$ are

$$\frac{\alpha}{(1-\delta_j^r)\Delta} v'(L) \geq C_v'(L) \quad \text{and} \quad \frac{\alpha}{\delta_j^r \Delta} v'(L) \geq C_v'(L),$$

while the FOC for j is $\alpha v'(L) \geq \delta_j^r(1-\delta_j^r)\Delta C_v'(L)$ which is implied by the previous inequalities. These conditions are satisfied if and only if

$$\max\{\delta_j^r \Delta, (1-\delta_j^r)\Delta\} \leq \alpha\Psi(L). \tag{5}$$

Suppose first that $\delta_j^r \leq 1/2$. Then (5) is $(1-\delta_j^r)\Delta \leq \alpha\Psi(L)$, or substituting $\Delta = \alpha\Psi(L)(1+\varepsilon)$, $\delta_j^r \geq \varepsilon/(1+\varepsilon)$. When instead $\delta_j^r \geq 1/2$, then (5) is $\delta_j^r \Delta \leq \alpha\Psi(L)$, or substituting $\Delta = \alpha\Psi(L)(1+\varepsilon)$, $\delta_j^r \leq 1/(1+\varepsilon)$. Thus,

$$\frac{\varepsilon}{1+\varepsilon} \leq \delta_j^r \leq \frac{1}{1+\varepsilon}. \tag{6}$$

Note that since $\varepsilon > 0$, the interval defined in (6) is strictly included in (0, 1). Thus for relatively centrist entrants, $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = L$ is a joint best response provided that the incumbent parties choose not to quit campaigning. To insure that this is the case, it is enough to consider the case of δ_j^r approaching either 0 or 1.[10] Hence, a necessary

[10] The reason for this is that when δ_j^r reaches say its lower bound (i.e., when entry occurs in a right neighborhood of x_1), than necessarily $\hat{\theta}_1 = \hat{\theta}_j = L$ is optimal. Hence, the incumbent rents approaches x_1, which must be larger than $C_v(L)$ in order for party 1 to prefer not to quit campaigning. When instead δ_j^r is strictly bigger than zero, either $\hat{\theta}_1 = L$ is still optimal and necessarily $\Pi_1(\hat{\theta}_1)|_{\hat{\theta}_1 = L} \geq x_1$, or if $\hat{\theta}_1 < L$ is optimal then it must be that $\Pi_1(\hat{\theta}_1)|_{\hat{\theta}_1 < L} > \Pi_1(\hat{\theta}_1)|_{\hat{\theta}_1 = L} \geq x_1$.

and sufficient condition for party 1 not to quit campaigning upon entry is $x_1 \geq C_v(L)$. The case of δ_j^r reaching its upper bound is similar and yields $1 - x_2 \geq C_v(L)$ as a necessary and sufficient condition for party 2 to prefer not to quit campaigning. Since $x_1 + \Delta + 1 - x_2 = 1$, these conditions can be satisfied if and only if $\Delta \leq 1 - 2C_v(L)$.

Now, given that $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = L$ we have that $\Pi_j(\hat{\theta}_j) = \Delta/2 - C(L) < 0$ if and only if $\Delta < 2C(L)$. Next, consider entry such that $\delta_j^r > 1/(1+\varepsilon)$. Here j enters relatively close to $k = 1$, and the strategy profile in the continuation game in which all three parties choose L cannot be an equilibrium. Consider instead $\hat{\theta}_2 \in (0, 1)$, and $\hat{\theta}_1 = \hat{\theta}_j = L$. The necessary first-order condition for $k = 2$ is

$$\hat{\theta}_2 = \Psi^{-1}\left(\frac{\delta_j^r \Delta}{\alpha}\right) = \Psi^{-1}\left(\delta_j^r \Psi(L)(1+\varepsilon)\right) < L,$$

where the second equality follows from $\Delta = \alpha\Psi(L)(1+\varepsilon)$, and the inequality follows from the fact that $\delta_j^r > 1/(1+\varepsilon)$, and that $\Psi(\cdot)$ is decreasing. The FOC for j is not relevant. The FOC for $k = 1$ is $(1 - \delta_j^r)\Delta \leq \alpha\Psi(L)$, which is implied by $\delta_j^r > 1/(1+\varepsilon)$. We now need to show that

$$\Pi_j(\hat{\theta}_j) = \tilde{x}_{j2}(L, \hat{\theta}_2) - \frac{x_1 + x_j}{2} - C_v(L) - F < 0.$$

Now if $\tilde{x}_{1j}$ were fixed, j would be better off choosing $\tilde{\theta}_j = \hat{\theta}_2$ (as in the case of a two-parties equilibrium). But then $\hat{\Pi}_j < \Delta/2 - C_v(L) - F < 0$ from $\Delta < 2(C_v(L) + F)$. As before, we need to make sure that the incumbent parties choose not to quit campaigning in the continuation game. However, from the previous discussion, we know that there exist parameters values for which incumbent parties will not quit campaigning as long as $\Delta \leq 1 - 2C_v(L)$.

To deter entry in $[0, x_1)$ and $(x_2, 1]$ it is sufficient that (1) $\Delta \geq 2C_v(L)$ and (2) $\Delta \geq 1 - 2F$. Condition (1) guarantees that the incumbents are not quitting campaigning upon entry. Condition (2) is a sufficient condition for the existence of x_1 and x_2 such that $\max\{x_1, 1 - x_2\} \leq F$. The latter inequality is clearly enough to guarantee no entry of an extreme party, and since $x_1 + \Delta + 1 - x_2 = 1$, it can be satisfied if and only if $\Delta \geq 1 - 2F$. □

We conclude this section with two observations. First, while Theorem 1 refers to the case of $K \geq 2$, extending the result to a single party is straightforward. First, it is immediate to verify that there always exists a *short-run* electoral equilibrium with one party running uncontested. Furthermore, in the case of a single party, the condition $K \times F \geq 1/2$ becomes $F \geq 1/2$. Hence, if the incumbent is located at the preferred position of the median voter, $x = 1/2$, the potential entry of a challenger is always deterred. To see why this must be the case, note that the continuation game after entry of a challenger must be a two-party equilibrium, and therefore campaign competition will be neutral for electoral outcomes in that continuation game. Since the incumbent is located at the median, it follows that the challenger's vote share will always be smaller than 1/2. But then $F \geq 1/2$ implies that the entrant would earn negative total rents.[11]

Second, when voters are sufficiently responsive to campaign spending, some of the sufficient conditions we used in the proof of Theorem 1 become necessary. We can then show that as L increases the set of long-run two-party electoral equilibria converges to the equilibrium that maximizes *ideological representation*, i.e., the location of parties that minimizes the total distance between voters' ideological preferences and parties' positions. This finding echoes a well-known result in industrial organization since $x_1 = 1 - x_2 = 1/4$ is the socially optimal location of two competing shops in the unit interval, i.e., the one that minimizes buyers' transportation costs (see Hotelling 1929).

Proposition 4 *Suppose that the conditions of Theorem 1 are satisfied for $K = 2$. Then there exists an α^{**} such that for all $\alpha > \alpha^{**}$, as L increases the set of long-run two-party electoral equilibria converges to a single equilibrium $(x_1^{**}(\alpha), \Delta^{**}(\alpha), \theta^{**}(\alpha))$. In this equilibrium, parties are positioned in the unique symmetric configuration that maximizes voters' ideological representation.*

To sum up, in this section, we uncovered an important interaction between campaign spending and the competitiveness of the political arena. In particular, we argued that the overall competitiveness of the political sector depends on both the endogenous number of parties contesting the election and the endogenous level of campaign competition. Our model shows that these two dimensions are linked together through their combined effect on the total equilibrium level of rents. Campaign finance regulation crucially affects this mechanism. We have shown that regulating campaign spending may avoid a perverse anti-competitive effect that is due to the fact that increasing spending caps heightens campaign competition, reduces political rents, and as a result reduces the number of parties contesting the election.

4 Conclusion

After the last decades of advances in the theory of industrial organization, it is now evident that putting together price competition with entry of new firms is key for the analysis of regulatory and antitrust policies. Putting together campaign competition with entry of new parties is no less important for the regulation of electoral politics.

In this paper, we studied a model of elections in non-majoritarian systems that captures the link between competition in policies and competition in campaign spending. The main thrust of the paper is that it is crucial to consider the overall competitiveness of elections—resulting from both ideological differentiation and campaigning—when evaluating policies designed to regulate electoral competition. We illustrate this point analyzing the effect of tightening/relaxing campaign spending limits. We show that increasing the spending cap can reduce the set of alternatives available to voters. This result relies completely on the level of political rents up for grabs. In fact, we

[11] Note that since the incumbent obtains more than half of all votes, and $C_v(\theta) \leq 1/2$ for all $\theta \leq L$ by hypothesis, the incumbent has an incentive to campaign in any continuation game following entry.

show that this happens even when the equilibrium number of parties is unaffected by the responsiveness of voters to political campaigning.

The paper opens several avenues for future research. While in this paper we confined our analysis to abstract non-majoritarian electoral systems, the analysis can be extended to other electoral institutions (see Iaryczower and Mattozzi 2009). In addition, extending our static model to a dynamic framework would allow us to tackle how term limits and incumbency influence electoral outcomes through their effect on total political rents. The model also has important implications for applied research. Besides delivering a number of novel empirical implications, our analysis makes clear that generically, campaign spending and entry should be treated as jointly determined in equilibrium.

Appendix A: Proofs

Proof of Proposition 1 Recall that $\theta_j \in [0, L]$ denotes party j's feasible campaign spending. Consider a candidate equilibrium strategy profile in which campaign constraints are not binding, i.e., $\max\{\theta_1^*, \theta_2^*\} < L$. Recall that we defined $\tilde{x}_{12} \in \mathcal{R}$ as the unique value of x such that $u(\theta_1, x_1; x) = u(\theta_2, x_2; x)$, where $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$. Note that if $\tilde{x}_{12} \in (0, 1)$ the vote share mapping $m_1(\theta_1; \theta_2, x)$ is differentiable and the marginal vote share is given by

$$\frac{\partial m_1}{\partial \theta_1} = \frac{\alpha v'(\theta_1)}{\Delta},$$

where we defined $\Delta \equiv x_2 - x_1$. The fact that campaign constraints are not binding implies that the necessary first-order condition must be satisfied with equality, i.e. $\alpha v'(\theta_k^*) = \Delta C_v'(\theta_k^*)$ for $k = 1, 2$ or, given $\Psi(\cdot) \equiv v'(\cdot)/C_v'(\cdot)$,

$$\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1}(\Delta/\alpha). \tag{7}$$

This implies that in any equilibrium with two parties in which they are not campaign constrained, there cannot be a differential in campaign investments. Moreover, it also implies that in any two-party equilibrium in which parties are not campaign constrained, they must be sufficiently ideologically differentiated, i.e. $\Delta \geq \alpha\Psi(L)$.[12]

Since $\theta_1^* = \theta_2^*$, candidates' vote shares are $m_1 = x_1 + \Delta/2$, and $m_2 = 1 - x_1 - \Delta/2$, and therefore $\Pi_1^* = x_1 + \Delta/2 - C(\theta^*)$ and $\Pi_2^* = 1 - x_1 - \Delta/2 - C(\theta^*)$ are the equilibrium rents, where recall $C(\theta) = C_v(\theta) + F$. The equilibrium requirement that parties earn non-negative rents implies that $(x_1, \Delta) \in A$, where

$$A \equiv \{(x_1, \Delta) : C(\Psi^{-1}(\Delta/\alpha)) - \Delta/2 \leq x_1 \leq 1 - \Delta/2 - C(\Psi^{-1}(\Delta/\alpha))\}.$$

[12] For any given θ_2, 1's vote share mapping $m_1(\theta_1; \theta_2, x)$ has two kinks, one at $\underline{t}$ such that $m_1(\underline{t}; \theta_2, x) \equiv 0$ and one at $\bar{t}$ such that $m_1(\bar{t}; \theta_2, x) \equiv 1$. In fact $\underline{t} = v^{-1}\left(v(\theta_2) - \Delta^2/\alpha\right) < \theta_2$ and $\bar{t} = v^{-1}(v(\theta_2) + \Delta(1 - \Delta)/\alpha) > \theta_2$. Thus, marginal rent is well defined, continuous and decreasing at all points $\theta_1 \in (\underline{t}, \bar{t})$. Since the condition for non-negative rents is also imposed for equilibrium, we know that $\theta_1^* = \theta^*$ is indeed a best response.

Note that if there exist some pair of candidate locations $(x_1, \Delta) \in A$, then the symmetric configuration $x_1' = 1 - x_1$ belongs to A as well. It then follows from the above inequalities that the set A is non-empty (i.e., there exists a pair of candidate locations (x_1, Δ) such that both parties earn non-negative rents in equilibrium) if and only if $C(\theta^*) \leq 1/2$. Now, suppose first that $C(L) \leq 1/2$. Then $C(\theta) \leq 1/2$ for all θ, and thus the fact that A is non-empty follows immediately. Thus, the necessary condition for interior campaigning $\alpha\Psi(L) \leq \Delta$ is the binding constraint. As a result, when $C(L) \leq 1/2$ there is an equilibrium with two parties unconstrained in campaigning as long as $\alpha\Psi(L) \leq 1$. So suppose instead that $C(L) > 1/2$. Substituting θ^*, we can write $C(\theta^*) \leq 1/2$ as $\Delta > \alpha\Psi(C^{-1}(1/2))$. Thus, there is an equilibrium with two parties unconstrained in campaigning if $\alpha\Psi(L) \leq 1$ and $\alpha\Psi(C^{-1}(1/2)) \leq 1$. But note that $C(L) > 1/2$ implies that $\alpha\Psi(C^{-1}(1/2)) > \alpha\Psi(L)$. Thus, when $C(L) > 1/2$, the requirement of non-negative rents is the binding constraint, and there is an equilibrium with two parties unconstrained in campaigning if $\alpha\Psi(C^{-1}(1/2)) \leq 1$.

Finally, to obtain the lower bound on differentiation, $\underline{\Delta}$, note that we have shown that when $C(L) \leq 1/2$, the binding constraint on differentiation is $\Delta \geq \underline{\Delta} \equiv \alpha\Psi(L)$, and when $C(L) > 1/2$, the binding constraint on differentiation is $\Delta > \underline{\Delta} \equiv \alpha\Psi(C^{-1}(1/2))$. Thus $\underline{\Delta}$ is increasing in α. □

Proof of Proposition 2 Consider first $C(L) \leq 1/2$. Then $b = 1/\Psi(L)$ and $\alpha > b$ implies that there does not exist $\Delta \in (0, 1)$ such that $\Delta \geq \alpha\Psi(L)$. It follows that in any equilibrium $\alpha\Psi(L) > \Delta$, and therefore—as long as both parties earn non-negative rents when constrained in campaigning—it must be that $\theta_1^* = \theta_2^* = L$. Recall that the condition $C(L) \leq 1/2$ guarantees that the set A defined in the proof of Proposition 1 is non-empty. In particular, then, if $C(L) \leq 1/2$ and $\alpha\Psi(L) > 1$ (i.e., $\alpha > b$) then in all equilibria the two parties are campaign constrained. Consider now the case of $C(L) > 1/2$. Then, $b = 1/\Psi(C^{-1}(1/2))$ and $\alpha > b$ implies that there does not exist $\Delta \in (0, 1)$ such that $\Delta \geq \alpha\Psi(C^{-1}(1/2))$. But $\Delta < \alpha\Psi(C^{-1}(1/2))$ is equivalent to $C(\theta^*) > 1/2$. It follows that if $C(L) > 1/2$ and $\alpha > b$ the set A is empty, and there cannot be an equilibrium in pure strategies with more than one party. □

Lemma 3 *In (any voting subgame of) any electoral equilibrium, voters vote for their preferred candidate.*

Proof of Lemma 3 Suppose voter i's preferred party is $k^*(i) \in \mathcal{K}$, and that $\tilde{k} \in \mathcal{K}$ and $\tilde{k} \neq k^*(i)$. Let $t_k(\sigma_{-i}^v)$ denote the number of votes for party k given a voting strategy profile σ_{-i}^v for all voters other than i. The payoff for i of voting for $\tilde{k}$ given σ_{-i}^v, $U(\tilde{k}; \sigma_{-i}^v)$, is

$$\sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{[t_{\tilde{k}}(\sigma_{-i}^v) + 1]}{N} u(x_{\tilde{k}}; z^i) + \frac{t_{k^*(i)}(\sigma_{-i}^v)}{N} u(x_{k^*(i)}; z^i).$$

Similarly, the payoff for i of voting for $k^*(i)$ given σ_{-i}^v, $U(k^*(i); \sigma_{-i}^v)$, is

$$\sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{t_{\tilde{k}}(\sigma_{-i}^v)}{N} u(x_{\tilde{k}}; z^i) + \frac{[t_{k^*(i)}(\sigma_{-i}^v) + 1]}{N} u(x_{k^*(i)}; z^i).$$

Thus

$$U(k^*(i);\sigma^v_{-i}) - U(\tilde{k};\sigma^v_{-i}) = \frac{1}{N}[u(x_{k^*(i)};z^i) - u(x_{\tilde{k}};z^i)],$$

which is positive by definition of $k^*(i)$. Since σ^v_{-i} was arbitrary, this shows that voting sincerely strictly dominates voting for any other available party and is thus a dominant strategy for voter i. It follows that in all Nash equilibria in the voting stage, voters vote sincerely among parties contesting the election. □

Proof of Lemma 1 Consider first interior parties $k = 2, \ldots, K-1$ and note that $m_k(\theta, x) \geq 0$ if and only if

$$\theta_k \geq v^{-1}\left(\left(\delta^r_k v(\theta_{\ell(k)}) + (1-\delta^r_k)v(\theta_{r(k)})\right) - \frac{\delta^r_k(1-\delta^r_k)(\Delta^T_k)^2}{2\alpha}\right) \equiv \underline{\theta}_k(\theta_{-k}, x). \quad (8)$$

But then $\underline{\theta}_k(\theta_{-k}, x) < \theta$ if $\theta_{\ell(k)} = \theta_{r(k)} \leq \theta$, and this is always the case in a LS equilibrium with $\theta_k < L$ for interior parties whose neighbors are themselves interior parties since $\theta_{\ell(k)} = \theta_{r(k)} = \theta_k$. Similarly, for interior parties with one extreme neighbor, say $k = K-1$, it must be that $\theta_k = \theta_{\ell(k)} > \theta_{r(k)}$ ($\theta_k = \theta_{r(k)} > \theta_{\ell(k)}$ for $k = 2$). For extreme parties, this is also true since in equilibrium they earn non-negative rents, which can only happen if they choose a campaign investment above the lower discontinuity point. Now consider the upper discontinuity point and let $j > \ell > k$. We want to show that if $\theta_r = \theta$ for all $r \neq k$, then k's best response $\theta_k(\theta_{-k})$ is lower than the point $D^{\ell,j}_k(\theta_{-k})$ at which $\tilde{x}_{k\ell} = \tilde{x}_{j\ell}$. Recall that for $r > k$, $\tilde{x}_{kr} = (x_k + x_r)/2 + \alpha(v(\theta_k) - v(\theta_r))/|x_r - x_k|$. After some algebra, we obtain

$$D^{\ell,j}_k(\theta_{-k}) = v^{-1}\left(\frac{|x_\ell - x_k||x_j - x_k|}{2\alpha} - \frac{|x_\ell - x_k|v(\theta_j) - |x_j - x_k|v(\theta_\ell)}{x_j - x_\ell}\right) \quad (9)$$

and if $\theta_r = \theta$ for all $r \neq k$, (9) simplifies to

$$D^{\ell,j}_k(\theta) = v^{-1}\left(v(\theta) + \frac{|x_\ell - x_k||x_j - x_k|}{2\alpha}\right) > \theta.$$

Hence, it follows from (8) and (9) that the vote share is differentiable in $[\underline{\theta}_k(\theta, x), D_k(\theta)]$. The same logic holds for extreme parties. Suppose all interior parties $k = 2, \ldots, K-1$ choose in equilibrium $\theta^* < L$. Consider k's problem. Note that since $\theta^*_j = \theta^*_r$ for all $j, r \neq k$, then k's FOC is given by $2\alpha v'(\theta^*_k) = \Delta C'_v(\theta^*_k)$. By our previous argument, this is well defined, and since the marginal vote share is decreasing above $\underline{\theta}_k(\theta, x)$, then the sufficient second order condition is satisfied. Therefore

$$\theta^*_k = \theta^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \quad \text{for all } k = 2, \ldots, K-1. \quad (10)$$

Finally, given that interior parties are choosing $\theta_k < L$, then optimal campaign spending by extreme parties must be strictly smaller that L as well. In particular, it must be that $\theta_1^* = \theta_K^* = \Psi^{-1}(\Delta/\alpha)$. This completes the proof. □

Proof of Proposition 3 Fix $K \geq 3$, and assume that $K \times C(L) \leq 1$. Lemma 4 shows that if $\alpha \geq \overline{a} \equiv C(L)/\Psi(L)$, there exists a short-run LS equilibrium with $K \leq 1/C(L)$ parties, such that $\theta_1^*(\alpha) = \theta^*(\alpha) = L$. Recall that in a LS equilibrium, we have $x_{k+1} - x_k = x_{j+1} - x_j \equiv \Delta$ for any $k = 1, \ldots, K-1$ and $j = 1, \ldots, K-1$. Lemma 5 shows that if $C(L)/2\Psi(L) \equiv \underline{a} \leq \alpha \leq \overline{a}$, there exists a short-run LS equilibrium with $K \leq 1/C(L)$ parties in which $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = L$, Lemma 6 shows that if $\alpha \leq \underline{a}$, there exists a short-run LS equilibrium with $K \leq 1/C(L)$ parties in which $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta/2\alpha) < L$. □

Lemma 4 *Take $K \geq 3$ as given and suppose $K \times C(L) \leq 1$. Then for any $\alpha \geq \overline{a}$, there exists a short-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with K parties such that $\theta^*(\alpha) = \theta_1^*(\alpha) = L$, and all parties earn positive rents.*

Proof of Lemma 4 Suppose that in equilibrium $\theta_k^* = L$ for $k = 2, \ldots, K-1$. For L to be optimal for k it must be that the marginal vote share given that the other parties are also choosing L is higher than the marginal cost at L; i.e., $2\alpha v'(L)/\Delta \geq C_v'(L)$, or $\Delta \leq 2\alpha\Psi(L)$. For non-negative rents, we must have $\Pi_k^* = \Delta - C(L) \geq 0$, or $\Delta \geq C(L)$. Now consider the extreme parties. For $\theta_1^* = \theta_K^* = L$, it is necessary that $\Delta \leq \alpha\Psi(L)$. For non-negative rents it is necessary that $\Pi_1^* = x_1 + \Delta/2 - C(L) \geq 0$, and since $\Delta \geq C(L)$ it is sufficient that $x_1 \geq \Delta/2$. Now $2x_1 + (K-1)\Delta = 1$, so $x_1 = (1 - (K-1)\Delta)/2$. Substituting, $x_1 \geq \Delta/2$ becomes $\Delta \leq 1/K$. Putting everything together implies that in equilibrium $\Delta^* \in A_T$, where

$$A_T \equiv \{\Delta : C(L) \leq \Delta \leq \min\{\alpha\Psi(L), 1/K\}\}$$

The set A_T is non-empty if and only if $K \times C(L) \leq 1$ and $\alpha \geq C(L)/\Psi(L) = \overline{a}$. Finally, note that if Δ^* is in the interior of A_T then all parties earn positive rents. □

Lemma 5 *Take $K \geq 3$ as given and suppose $K \times C(L) \leq 1$. Then for any $\alpha : \underline{a} \leq \alpha \leq \overline{a}$, there exists a short-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with K parties such that $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = L$, and all parties earn positive rents.*

Proof of Lemma 5 The first part of the proof is identical to the proof of Lemma 4. For L to be optimal for $k = 2, \ldots, K-1$ it must be that $\Delta \leq 2\alpha\Psi(L)$, and for non-negative rents for interior parties, we must have $\Delta \geq C(L)$. For extreme parties to choose interior campaign spending, i.e., $\theta_1^* = \theta_K^* = \Psi^{-1}(\Delta/\alpha) < L$, it must be that $\Delta > \alpha\Psi(L)$. For non-negative rents, we need

$$\Pi_1^* = \Pi_K^* = x_1 + \frac{\Delta}{2} - \frac{\alpha}{\Delta}[v(L) - v(\theta_1^*)] - C(\theta_1^*) \geq 0.$$

Since θ_1^* maximizes $\Pi_1(\theta_1)$, then $\Pi_1(\theta_1^*) \geq \Pi_1(\theta_1)$ for all $\theta_1 \neq \theta_1^*$, and thus it is enough to show that $\Pi_1(L) \geq 0$. But this is $x_1 + \Delta/2 \geq C(L)$, which holds whenever

$x_1 \geq \Delta/2$. As before, this implies $\Delta \leq 1/K$. Putting everything together implies that in equilibrium $\Delta^* \in A_M$, where

$$A_M \equiv \{\Delta : \max\{C(L), \alpha\Psi(L)\} \leq \Delta \leq \min\{2\alpha\Psi(L), 1/K\}\}$$

The set A_M is non-empty if and only if (1) $K \leq 1/C(L)$, (2) $\alpha \geq C(L)/2\Psi(L)$, and (3) $K \leq 1/\alpha\Psi(L)$. But $\alpha \leq \overline{a}$ implies that $C(L) \geq \alpha\Psi(L)$, and the result follows. As in the previous lemma, note that if Δ^* is in the interior of A_M, then all parties earn positive rents. □

Lemma 6 *Take $K \geq 3$ as given, and suppose $K \times C(L) \leq 1$. Then*

1. *For any $\alpha \leq \underline{a}$, there exists a short-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with K parties such that $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta/2\alpha) < L$, and all parties earn positive rents.*
2. *For any $\alpha \leq 1/(6\Psi(C_v^{-1}(1/3 - F)))$, there exists a short-run LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with at least three parties in which $\theta_k^*(\alpha) < L$ for all k, and all interior parties earn zero rents.*

Proof of Lemma 6 Consider first the interior parties $k = 2, \ldots, K-1$. If $\theta_j^* = \theta_r^* < L$ for all $j, r \neq k$, then Lemma 1 implies that k's marginal vote share is differentiable, and k's FOC is given by $2\alpha v'(\theta_k^*)/\Delta = C_v'(\theta_k^*)$. Therefore,

$$\theta_k^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \quad \text{for all } k = 2, \ldots, K-1.$$

Moreover, since $\theta^* \leq L$, it must be that $\Delta \geq 2\alpha\Psi(L)$. Non-negative rents for interior parties require that $\Pi_k^* = \Delta - C(\theta^*) \geq 0$ or $\theta^* \leq C_v^{-1}(\Delta - F)$. Substituting θ^* we get $\Delta \geq 2\alpha\Psi(C_v^{-1}(\Delta - F))$. And note that $2\alpha\Psi(C_v^{-1}(\Delta - F)) \geq 2\alpha\Psi(L)$ if and only if $\Delta \leq C(L)$. Then, as long as in equilibrium $\Delta \leq C(L)$ (i.e., $\Pi_k(L) \leq 0$ for $k = 2, \ldots, K-1$), $\Delta \geq 2\alpha\Psi(C_v^{-1}(\Delta - F))$ implies $\Delta \geq 2\alpha\Psi(L)$. That is, if interior parties earn non-negative rents, they are choosing $\theta^* < L$. Therefore in equilibrium either $2\alpha\Psi(C_v^{-1}(\Delta - F)) \leq \Delta \leq C(L)$ or $\Delta \geq \max\{C(L), 2\alpha\Psi(L)\}$. Consider next optimality and non-negative rents for extreme parties. Note first that given that interior parties are choosing $\theta^* < L$, then optimal campaign spending by extreme parties must be interior as well. For non-negative rents, we need $\Pi_1^* = x_1 + \Delta/2 - \alpha/\Delta(v(\theta^*) - v(\theta_1^*)) - C(\theta_1^*) \geq 0$. Since Π_1^* is maximized at θ_1^*, then $\Pi_1^*(\theta_1^*) \geq \Pi_1^*(\theta_1)$ for all $\theta_1 \neq \theta_1^*$ and, as a result, it suffices to show that $\Pi_1^*(\theta^*) > 0$, or equivalently, $(K-2)\Delta/2 + C(\theta^*) \leq 1/2$. But since in equilibrium it must be that $\Delta \geq C(\theta^*)$, then it is sufficient that $\Delta \leq 1/K$.

Putting everything together, then in equilibrium either $2\alpha\Psi(C_v^{-1}(\Delta - F)) \leq \Delta \leq \min\{C(L), 1/K\}$ or $\max\{C(L), 2\alpha\Psi(L)\} \leq \Delta \leq 1/K$. To conclude the proof of part (1), consider the latter case. There exists such a Δ iff (1) $K \leq 1/C(L)$, and (2) $K \leq 1/(2\alpha\Psi(L))$. But $\alpha \leq \underline{a}$ implies $C(L) \geq 2\alpha\Psi(L)$, and the result follows. Moreover, if the above inequalities are strict, then all parties earn positive rents. To conclude the proof of part (2), consider instead an equilibrium in which $2\alpha\Psi(C_v^{-1}(\Delta - F)) \leq \Delta \leq \min\{C(L), 1/K\}$. There exists such a Δ iff (1) $K \leq 1/C(L)$, and (2)

$2\alpha\Psi(C_v^{-1}(\Delta - F)) \leq \Delta$. Note that the right-hand side is increasing in Δ, and the left-hand side is decreasing in Δ. With $\Delta = 1/K$, this is $2\alpha\Psi(C_v^{-1}(1/K - F)) \leq 1/K$. Note that if this is satisfied for some $K \geq 3$, it is satisfied for $K = 3$. Then we need $2\alpha\Psi(C_v^{-1}(1/3 - F)) \leq 1/3$. But this is the same as $\alpha \leq 1/(6\Psi(C_v^{-1}(1/3 - F)))$. □

Proof of Theorem 1 The proof follows from Propositions 5 and 6. □

Proposition 5 *Suppose that* $2 \times C(L) \leq 1$ *and* $2 \times F \geq 1/2$. *Then there exists a long-run electoral equilibrium with two parties. Moreover, there exist a threshold* $\overline{\alpha}_{2P}$ *and, for any* α, *a long-run two-party electoral equilibrium* $(x_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ *such that*

1. *If* $\alpha \leq \overline{\alpha}_{2P}$, *then both parties campaign unconstrained, and campaign spending is decreasing in parties' ideological differentiation and voters' ideological focus i.e.,* $\theta^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < L$.
2. *If* $\alpha > \overline{\alpha}_{2P}$, *then both parties are campaign constrained, i.e.,* $\theta^*(\alpha) = L$.

Proof of Proposition 5 Follows from Lemma 2 (in the text) and Lemma 7. □

Lemma 7 *Suppose that* $2 \times C(L) \leq 1$ *and* $2 \times F \geq 1/2$. *Then for any* $\alpha > \overline{\alpha}_{2P}$, *there exists a long-run two-party electoral equilibrium* $(x_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ *such that both parties are campaign-constrained; i.e.,* $\theta^*(\alpha) = L$.

Proof of Lemma 7 We showed in the proof of Proposition 2 that if two parties compete for votes in the election, and (i) voters vote for their preferred party, (ii) both parties are constrained in campaign spending; i.e., $\theta^* = L$, and (iii) parties' ideological differentiation is $\Delta < \alpha\Psi(L)$, then there is a location of the left party x_1 such that a short-run electoral equilibrium exists. We next show that if conditions (i) and (ii) are satisfied, and parties' ideological differentiation $\Delta \in B_{2P}^*$, where

$$B_{2P}^* \equiv \{\Delta : 1 - 2F \leq \Delta \leq \min\{\alpha\Psi(L), 2C(L), 1 - 2C_v(L)\}\},$$

then there is a location of the left party x_1 such that a long-run electoral equilibrium exists. It is then easy to check that the conditions in the hypothesis imply that B_{2P}^* is non-empty. Consider then the threat of entry. Suppose j enters at $x_j \in (0, x_1)$. As before, $\hat{\theta}_1 = \hat{\theta}_2 = L$ is a mutual best response if $\Delta/2 \geq C_v(L)$. Now, given that $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = L$ we have that $\tilde{r}_j = (x_j + x_2)/2$ and $\tilde{\ell}_j = (x_1 + x_j)/2$. But then

$$\Pi_j(\hat{\theta}_j) = \frac{\Delta}{2} - C(L) < 0 \Leftrightarrow \Delta < 2C(L).$$

Since $\hat{\theta}_1 = \hat{\theta}_2 = L$, a sufficient condition to deter entry in $[0, x_1]$ is $x_1 \leq F$. Similarly $1 - x_2 \leq F$ prevents entry in $[1 - x_2, 1]$. Since $\Delta = 1 - [x_1 + (1 - x_2)]$, this requires $\Delta \geq 1 - 2F$. This gives B_{2P}^*. □

Proposition 6 *Take* $K \geq 3$ *as given. If* $K \times C(L) \leq 1$, *and* $K \times F \geq 1/2$ *there exists a long-run LS equilibrium with* K *parties. Moreover, there exist* $\underline{\alpha}$, $\overline{\alpha}$ *and, for any* α *a long-run LS equilibrium* $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ *such that*

1. *If $\alpha \leq \underline{\alpha}$, then all parties campaign unconstrained, and for every party k, campaign spending is decreasing in parties' ideological differentiation and voters' ideological focus i.e.,* $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta/2\alpha) < L$.
2. *If* $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$, *then only interior parties are campaign constrained, i.e.,* $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = L$, *and if* $\alpha \geq \overline{\alpha}$ *all parties are campaign constrained, i.e.,* $\theta_1^*(\alpha) = \theta^*(\alpha) = L$.

Proof of Proposition 6 Fix $K \geq 3$, and assume that $K \times C(L) \leq 1$ and $K \times F \leq 1/2$. Lemma 8 extends Lemma 4 to long-run equilibria, and shows that if

$$\alpha \geq \max\left\{\frac{C(L)}{\Psi(L)}, \frac{1-2F}{(K-1)\Psi(L)}\right\} \equiv \overline{\alpha},$$

there exists a long-run LS equilibrium with K parties, such that $\theta_1^*(\alpha) = \theta^*(\alpha) = L$. Similarly, Lemma 9 extends Lemma 5 to long-run equilibria, and shows that if

$$\underline{\alpha} \equiv \max\left\{\frac{1-2F}{2\Psi(L)(K-1)}, \frac{C(L)}{2\Psi(L)}\right\} \leq \alpha \leq \overline{\alpha},$$

there exists a long-run LS equilibrium with K parties in which $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = L$. Finally, Lemma 10 extends the corresponding Lemma 6 to long-run equilibria and shows that if $\alpha \leq \underline{\alpha}$, there exists a long-run LS equilibrium with K parties in which $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta/2\alpha) < L$. □

Lemma 8 *Take* $K \geq 3$ *as given. If* $K \times C(L) \leq 1$ *and* $K \times F \geq 1/2$, *then for any* $\alpha \geq \overline{\alpha}$ *, there exists a LS equilibrium* $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ *with K parties such that* $\theta^*(\alpha) = \theta_1^*(\alpha) = L$, *and all parties earn positive rents.*

Proof of Lemma 8 We showed in the proof of Lemma 4 that if (i) voters vote for their preferred party, (ii) all parties are campaign constrained, i.e., $\theta_k^* = L$ for all k, and (iii) parties' ideological differentiation $\Delta \in A_T = \{\Delta : C(L) \leq \Delta \leq \min\{\alpha\Psi(L), 1/K\}\}$, then a short-run LS electoral equilibrium exists. We show below that if conditions (i) and (ii) are satisfied, and (iii') parties' ideological differentiation $\Delta \in A_T^* \subset A_T$, where

$$A_T^* \equiv \left\{\Delta : \max\left\{\frac{1-2F}{(K-1)}, C(L)\right\} \leq \Delta \leq \min\left\{2C(L), \alpha\Psi(L), \frac{1}{K}\right\}\right\},$$

then a long-run electoral equilibrium exists. Moreover, if the inequalities are strict (if Δ is in the interior of A_T^*), then all parties earn positive rents. Note that A_T^* is non-empty iff

$$\alpha \geq \max\left\{\frac{C(L)}{\Psi(L)}, \frac{1-2F}{K-1\Psi(L)}\right\} = \overline{\alpha} \tag{11}$$

and

$$\max\left\{\frac{C_v(L)+1/2}{C(L)}, \frac{1}{2F}\right\} \leq K \leq \frac{1}{C(L)}. \tag{12}$$

Now, $(C_v(L) + 1/2)/C(L) \leq 1/2F$ if and only if $F \leq 1/2$, but this must surely be the case, since for $K \geq 3$, the RHS of (12) implies $C(L) \leq 1/3$. Thus (12) boils down to $KC(L) \leq 1$ and $KF \geq 1/2$. Consider then the entry of j at $x_j \in (x_k, x_{k+1})$ for $k = 1, \ldots, K-1$. Since in equilibrium $\theta_k^* = L$ for $k = 1, \ldots, K$, then as long as incumbents do not prefer to quit campaigning after j's entry, we can always sustain in the continuation game an equilibrium such that $\hat{\theta}_j = \hat{\theta}_k = L$ for all j, k. For incumbents to prefer not to quit campaigning it is enough that $\Delta/2 \geq C_v(L)$, or $\Delta \geq 2C_v(L)$. But since $C_v(L) < F$ by hypothesis, this is implied by $\Delta > C(L)$. Given $\hat{\theta}_j = \hat{\theta}_k = L$, j's entry is not profitable if $\hat{\Pi}_j = \Delta/2 - C(L) < 0$, or equivalently $\Delta < 2C(L)$. For no profitable entry of (more) extreme parties, it is enough that $x_1 \leq F$, since $\hat{\theta}_j \leq \hat{\theta}_1$ implies that in the continuation after entry $\tilde{x}_{1j} < (x_1 + x_j)/2 < x_1$. Substituting $x_1 = 1 - x_K = (1 - (K-1)\Delta)/2$, this is $\Delta \geq (1 - 2F)/(K-1)$. Together, these conditions imply that in equilibrium it must be that $\Delta \in A_T^*$. □

Lemma 9 *Take $K \geq 3$ as given. If $K \times C(L) \leq 1$ and $K \times F \geq 1/2$, then for any $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, there exists a LS equilibrium $(x_1^*(\alpha), \theta_1^*(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with K parties such that $\theta_1^*(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = L$, and all parties earn positive rents.*

Proof We showed in Lemma 5 that if (i) voters vote for their preferred party, (ii) parties are campaign constrained if and only if they are not extreme, i.e., $\theta_k^* = L$ if and only if $k = 2, \ldots, K-1$, and (iii) parties' ideological differentiation $\Delta \in A_M = \{\Delta : \max\{C(L), \alpha\Psi(L)\} \leq \Delta \leq \min\{2\alpha\Psi(L), 1/K\}\}$, then a short-run LS electoral equilibrium exists. We next show that if conditions (i) and (ii) are satisfied, and parties' ideological differentiation $\Delta \in A_M^* \subset A_M$, where

$$A_M^* \equiv \left\{\Delta : \max\left\{\frac{1-2F}{K-1}, C(L), \alpha\Psi(L)\right\} \leq \Delta \right.$$
$$\left. \leq \min\left\{2C(L), \frac{1-2C_v(L)}{K-1}, \frac{1}{K}, 2\alpha\Psi(L)\right\}\right\},$$

then a long-run electoral equilibrium exists. Moreover, if the inequalities are strict (if Δ is in the interior of A_M^*), then all parties earn positive rents. Note that since $C(L) \leq 1/K$ and $F \geq 1/2K$ imply that $(1 - 2C_v(L))/(K-1) \geq 1/K$, the conditions defining A_M^* are equivalent to those defining A_T^* in all but the α terms. It follows from this that A_M^* is non-empty if and only if

$$\frac{1}{2F} \leq K \leq \frac{1}{C(L)}$$

and

$$\max\left\{\frac{(1-2F)}{2\Psi(L)(K-1)}, \frac{C(L)}{2\Psi(L)}\right\} \leq \alpha \leq \min\left\{\frac{1}{K\Psi(L)}, \frac{1-2C_v(L)}{(K-1)\Psi(L)}, \frac{2C(L)}{\Psi(L)}\right\}. \tag{13}$$

Now, the LHS of (13) is equal to $\underline{\alpha}$ by definition. Simple algebra shows (using the conditions $C(L)K \leq 1$ and $KF \geq 1/2$) that the RHS of (13) is bigger than $\overline{\alpha}$.

Consider then the threat of entry. As in the previous lemma, to deter entry by j at $x_j \in (x_k, x_{k+1})$ it is enough that $2C_v(L) \leq \Delta < 2C(L)$ and $C_v(L) \leq x_1$. To deter entry of more extreme parties, a sufficient condition is $x_1 \leq F$, and $2C_v(L) \leq \Delta$ (recall that this guarantees that the extreme incumbent parties will not quit campaigning in the continuation game). Since $C_v(L) \leq F$ by hypothesis, this is implied by $\Delta \geq C(L)$. Substituting $\Delta_0 = (1-(K-1)\Delta)/2$ and collecting the relevant inequalities gives A^*_M. □

Lemma 10 *Take $K \geq 3$ as given. If $K \times C(L) \leq 1$ and $K \times F \geq 1/2$, then for any $\alpha \leq \underline{\alpha}$, there exists a LS equilibrium $(x^*_1(\alpha), \theta^*_1(\alpha), \Delta^*(\alpha), \theta^*(\alpha))$ with K parties such that $\theta^*_1(\alpha) = \Psi^{-1}(\Delta/\alpha) < \theta^*(\alpha) = \Psi^{-1}(\Delta/2\alpha) < L$, and all parties earn positive rents.*

Proof We showed in Lemma 6 that if (i) voters vote for their preferred candidate, (ii) parties are unconstrained in campaign spending and choose

$$\theta^*_k = \theta^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \quad \text{for all } k = 2, \ldots, K-1, \text{ and } \theta^*_1 = \theta^*_K = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right),$$

and (iii) candidate's differentiation $\Delta \in A_L \equiv \{\Delta : \max\{C(L), 2\alpha\Psi(L)\} \leq \Delta \leq 1/K\}$, the polity is in a short-run LS electoral equilibrium. We next show that if conditions (i) and (ii) are satisfied, and (iii') candidate's differentiation $\Delta \in A^*_L \subset A_L$, where

$$A^*_L \equiv \left\{\left\{2C_v(L), \frac{1-2F}{K-1}, C(L), 2\alpha\Psi(L)\right\} \leq \Delta \leq \left\{2C(L), \frac{1-2C_v(L)}{K-1}, \frac{1}{K}, 2F\right\}\right\}$$

then a long-run electoral equilibrium. Moreover, if the inequalities are strict (if Δ is in the interior of A^*_M), then all parties earn positive rents. Note that the conditions defining A^*_L are equivalent to those defining A^*_M in all, but the α terms and the $2F$ term on the RHS (recall that $C(L) \leq 1/K$ and $F \geq 1/2K$ imply $2C_v(L) < C_(L)$). Thus, it follows that A^*_L is non-empty if

$$\frac{1}{2F} \leq K \leq \frac{1}{C_v(L)+F},$$

and

$$\alpha \leq \min\left\{\frac{C(L)}{\Psi(L)}, \frac{1-2C_v(L)}{2(K-1)\Psi(L)}, \frac{1}{2K\Psi(L)}, \frac{F}{\Psi(L)}\right\}. \tag{14}$$

Simple algebra shows (using the conditions $C(L)K \leq 1$ and $KF \geq 1/2$) that the RHS of (14) is bigger than $\underline{\alpha}$. Consider then the threat of entry. To deter entry of more extreme parties, it is sufficient that $x_1 < F$, and since $x_1 = (1-(K-1)\Delta)/2$ this can be written as $\Delta > (1-2F)/(K-1)$. So suppose that j enters at $x_j \in (x_k, x_{k+1})$ for $k = 1, \ldots, K-1$, and define $\delta^r_j \equiv (x_{k+1} - x_j)/\Delta$. Suppose first that in the continuation $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = L$. Then it must be that

$$\alpha v'(L)\left[\frac{1}{\delta_j^r \Delta} + \frac{1}{\Delta}\right] \geq C_v'(L) \quad \text{and} \quad \alpha v'(L)\left[\frac{1}{(1-\delta_j^r)\Delta} + \frac{1}{\Delta}\right] \geq C_v'(L).$$

Then if $\delta_j^r \geq 1/2$ (j enters in (x_k, x_{k+1}) closer to x_k than to x_{k+1}), the first two inequalities above hold if and only if $\Delta \leq \alpha\Psi(L)(1 + 1/\delta_j^r)$, or $\delta_j^r \leq \alpha\Psi(L)/(\Delta - \alpha\Psi(L))$. Thus, the continuation strategy profile is a Nash equilibrium for $1/2 \leq \delta_j^r \leq \alpha\Psi(L)/(\Delta - \alpha\Psi(L))$, which is feasible if and only if $\Delta \leq 3\alpha\Psi(L)$. When instead $\delta_j^r \leq 1/2$ (j enters closer to x_k), then we need $\Delta \leq \alpha\Psi(L)(1 + 1/(1-\delta_j^r))$, or $\delta_j^r \geq (\Delta - 2\alpha\Psi(L))/(\Delta - \alpha\Psi(L))$. Thus, the continuation strategy profile is a Nash equilibrium for $(\Delta - 2\alpha\Psi(L))/(\Delta - \alpha\Psi(L)) \leq \delta_j^r \leq 1/2$, which is feasible if and only if $\Delta \leq 3\alpha\Psi(L)$. Therefore, the strategy profile $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = L$ is a Nash equilibrium in the continuation for entrants such that

$$\frac{\Delta - 2\alpha\Psi(L)}{\Delta - \alpha\Psi(L)} \leq \delta_j^r \leq \frac{\alpha\Psi(L)}{\Delta - \alpha\Psi(L)}, \tag{15}$$

where $2\alpha\Psi(L) < \Delta \leq 3\alpha\Psi(L)$. Since the entrant in this case obtains $\Pi_j(\hat{\theta}_j) = \Delta/2 - C(L)$, then as long as in equilibrium $\Delta < 2C(L)$, entry in an "interior" region as in (15) is not profitable. It should be clear that this rules out "interior" entrants only, since $2\alpha\Psi(L) < \Delta$ with (15) implies that $\delta_j^r \in (0, 1)$. Consider then $\delta_j^r > (\alpha\Psi(L))/(\Delta - \alpha\Psi(L))$ (j enters close to x_k; the other case is symmetric). Consider the continuation $\hat{\theta}_k = \hat{\theta}_j = L, \hat{\theta}_{k+1} = \Psi^{-1}(\delta_j^r\Delta/((1+\delta_j^r)\alpha)) < L$. This is clearly an equilibrium in the continuation (j and k have even a greater incentive to choose L than in the previous case since they are now closer substitutes). For entry not to be profitable, we need

$$\Pi_j(\hat{\theta}_j) = \frac{\Delta}{2} + \frac{\alpha}{\delta_j^r\Delta}[v(L) - v(\hat{\theta}_{k+1})] - C(L) < 0.$$

A sufficient condition for the above inequality to be true is $\Delta \leq 2F$. To see this, suppose that the division of the electorate between k and j were fixed, with cutpoint $\tilde{x}_{kj} = (x_k + x_j)/2$. Then j would optimally choose $\tilde{\theta}_j = \Psi^{-1}(\delta_j^r\Delta/\alpha) < \hat{\theta}_{k+1}$, and we have that

$$\Pi_j(\hat{\theta}_j) \leq \frac{\Delta}{2} - \frac{\alpha}{\delta_j^r\Delta}[v(\hat{\theta}_{k+1}) - v(\tilde{\theta}_j)] - [C_v(\tilde{\theta}_j) + F] < \frac{\Delta}{2} - [C_v(\tilde{\theta}_j) + F].$$

To assure that all incumbent parties do not prefer to quit campaigning upon entry in any continuation, it is sufficient that $\min\{x_1, \Delta/2\} \geq C_v(L)$. Since $2x_1 + (K-1)\Delta = 1$, then $x_1 = (1 - (K-1)\Delta)/2$, and the previous condition can be written as

$$2C_v(L) \leq \Delta \leq \frac{1 - 2C_v(L)}{K - 1}.$$

Collecting the relevant inequalities gives A_L^*. □

Proof of Proposition 4 In the proof of Lemma 2, we showed that $\Delta < 1 - 2C_v(L)$ is a sufficient condition to guarantee that incumbent parties do not quit campaigning upon entry of a centrist challenger. It turns out that in all electoral equilibria where incumbent parties earn positive rents, this condition is also necessary for existence of a long-run pure strategy electoral equilibrium. To see why this is the case, first notice that the definition of long-run equilibrium requires incumbent parties to deter the entry of *any* challenger and therefore also the entry of a centrist challenger championing an ideological position x_j arbitrarily close to an incumbent's position. For example, suppose that $x_j > x_1$ but $x_j \rightarrow x_1$, and therefore the entrant and party 1 are almost "perfect substitutes" in the voters' eyes. In this case, it cannot be that both j and 1 choose an interior campaign spending in the continuation game. In fact, if that were the case either party could deviate and increase discretely its vote share by increasing its campaign spending slightly above the opponent's level. As a consequence, in any continuation equilibrium in pure strategies upon entry of a close centrist challenger, either both parties are campaign constrained at L or the incumbent keeps spending at his equilibrium level θ^* and the entrant optimally chooses not to campaign. The latter case, however, cannot be optimal either when the entrant is arbitrarily close to an incumbent party earning positive rents in equilibrium. In fact the entrant can always choose a campaign level slightly above its closest competitor and *de facto* attract the votes of all the incumbent supporters. Since the incumbent was earning strictly positive rents in equilibrium, this strategy is indeed profitable for the entrant. Summarizing, when incumbent parties are earning positive rents in equilibrium, it must be the case that upon entry of a centrist challenger very close to an incumbent party, the only continuation equilibrium in pure strategies has both parties constrained in campaigning, which implies that $\Delta < 1 - 2C_v(L)$ becomes a necessary condition to guarantee that incumbent parties do not quit campaigning in the continuation game.

Since the necessary condition $\Delta < 1 - 2C_v(L)$ provides an upper bound on differentiation as a function of the level of campaign regulation, it is natural to ask whether there exist also a lower bound on ideological differentiation between parties. To answer this question, notice that when $\Delta < \alpha\Psi(L)$ it must be the case that parties are costrained in campaigning in equilibrium. This implies that when incumbents are relatively close ideologically, we can use an argument similar to the one above and conclude that upon entry of an extreme challenger ($x_j < x_1$ or $x_2 > x_j$) that is arbitrarily close to an incumbent, $\Delta > 2C_v(L)$ guarantees that incumbents do not quit campaigning in any continuation game. When voters are sufficiently responsive to campaign, i.e., when $\alpha > 2C_v(L)/\Psi(L)$, we have that $2C_v(L) < \alpha\Psi(L)$ and hence $\Delta > 2C_v(L)$ becomes a necessary condition when parties are relatively close ideologically, i.e., when $\Delta < \alpha\Psi(L)$. Summarizing, when incumbent parties are earning positive rents in equilibrium and voters' are sufficiently responsive to campaigning it must be that $2C_v(L) < \Delta < 1-2C_v(L)$ and, as campaign limits become less and less stringent, it is immediate to verify that parties' ideological positions must converge to $x_1 = 1 - x_2 = 1/4$. □

Appendix B

Consider two parties k and $j > k$ representing policy positions x_k and $x_j > x_k$ and investing θ_k and θ_j, respectively. Recall that there always exists a unique policy $\tilde{x}_{kj}$ such that a voter i with ideal policy $z^i = \tilde{x}_{kj}$ would be completely indifferent between parties k and j,

$$\tilde{x}_{kj} = \frac{x_k + x_j}{2} + \alpha \frac{v(\theta_k) - v(\theta_j)}{|x_j - x_k|}. \tag{16}$$

For $k < K$, let $r_k(\theta, x) \equiv \arg\min_{j>k}\{\tilde{x}_{kj}(\theta, x)\}$ denote the identity of k's relevant competitor to the right given (x, θ), and let $\tilde{r}_k(\theta, x) \equiv \min_{j>k}\{\tilde{x}_{kj}(\theta, x)\}$ denote the position of the voter that is indifferent between k and $r_k(\theta, x)$. Similarly, for $k > 1$, define $\ell_k(\theta, x) \equiv \arg\max_{j<k}\{\tilde{x}_{jk}(\theta, x)\}$ and $\tilde{\ell}_k(\theta, x) \equiv \max_{j<k}\{\tilde{x}_{jk}(\theta, x)\}$. For $k = 1$, let $\tilde{\ell}_k(\theta, x) \equiv 0$, and for $k = K$, let $\tilde{r}_k(\theta, x) \equiv 1$. Let then $\Delta_k(\theta, x) \equiv x_{r(k)} - x_{\ell(k)}$ denote the distance between the policy positions represented by $r_k(\theta, x)$ and $\ell_k(\theta, x)$, and let $\delta_k^r \equiv (x_{r(k)} - x_k)/\Delta_k$. Then for any given (θ, x), as long as $0 < \tilde{\ell}_k(\theta, x) < \tilde{r}_k(\theta, x) < 1$, party k's vote share is given by

$$m_k(\theta, x) = \tilde{r}_k(\theta, x) - \tilde{\ell}_k(\theta, x).$$

As we already mentioned, the vote share m_k as a function of θ_k will typically have one or more points of non-differentiability. Clearly, the first such point is at the value $\underline{\theta}_k(\theta_{-k}, x)$ for which $m_k(\theta, x) = 0$ for $\theta_k < \underline{\theta}_k(\theta_{-k}, x)$ and $m_k(\theta, x) > 0$ for $\theta_k \geq \underline{\theta}_k(\theta_{-k}, x)$; i.e., the minimum campaign investment at which k obtains some votes. However, provided that $\theta_k \geq \underline{\theta}_k(\theta_{-k}, x)$ and *given the identity* of k's relevant competitors for (x, θ_{-k}), the vote share of an interior party $1 < k < K$ is

$$\begin{aligned} m_k(\theta_k; \theta_{-k}, x) &= \frac{\Delta_k}{2} + \frac{\alpha}{\Delta_k}\left(\frac{v(\theta_k) - v(\theta_{r(k)})}{\delta_k^r} + \frac{v(\theta_k) - v(\theta_{\ell(k)})}{(1 - \delta_k^r)}\right) \\ &= \frac{\Delta_k}{2} + \frac{\alpha}{\Delta_k}\frac{v(\theta_k) - [(1 - \delta_k^r)v(\theta_{r(k)}) + \delta_k^r v(\theta_{\ell(k)})]}{(1 - \delta_k^r)\delta_k^r}, \end{aligned} \tag{17}$$

which is only a function of the distance between the policy represented by k and that of its relevant neighbors, $\delta_k^r\Delta_k$ and $\delta_k^\ell\Delta_k$, and the campaign investment of k and its relevant neighbors $\theta_{r(k)}$ and $\theta_{\ell(k)}$. Thus given the identity of k's relevant competitors for (x, θ_{-k}), the vote share mapping $m_k(\theta_k; \theta_{-k}, x)$ is differentiable at $\theta_k \geq \underline{\theta}_k(\theta_{-k}, x)$, and the marginal vote share is given by

$$\frac{\partial m_k}{\partial \theta_k} = \alpha v'(\theta_k)\left(\frac{1}{\Delta_k^r} + \frac{1}{\Delta_k^\ell}\right) = \frac{\alpha}{\delta_k^r(1 - \delta_k^r)\Delta_k^T}v'(\theta_k).$$

In particular, the marginal impact of campaigning on vote share given the identity of k's relevant competitors is well defined, and increases the larger is α, the smaller is Δ_k

and—given Δ_k—the larger is $|\delta_k^r - 1/2|$. Generically, however, $m_k(\theta_k; \theta_{-k}, x)$ will not be differentiable at all $(\theta_k; \theta_{-k}, x)$. To see why this is the case note that

$$\frac{\partial \tilde{x}_{kn}}{\partial \theta_k} = \frac{\alpha}{x_n - x_k} v'(\theta_k) > \frac{\alpha}{x_m - x_k} v'(\theta_k) = \frac{\partial \tilde{x}_{km}}{\partial \theta_k}$$

whenever $x_m > x_n$. Since parties k and n are closer substitutes for voters than parties k and m, an increase in θ_k has a larger impact in how the electorate divides among k and n than in how the electorate divides between k and m. We have then two possibilities. If k's relevant competitor at θ_k is m, then n will not be the relevant competitor at $\theta_k' > \theta_k$, and in this case there are no discontinuities in the marginal vote share. But if n is k's relevant competitor at θ_k, then it is possible that for sufficiently high θ_k', m becomes k's relevant competitor, "squeezing" n. In this case, the change in the identity of the relevant competitor $r_k(\theta, x)$ forces an (upward) jump in $\Delta_k^r(\theta, x) \equiv x_{r(k)} - x_k$, and therefore a downward jump in $\partial m_k / \partial \theta_k$ (see Fig. 1 below).

It is apparent from Fig. 1 that relying on the first-order condition and a (local) second order condition can potentially be very misleading. To see this, consider k's best response from this first-order approach, which is given by

$$\theta_k = \Psi^{-1}\left(\frac{\delta_k^r (1 - \delta_k^r) \Delta_k^T}{\alpha}\right). \tag{18}$$

Now suppose that $x_k \to x_{k+1}$. Then $\delta_k^r \to 0$ and (18) imply that unless the cost of campaigning increases very sharply, θ_k will eventually hit its upper bound. This logic, however, is not necessarily correct. While k and $k+1$ are close substitutes, and therefore voters who rank k and $k+1$ highest are very sensitive to differences in campaigning among these candidates, the "local market" can very well be small. In this case, while k's marginal vote share can be very high for a small interval of θ_k, it will then drops to a much smaller level as soon as k's relevant competitor changes from $k+1$ to the more distant $k+2$, a much worst substitute to k in the eyes of voters. This is illustrated in Fig. 2, which shows that in this example the second discontinuity of the marginal vote share function would hit earlier than the intersection with the marginal

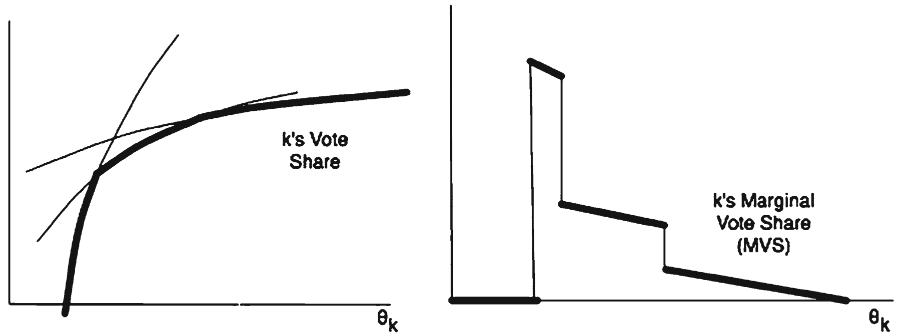

Fig. 1 k's Vote share and marginal vote share

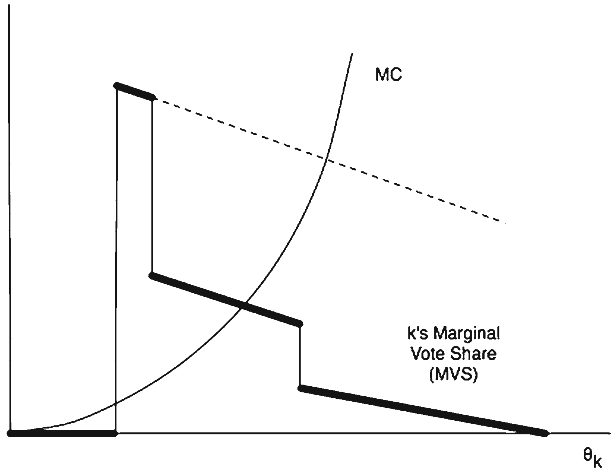

Fig. 2 k’s Marginal vote share and marginal cost of campaigning (MC)

cost schedule. Hence, the intersection of marginal cost and marginal vote share would be at a lower θ_k than in the absence of discontinuities. Under some conditions, however, the action identified as optimal by the first-order condition will indeed be a best response. Consider for example the case of interior equilibria with two parties running for office (Proposition 1). In this case, the identity of the relevant competitor is fixed by construction and therefore for any given θ_2, 1’s vote share mapping $m_1(\theta_1; \theta_2, x)$ has two kinks, one at $\underline{t}$ such that $m_1(\underline{t}; \theta_2, x) \equiv 0$ and one at $\bar{t}$ such that $m_1(\bar{t}; \theta_2, x) \equiv 1$. In fact $\underline{t} = v^{-1}\left(v(\theta_2) - \Delta^2/\alpha\right) < \theta_2$ and $\bar{t} = v^{-1}\left(v(\theta_2) + \Delta(1-\Delta)/\alpha\right) > \theta_2$. Thus, marginal rent is well defined, continuous and decreasing at all points $\theta_1 \in (\underline{t}, \bar{t})$. Since the first-order conditions for 1 and 2 imply $\theta_1^* = \theta_2^* = \theta^*$, the kinks are not relevant. The same result holds for all location-symmetric (LS) electoral equilibria as we show in Lemma 1.

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