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## Long Run versus Short Run Player

## Moral Hazard

choose $a^{i} \in A$
observe $y \in Y$
$\rho(y \mid a)$ probability of outcome given action profile
$u^{i}\left(a^{i}, y\right)$ utility
private history: $h^{i}=\left(a_{1}^{i}, a_{2}^{i}, \ldots\right)$
public history: $h=\left(y_{1}, y_{2}, \ldots\right)$
strategy $\sigma^{i}\left(h^{i}, h\right) \in \Delta\left(A^{i}\right)$
"public strategies" , perfect public equilibrium

## Moral Hazard Example

mechanism design problem
each player is endowed with one unit of income
players independently draw marginal utilities of income $\eta \in\{\bar{\eta}, \underline{\eta}\}$
player 2 (SR) has observed marginal utility of income player 1 (LR) has unobserved marginal utility of income
player 2 decides whether or not to participate in an insurance scheme
player 1 must either announce his true marginal utility or he may announce $\bar{\eta}$ independent of his true marginal utility
non-participation: both players get $\gamma=\frac{\bar{\eta}+\underline{\eta}}{2}$
participation: the player with the higher marginal utility of income gets both units of income
normal form
non-participation participate
truth
lie

| $\gamma, \gamma$ | $\frac{\bar{\eta}+\gamma}{2}, \frac{\bar{\eta}+\gamma}{2}$ |
| :--- | :--- |
| $\gamma, \gamma$ | $\frac{3 \gamma}{2}, \frac{\bar{\eta}}{2}$ |

$p^{*}=\frac{\eta}{\gamma}$ makes player 2 indifferent

$$
\left\{\begin{array}{l}
\max u^{1}(a)=\frac{3 \gamma}{2} \\
\text { mixed precommitment/Stackelberg }=\frac{\bar{\eta}+\gamma}{2}+\left(1-\frac{\eta}{\gamma}\right) \frac{\eta}{2} \\
\bar{v}^{1} \text { best dynamic equilibrium }=\frac{\bar{\eta}+\gamma}{2} \\
\text { pure precommitment/Stackelberg }=\frac{\bar{\eta}+\gamma}{2} \\
\begin{array}{l}
\text { Set of dynamic } \\
\text { equilibria }
\end{array} \\
\text { static Nash= } \gamma \\
v^{1} \text { worst dynamic equilibrium }=\gamma \\
\min u^{1}(a)=\gamma, \text { minmax }=\gamma
\end{array}\right.
$$

## moral hazard case

player 1 plays "truth" with probability $p^{*}$ or greater player 2 plays "participate"

$$
\begin{aligned}
& \bar{v}=(1-\delta) \frac{\bar{\eta}+\gamma}{2}+\delta\left(\frac{1}{2} w(\underline{\eta})+\frac{1}{2} w(\bar{\eta})\right) \\
& \bar{v} \geq(1-\delta) \frac{3 \gamma}{2}+\delta w(\bar{\eta}) \\
& \bar{v} \geq w(\underline{\eta}), w(\bar{\eta})
\end{aligned}
$$

$w(\bar{\eta})$ must be as large as possible, so inequality must bind; $w(\underline{\eta})=\bar{v}$

$$
\bar{v}=(1-\delta) \frac{3 \gamma}{2}+\delta w(\bar{\eta})
$$

solve two equations

$$
\begin{aligned}
& \bar{v}=\bar{\eta}-\frac{\gamma}{2} \\
& w(\bar{\eta})=\frac{\bar{v}-(1-\delta) 3 \gamma / 2}{\delta}
\end{aligned}
$$

check that $w(\bar{\eta}) \geq \gamma$
leads to $\delta \geq 2\left(2-\frac{\bar{\eta}}{\gamma}\right)$
from $\delta<1$ this implies
$\bar{\eta}>3 \underline{\eta}$

## Moral Hazard Mixing Games

$$
\rho(y \mid a)>0
$$

then $\bar{v}_{M H}<\bar{v}_{P O}$
moral hazard worse than perfect observability

