Long Run versus Short Run Player

Moral Hazard

choose \( a^i \in A \)
observe \( y \in Y \)
\( \rho(y|a) \) probability of outcome given action profile
\( u^i(a^i, y) \) utility

private history: \( h^i = (a_1^i, a_2^i, \ldots) \)
public history: \( h = (y_1, y_2, \ldots) \)

strategy \( \sigma^i(h^i,h) \in \Delta(A^i) \)
“public strategies”, perfect public equilibrium
**Moral Hazard Example**

mechanism design problem

each player is endowed with one unit of income

players independently draw marginal utilities of income $\eta \in \{\bar{\eta}, \eta\}$

player 2 (SR) has observed marginal utility of income
player 1 (LR) has unobserved marginal utility of income

player 2 decides whether or not to participate in an insurance scheme
player 1 must either announce his true marginal utility or he may announce $\eta$ independent of his true marginal utility

non-participation: both players get $\gamma = \frac{\eta + \eta}{2}$

participation: the player with the higher marginal utility of income gets both units of income
normal form

<table>
<thead>
<tr>
<th></th>
<th>non-participation</th>
<th>participate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>truth</strong></td>
<td>$\gamma, \gamma$</td>
<td>$\frac{\bar{\eta} + \gamma}{2}, \frac{\bar{\eta} + \gamma}{2}$</td>
</tr>
<tr>
<td><strong>lie</strong></td>
<td>$\gamma, \gamma$</td>
<td>$\frac{3\gamma}{2}, \frac{\bar{\eta}}{2}$</td>
</tr>
</tbody>
</table>

$p^* = \frac{\eta}{\gamma}$ makes player 2 indifferent
\[
\max u^1(a) = \frac{3\gamma}{2}
\]

Mixed precommitment/Stackelberg:

\[
\frac{\eta + \gamma}{2} + (1 - \frac{\eta}{\gamma}) \frac{\eta}{2}
\]

\(\bar{v}^1\) best dynamic equilibrium:

\[
\frac{\eta + \gamma}{2}
\]

Pure precommitment/Stackelberg:

\[
\frac{\eta + \gamma}{2}
\]

Set of dynamic equilibria

Static Nash:\n
\[
\gamma
\]

\(v^1\) worst dynamic equilibrium:\n
\[
\gamma
\]

\[
\min u^1(a) = \gamma, \text{ minmax } = \gamma
\]
**moral hazard case**

player 1 plays “truth” with probability $p^*$ or greater

player 2 plays “participate”

$$
\bar{v} = (1 - \delta) \frac{\eta + \gamma}{2} + \delta \left( \frac{1}{2} w(\eta) + \frac{1}{2} w(\bar{\eta}) \right)
$$

$$
\bar{v} \geq (1 - \delta) \frac{3\gamma}{2} + \delta w(\bar{\eta})
$$

$$
\bar{v} \geq w(\eta), w(\bar{\eta})
$$

$w(\bar{\eta})$ must be as large as possible, so inequality must bind; $w(\eta) = \bar{v}$
\[ v = (1 - \delta) \frac{3\gamma}{2} + \delta w(\eta) \]

solve two equations

\[ \bar{v} = \bar{\eta} - \frac{\gamma}{2} \]

\[ w(\bar{\eta}) = \frac{v - (1 - \delta)3\gamma/2}{\delta} \]
check that $w(\eta) \geq \gamma$

leads to $\delta \geq 2 \left( 2 - \frac{\eta}{\gamma} \right)$

from $\delta < 1$ this implies

$\overline{\eta} > 3\eta$
Moral Hazard Mixing Games

\[ \rho(y \mid a) > 0 \]

then \[ \bar{v}_{MH} < \bar{v}_{PO} \]

moral hazard worse than perfect observability