Decision Theory: Time

Impatience

infinite discounted utility

$\sum_{t=1}^{\infty} \delta^{t-1} u_t$

average discounted utility

$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_t$

note that average present value of 1 unit of utility per period is 1

The real equity premium puzzle

Utility $u(x) = \frac{x^{1-\rho}}{1-\rho}$, $\sum_{t=1}^{\infty} \delta^{t-1} u_t$

Consumption grows at a constant rate $x_t = \gamma^t$

$u'(x) = x^{-\rho}$

marginal rate of substitution $\frac{1}{1+r} = \frac{\delta u'(x_{t+1})}{u'(x_t)} = \frac{\delta \gamma^{-\rho(t+1)}}{\gamma^{-\rho t}} = \delta \gamma^{-\rho}$

1889-1984 from Shiller [1989]

average real US per capita consumption growth rate 1.8%

$\rho = 8.84 \;\; r = 17\%$

Mean real return on bonds 1.9%; Mean real return on S&P 7.5%

http://www.dklevine.com/econ201/interest.xls

How does the market react to good news?

Value of claims to future consumption relative to current consumption

$$x_1 = 1$$

$$\frac{\sum_{t=2}^{\infty} \delta^{t-1} u'(x_t) x_t}{u'(1)}$$

$$\sum_{t=2}^{\infty} \delta^{t-1} \gamma^{-(t-1)\rho} \gamma^{t-1} = \sum_{t=1}^{\infty} \left[\delta \gamma^{1-\rho} \right]^t = \frac{\delta \gamma^{1-\rho}}{1 - \delta \gamma^{1-\rho}}$$

to be finite we need $\delta \gamma^{-\rho} < 1$

$$\frac{\partial}{\partial \gamma} \frac{\delta \gamma^{1-\rho}}{1 - \delta \gamma^{1-\rho}} = \frac{\delta (1-\rho) \gamma^{-\rho}}{(1 - [\delta \gamma^{-\rho}])^2}$$

$\rho > 1$ this is negative

Hyperbolic Discounting

(based on Villaverde and Mukherji [2001])

Q1: would you like \$10 today or \$15 tomorrow?

Q2: would you like \$10 100 days from now or \$14 101 days from now?

Some people answer prefer \$10 in Q1 and \$14 in Q2. This is inconsistent with (geometric) discounting and a time and risk invariant marginal rate of substitution between days.

Note that (because of asset markets) this makes little sense when expressed in terms of money. So let us suppose that the “paradox” refers to consumption.

One explanation: “hyperbolic discounting” meaning preferences of the form $u(c_1) + \theta \sum_{t=2}^{\infty} \delta^{t-1} u(c_t)$

Another Explanation

Uncertainty about preferences 100 days from now.

Suppose marginal utility of consumption can take on two values 1 or 2 with equal probability and that the daily subjective discount factor is to a good approximation 1.

Today the value of todays and tomorrow's marginal utility is know with certainty. Hence the subjective interest rate can take on the values of 1, 0 or -½ with probabilities .25, .5 and .25. Expected subjective interest rate is .125 = 1/8. If you are offered 10 today versus 15 tomorrow, you take 10 today with probability .25.

Suppose on the other hand, suppose that preferences 100 days from now are unknown. Ratio of expected utilities is 1, so subjective interest rate is 0. If you are offered 10 in 100 days versus 14 in 101 days you always take 14.

Notice that pigeons have apparently figured this out correctly.

Demand for commitment? 13%

Time and Uncertainty

able 1 – Dynamic Preference Reversal

		Probability of reward[1]	
Scenario		1.0 (60)	0.5 (100)
1	S $175 now	0.82	0.39
	L $192 4 weeks	0.18	0.61
2	S $175 26 weeks	0.37	0.33
	L $192 30 weeks	0.63	0.67

Keren, G. and P. Roelofsma [1995], "Immediacy and Certainty in Intertemporal Choice," *Organizational Behavior and Human Decision Making*, **63** 297-297.

[1] Sample size in parentheses.

Interpersonal Preferences

Experimental results

Roth et al [1991]

US $10.00 stake games, round 10

Second and final round of bargaining game:

Player may take x or reject it and get nothing.

The other player gets $10-x

5 of 27 offers with x>0 are rejected

5 of 14 offers with 5>x>0 are rejected

x	Offers	Rejection Probability
$2.00	1	100%
$3.25	2	50%
$4.00	7	14%
$4.25	1	0%
$4.50	2	100%
$4.75	1	0%
$5.00	13	0%
total	27	