When Pollsters are Wrong and Lobbyists Win: Economic Sociology and Political Economy☆

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Preface

This is a book about democracy and lobbying. The book has three goals:

1. To shed light on how large interest groups may succeed in influencing political elections yet be undermined by small groups in lobbying for policies. We examine issues such as voter turnout, the efficacy of interest groups, and the rise of populism.

2. To introduce new tools for incorporating sociological elements of peer pressure and social networks into modeling the behavior of interest groups. In doing so we incorporate standard elements of economic theory into the analysis of political contests: especially incentive constraints and auction theory.

3. To present parts of the large existing literature on political contests, voting, and lobbying from a unified perspective.

The book is designed for advanced economics undergraduates and graduate students in economics and related disciplines such as political science or sociology. The basic prerequisite is familiarity with calculus, basic non-cooperative game theory and especially Nash equilibrium.

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This book is dedicated to our partners who put up with much in the writing of the book and the many papers and revisions of papers that form its basis: to Catharina Tilmans, Daniela Iorio, and Marta Terranova.

1. Lobbies and Democracy

In March of 1998 both houses of the U.S. Congress enacted by voice vote and without dissent Public Law 105-298. The law extended copyright protection for books produced before 1978 from the original 56 years to 95 years.[4] The reason for a public interest in copyright is stated in the U.S. Constitution: "to promote the progress of science and the useful arts." However, it does not require a Nobel prize in economics to understand that increasing the length of copyright in 1998 on books produced before 1978 in no way changes the fact that they already exist.[5] On the contrary - the obvious effect is to take money out of the pocket of the many who do not hold those copyrights and put them in the pocket of the few who do - with some money being lost along the way.

As a modest illustration of the point, consider two children's novels, *Winter Holiday* and *Coot Club*, published by Sir Arthur Ransome in the early 1930s. At that time the copyrights were scheduled to expire by 1990. They have not. On the first day of 2017 one was available for the Kindle at a price of $6.78 and the other not available at all. By contrast a novel of similar popularity *The League of the Scarlet Pimpernell*, published in 1919 by the Baroness Emmuska Orczy, had copyright expire in 1975 and it is currently available for free. So the $6.78 for the *Winter Holiday* is money to the pocket of the copyright holder - not the author because he died in 1967 thinking that his copyright would expire by 1990. As for the *Coot Club* - the one which is not available - this represents a loss with no corresponding gain.

Now you may feel that the distant heirs of distinguished but long dead people should live off the proceeds of the hard work of their distinguished forebearers rather than earning a living like the rest of us. Indeed, as an example of how a law favors the few at the expense of the many the fate of *Coot Club* is trivial - perhaps even when multiplied by the many millions of works in the same category. It does, however, cleanly illustrate how rent-seeking by lobbyists serves to shift money to their own pockets while losing some of the money in the process. Copyright lobbying, moreover, has consequences beyond the fate of a single children's novel. Copyright substantially limits the usefulness of the internet: while it is technically feasible that we can have all book, movies, music, technical research and so forth available freely and nearly instantaneously wherever we might be, this is an ideal far from achievement - and only because of the copyright lobby.

Moving outside of retroactive extension of copyright, the consequences of lobbying are murkier - but much more substantial. For example, many of AIDS deaths in Africa are directly attributable to lobbying over patent rights - this is not trivial. Nor is the $180 billion in taxpayer money used to bail out Goldman Sachs in 2008. This money was not only taken from the pockets of the many for the few, but for a few who are already very rich few. Clearly, it is not exactly

[4] https://www.copyright.gov/circs/circ15a.pdf

[5] For the statement of the Nobel Prize winning economists concerning retroactive copyright, see: https://cyber.harvard.edu/openlaw/eldredvashcroft/supct/amici/economists.pdf.

news that lobbyists thrive by picking the pockets of the taxpayer. Nor yet is it news that money has a corrupting influence on government. Yet how can this happen in a democracy? Do not the interests of the many outweigh the interests of the few? And is corruption a big problem or a small problem? And what to do about it?

A common view is that corruption is a big problem and that the corrupting influence of money in U.S. politics comes about because political campaigns are financed by rich lobbies. A common solution - one championed by Larry Lessig among others - is that we need to have public financing of political campaigns so that politicians are not dependent on donations.[6] The problem with this is that the U.S. system of expensive and privately financed political campaigns is relative unique - yet political corruption is by no means limited to the United States. Take Ireland where political campaigns are publicly financed: in September 2008 the Irish finance minister used 64 billion Euros of taxpayer money to bail out banks that - like Goldman Sachs - had made some bad bets. Or take Italy where public financing of political campaigns has been introduced in 1973 and abolished in 1993 with a national referendum in the aftermath of "Tangentopoli" - the biggest investigation on political corruption in the Italian porstwar period.[7]

One reason keeping private money out of campaigns is not likely to have much impact on government corruption is that a great deal of corruption is due to appointed or civil service officials and not only to elected officials. Indeed, Larry Lessig's view is somewhat ironic in this respect: he began his campaign not after the corrupt passage of the copyright extension law by Congress and the President, but after the equally corrupt rejection by the Supreme Court of his legal challenge to that law. Yet campaign contributions surely play little role in the corruption of the Supreme Court. A different argument is that bribing politicians through campaign contributions is only the tip of the iceberg. Now and historically a simple and effective form of bribery is to give money to the family or to give money after departing office. When he was a Senator, Chris Dodd was famous for carrying the water of the motion picture industry. If the industry wanted the internet shut down so that their films could not be pirated, he was there to fight for them. After he left office in 2011 he took a several million a year job as the CEO of the Motion Picture Association of America. When as a sleek lobbyist Chris Dodd appears in the office of one of his former colleagues, do you suppose the message he brings is "this copyright restriction is good for your constituents for the following reasons?" Or do you suppose his message is "look how rich I am - if you play ball like I did you too can one

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[6]Larry Lessig is the Roy L. Furman Professor of Law at Harvard Law School. Like one of the authors of this book he became interested in political corruption because of the brutality with which the copyright lobby has pushed aside the public interest. He was a candidate for the Democratic Party's nomination in the 2016 U.S. presidential election running on an anti-corruption platform, but withdrew before the primaries.

[7]Public financing of political campaigns has been reintroduced in Italy in 1993 and abolished again in 2013.

day be a rich and sleek lobbyist like me?" Or, not to be U.S. centric, take Jose Barroso, the Portuguese politician who was the 11th President of the European Commission - and no sooner stepped down than he was named Chairman at Goldman Sachs International. No doubt because of the investment skills he acquired in his years of politics. How many 31 year old's fresh out of school whose father is not a former President and whose mother is not the Secretary of State are offered a $600,000 a year job as "special correspondent?" And so forth and so on.

If lobbyists take the long view it is hard to legislate against them: Do we pass a law that anyone who has ever worked in government, is likely ever to work in government or who is related to such a person is unemployable? It is a possible solution - and one that has been tested and proven effective in the past. In Imperial China and in the Ottoman Empire high ranking government officials were castrated male slaves separated from their families at an early age. This solution seems unlikely to be acceptable in the current social environment.

And if lobbyists are so effective why do they sometimes lose? Why is the Disney Corporation so effective in getting retroactive copyright extensions whenever their Mickey Mouse copyright is due to expire - but large pharmaceutical companies have never managed to get a retroactive patent extension when their blockbuster drug patents are due to expire? Why does the copyright industry sometimes lose in Congress as it did when it proposed the "Stop Online Piracy Act?" Why if small special interest groups are so effective did it take decades for minorities such as blacks and gays to succeed with their agendas of equal rights? Are we - as Mancur Olson who documented the effectiveness of small special interest groups argued - doomed to ever increasing lobbying until the economy is choked and we find ourselves as an Argentina, Brazil or Greece? And if so - why did the previous gilded age of railroad barons bribing politicians to give them public land not end with a catastrophic collapse? Most would say our best times came after the gilded age and not before.

It seems that to understand what practical solutions might be we need not only to look to the past, but also to understand why lobbyists do and do not succeed and how their efforts interact with the political system as a whole. Moreover not all special interests are bad: ideally we look for institutions that protect the good while blocking the bad.

1.1. Costs of Rent Seeking

Inspired by the economic failure of such diverse countries as Argentina, Brazil, Greece, India, Italy and Zimbabwe, economists have great concern with rent-seeking and the inefficiencies that occur when lobbying and corruption are used to limit markets and restrict competition. Nationalization of firms, restrictions on foreign trade in goods and services, limitation of foreign investment, restrictions on immigration, unionization, cartelization, product and price regulation, and restrictive labor contracts are all among the problems that economists have identified and studied. To that list we would add the monopolies implicit in intellectual property such as copyrights and patents. Subsidies and transfer payments ranging from farm subsidies to bank bailouts all create inefficiencies.

The political conflicts that lead to these bad outcomes are themselves costly in resources. Even worse: debt arising from rent-seeking leads to financial crises - and this in turn can lead to political instability. It is perhaps worth remembering that the French revolution occurred because of excessive debt. Yet we should keep in mind: not all of these bad policies are due to small groups lobbying. Restrictive policies are sometimes the outcome of majority voting. In the 2016 U.S. elections, Donald Trump won the presidency on the promise of restricting both trade and competition.

1.2. Good Corruption Versus Bad Corruption

By selling favors, corrupt public officials may subvert the intention of the voters who directly elect them or are indirectly responsible for their appointment. It is natural to see this as undemocratic and therefore bad. But democracy has no moral claim: what moral principle says the 51% of a particular group of people may do as they wish with the other 49%? More to the point, as economists we are interested in the efficiency of outcomes: do democratic or non-democratic systems with or without corruption lead to good or bad outcomes? We know from the phenomenon of Condorcet cycles and the Arrow Impossibility Theorem that providing "democratic" methods of ranking alternatives is problematic. A relatively simple example illustrates the issue.

Voting over an Externality. Suppose that there are $N > 3$ people $i = 1, 2, \ldots, N$ in a cold country. Each of them can light a fire that can keep them warm but at some cost to the others, who must suffer the smoke from the fire. There are N regulations $j = 1, 2, \ldots, N$, with regulation j banning person j from lighting a fire. Let us suppose that the benefit of a fire to the person lighting it is 1 and the cost to each other person is $v > 0$. Let us consider a sequential model where in each period there is an existing status quo of regulations in force and that one regulation is chosen at random. If the regulation chosen is already in effect there will be a vote on whether to remove the regulation or maintain the status quo and if the regulation chosen is not already in effect the vote is on whether to add the regulation to those already in effect or maintain the status quo. Majority rules with the status quo remaining in case of a tie. Let us assume that people vote sincerely, meaning that the person affected by the regulation will always vote against it and everyone else in favor of it. Every proposal to remove a regulation will be defeated by a vote of $N - 1$ to 1, while every proposal to add a regulation will win by the same vote of $N - 1$ to 1, so, after some period of time, all N regulations will be in effect.

Is it a good thing or a bad thing that all N regulations will be in effect? The social cost of a regulation is 1 and the social benefit is $(N-1)v$. If $1 < (N-1)v$ then the regulation is efficient and having all N regulations in effect is a good thing: everyone agrees that they would be willing to give up their own fire in order to avoid the smoke of the others. Democracy works. On the other hand if $1 > (N-1)v$ the regulation is inefficient: everyone agrees they would be willing to suffer the smoke of the others in order to keep warm and the outcome in which there is no regulation Pareto dominates the democratic outcome in which fires are banned. The electoral system here delivers exactly the same result - the banning of fires - regardless of whether it is a good or a bad idea. There is no “invisible hand” for democracy guaranteeing that it delivers good outcomes.

Now suppose that there is a corrupt official charged with enforcing the regulations. The official can be bribed by an amount b to look the other way and permit a fire. If $b > 1$ nobody will pay the bribe, so corruption does not matter. If $b < 1$ everyone will pay the bribe and despite the electoral outcome everyone will have a fire. If the regulation is efficient because $1 < (N-1)v$ this is unambiguously a bad thing: everyone is worse off, plus they all pay a bribe.

If the regulation is inefficient, the case is less clear cut because it depends on how we count the bribe money. The bribe is a transfer payment to the public official so from an efficiency point of view ought not to matter, so let us take this as our model. Without corruption per capita utility is $(N-1)v - 1 < 0$, while with corruption per capita utility is $1 - (N-1)v > 0$. Hence corruption is good!

This is not purely a matter of theory: it is argued that many under-developed nations are over regulated and that corruption is the “grease for the wheels” that keeps the economy from choking to death. The reason corruption is potentially good is because democracy need not work very well. When regulation is inefficient the ranking of alternatives by majority voting exhibits a *Condorcet cycle*: one additional regulation will always win but when all regulations are in place no regulation at all will win.

One reason behind the inefficiency result has to do with the fact that majority voting does not allow the intensity of preferences to be registered. Suppose that instead of voting a system of bidding was used: everyone submits a dollar bid either for the status quo or the alternative, whichever alternative gets the highest aggregate bid wins and the winning bidders pay their bid. The proceeds of the bid are divided equally among everyone - with many people we may assume that individuals ignore the fact that they may get a tiny amount of the money back. This is an example of what is called a menu auction and in this setting we look for what is called a *truthful equilibrium* (see the Appendix to this section). Think of the population as being divided into two groups: those who favor the status quo and those who favor the alternative. Each individual has a positive value for their favored alternative, and each group has an aggregate value equal to the sum of these individual values. In this setting it is a truthful equilibrium for members of the group with the lower aggregate value to each bid their individual value, and for the group with the higher aggregate value to bid an amount less than or equal to their individual value and such that the sum of their bids equals the aggregate value of the other group. Since the group with the higher aggregate value wins, this guarantees that only efficient regulations are approved and that inefficient regulations are removed. One could argue that this system of bribery - because it allows the possibility of a bigger bribe when the benefit is greater - is better than democracy. However, real electoral systems do take account of the intensity of preferences in spite of the fact that voting, like lobbying, is a costly activity - people are more likely to vote if the benefit of winning is greater.

Direct Democracy and Rational Voter Ignorance. Concern over corruption has led to many policy proposals. We already discussed limitation on campaign contributions and why that is unlikely - for good or bad - to be effective. Another proposal championed by reform parties in Europe such as M5S in Italy and Podemos in Spain is for direct democracy. That is, rather than having decisions made by corrupt elected officials, policies should be decided by referendum. There are obvious reasons why this is problematic. First, whatever is decided in a referendum someone has to implement it - and that someone may well be corrupt. Second, since a government like a large business is a complicated organization that acquires many inputs and produces many outputs, to think that the average voter has any expertise in running such an organization is foolish. Moreover, to know what is a good public policy is not easy and requires a good deal of time and effort. Democracies face an enormous public goods problem: each individual has little influence on the electoral outcome and so has practically no incentive to become informed about issues - this is the curse of rational voter ignorance. This is why we have representative government: just as doctors delegate their investments to experts, so voters in democracies delegate policies to politicians. And just as there is an agency problem with dishonest financial advisors so there is an agency problem with corrupt politicians. But that does not mean the solution is to get rid of politicians.

The curse of rational voter ignorance is indeed one of the fundamental problem with democracy - direct or otherwise. Even after years of study resulting in a PhD in economics; even after years of academic or practical experience after a PhD, the consequences of economic policies are at best uncertain. While agreement among experts is greater than realized by the general public, there still is legitimate disagreement. It could not possibly be sensible for an individual voter - whose vote makes little difference - to invest years of effort in hopes of deciding what are the best economic policies. This is the public goods problem.

Because we cannot sensibly know ourselves what constitutes good policy we must rely on experts. Indeed, even those who denounce academic experts and pretend unwillingness to rely on experts, nevertheless follow like sheep their own "experts" like Rush Limbaugh, Michael Gove and Donald Trump. And therein also lies a dilemma: to know who is a true expert and who is a charlatan requires a lot of knowledge. Not perhaps a PhD in economics - maybe only a MA? And this is a problem that cannot be solved by direct democracy in which laws are passed by referendum. To know how to vote in a referendum we must trust experts to tell us.

Unfortunately there is more. Even if we can distinguish experts from charlatans, experts have their own agendas. Experts as well as politicians can be corrupted. For example, the famous academic economist Frederic Mishkin accepted \$124,000 from the Icelandic Chamber of Commerce to praise its regulatory and banking systems: this only two years before the Icelandic banks' catastrophic collapse. Nor to our knowledge has he been denounced by his colleagues for this. Owing to social pressure experts - like the police, doctors, or any other professions - rarely discipline within their own social networks. The supporters of populist movements are quite right to distrust experts!

Finally - there is the less obvious reason that direct democracy can be counterproductive since it lowers the cost of voting. This makes it harder for voters to register the intensity of their preferences, for when voting is costly only those voters with strong preferences are likely to vote. Other schemes designed to "encourage democracy" - such as mandatory voting - have the same problem.

The bigger message is: political systems have different parts that interact with each other. A system of corruption that is bad may be the cure for a system of election that is bad; fixing one without fixing the other may make things worse not better. One of the roles of constitutional government is to commit to non-democratic decision making processes in some domains: property cannot be taken simply by a majority vote - and one can imagine constitutional systems that implement menu auctions for a subset of regulation-related issues in place of voting. It is easy to chant "public campaign finance" or "direct democracy" or "jail corrupt bureaucrats" but we cannot think of fixing things or whether things can be or need to be fixed without understanding them first. That is what this book is about.

The Downside of Democracy. The example of the Condorcet cycle shows that democracy is far from perfect. It may well be as Winston Churchill once famously said "democracy is the worst form of government except all the others that have been tried," but it is important to recognize that from the perspective of economics it is welfare that matters and not "democracy." Indeed: is it right, or just, or moral that 51% of a group of eligible voters have a completely arbitrary rule over the other 49%? The writers of the US Constitution thought otherwise: institutional changes require a supermajority and an elaborate procedure; courts have substantial independence; certain rights may not be denied based on a majority vote, and so forth and so on. Does this mean the USA is not a democracy? And if so is that a good thing or a bad thing?

Acemoglu and Robinson (2012) argue that indeed democracy is the solution to problems big and small. They argue that there are two types of institutions: there are inclusive institutions that represent a wide range of interests and there are extractive institutions in which a few steal from the many. Inclusive institutions lead to economic success and extractive ones lead to economic failure. Democracy is good and autocracy is bad - not just for our liberty but for our economic well-being. But we must ask whether this is wishful thinking or if it is true.

The graph below shows per capita income for two countries since 1960: one starts low and ends much higher - it is an economic success as the other is a failure.

Which country has the more inclusive institutions? The economic failure is India: the shining success of democracy among post World-War II newcomers. The economic success is China: a country which has never known democracy. Notice that it is possible to create all sorts of comparisons - but the China-India comparison is a crucial one because the two countries contain about two and a half billion people - roughly a third of world population. By contrast those who point to places such as Hong Kong or Singapore must reflect on the fact that these are very small and special cases. There is no reason to believe that institutions that work well in Singapore - a rich city protected from greedy neighbors by strong geographical barriers - would work well in the hinterland of China.

The example of China and India is surely not an advertisement for the idea that the way to economic success is through inclusive institutions as Acemoglu and Robinson assert. More importantly, it highlights an important difference between democracy and autocracy, which is missing in the Acemoglu and Robinson narrative. China shows both the strength and weakness of autocracy. With bad rulers you get the horror of the cultural revolution. With good rulers you get the greatest growth in economic welfare in the history of humanity. Indeed: what if the economic success of China is *because* of the previous destruction of vested interests that took place during the cultural revolution? What moral calculus do we apply? Should we conclude that the cultural revolution was "worth it" to end world poverty? Far greater evils have been inflicted with far less beneficial long-term results. Or do we conclude that not even the greatest improvement in human welfare - an order of magnitude greater than any other in history - can justify the horror of the cultural revolution? By contrast India - with its well-functioning democracy and peaceful transitions of government - has held a more steady course: it had neither the ups nor the downs of China. Should we view this as failure? Or as success?

As many have argued, policy matters as well as institutions. "Good" institutions can generate bad policy as in India and "bad" institutions can generate good policy as in China. Nor is it the case, as some political scientists assert, that autocratic governments are condemned to short lives due to their inherent contradictions - that "coup proofing" by autocrats weakens institutions to the point at which they are doomed to fail. The history of China is a strong (and large) counter-example. China maintained autocratic rule through a professional bureaucracy for roughly 1300 years from 605 CE (some would argue 134 BC) until 1905 CE a period of generally high prosperity - and far longer than any democratic institutions have survived.

Corruption and Bribery Versus Lobbying. Systems - both democratic and autocratic - are subject to corruption. This is not a legal treatise: by corruption we do not mean "violation of a law against corruption." We mean the purchase of political decisions by money or favor rather than by voting. As we have indicated this is often legal - for example offering employment to a state official after they leave office. As economists we are not concerned with what the law is or how it is written but what it should be. Outlawing bribery is one of many institutional tools that may or may not be useful in improving political outcomes and economic welfare.

How does lobbying fit into this picture? Strictly speaking there is little wrong with lobbying - why should not individuals and groups be able to try to persuade state officials to their point of view? Indeed there is a literature on lobbying which views lobbying as primarily providing information to government officials. No doubt this happens. However, as we indicated, we think the most important message carried by a well-heeled lobbyist is not "your constituents would really benefit from this action" but "play ball and you can be rich like me." Naturally the information provided by lobbyists is biased: it is propaganda - studies and arguments biased to get a particular result. But - this is important as well - the mere fact that arguments are self-serving does not make them wrong.

What about bribery in the ordinary rather than legal sense of the word? This also involves the payment of money in exchange for political favor. There is clearly a range of activity at the one end of secret individual action such as directly paying off officials in cash for business licenses or other favoritism. On the other end is the more open form of bribery in which subtle hints of future consideration are exchanged for legislation that is public and visible. The difference between the two that is most important from our point of view is that the latter changes policy and is in effect a more acceptable contest between those who favor and those who oppose the legislation. In a sense both sides have the opportunity to compete by offering a suitable bribe. Moreover, typically this type of contest is between groups rather than a seedy meeting in which an individual bribes an official for a particular narrow consideration. It is the broad policy problem of contests between groups that is our primary focus. As we will see: some of the same considerations about groups apply also to more traditional bribery. For example, police corruption is largely possible because of the social norm called "the code of blue silence" in which police officers are expected to cover up for one another. In the other direction lobbying is not always open and visible - and naturally every effort is taken to mislead opponents about the nature and significance of proposals that are to their disadvantage.

1.2.1. Appendix: Menu Auctions

Menu auctions were introduced by Bernheim and Whinston (1986b) to study political influence and have been widely used for this purpose.

In a menu auction there is a finite set of alternatives $k = 1, 2, \ldots K$ and a finite set of bidders $i = 1, 2, \ldots, N$. If alternative k is chosen bidder i receives utility $u_i(k)$. Each bidder makes a bid $b_i(k)$ for each alternative. The auctioneer chooses the alternative that maximizes income $\sum_i b_i(k)$. Implicitly there is an endogenous tie-breaking rule: in case of a tie the auctioneer is free to choose the alternative that supports a particular equilibrium.

A *truthful equilibrium* consists of a choice k_0 by the auctioneer and bids such that the bid differentials reflect the utility differential between alternatives: $b_i(k) - b_i(k_0) = u_i(k) - u_i(k_0)$ (except that if this forces $b_i(k) < 0$ the bid is 0) and such that no bidder can improve their utility by an alternative bid. Notice that the condition $b_i(k) - b_i(k_0) = u_i(k) - u_i(k_0)$ has no implication for bids on the equilibrium alternative k_0, that is if $k_0 = k$ it reduces to an identity.

What does this mean in the case of two alternatives? This would be the case of a standard first price auction with two bidders or the example of the regulation game in section 1.2. To see what happens in this simple case, first normalize the payoffs so that for each bidder the less favored alternative yields zero utility. Nash equilibrium forces every member of the losing group - those that favor the alternative not selected - to bid zero for the winning alternative: bidding more loses money, and if bidding less changes the outcome it can only change it favorably. Truthfulness then forces each member of the losing group to bid their value for the losing alternative. To win the winning group must in aggregate bid at least this amount, which means that their value must be at least that of the losing group. Hence only efficient alternatives win when there are just two alternatives. Moreover, the aggregate bid of the winning group must equal the value of the losing group: otherwise each member of the winning group should reduce his bid. In the two bidder case this is exactly the same as would be the case for a second price sealed bid auction: the winner has the highest value and pays the lower value. The division of cost among the winners is indeterminate: no member can bid more than their value, but subject to this constraint - to bid less loses and there is no reason to bid more - any allocation of bids among the winners is an equilibrium. This also forces members of the winning group to bid zero for the losing alternative: since they bid less than or equal to their value for the winning alternative - $b_i(k_0) \leq u_i(k_0)$ - the differential with the alternative is non-positive so the corresponding bid must be zero: $b_i(k) = b_i(k_0) + u_i(k) - u_i(k_0) \leq u_i(k) = 0$.

Notice that the refinement of truthful equilibrium has bite: in the two bidder case there are many Nash equilibria - as long as the high valued bidder wins, bidding between the value of the low value bidder and his own value is an equilibrium. In the "non-truthful" equilibria the high value bidder "over-pays", and he is forced to do so by the somewhat foolish strategy of the low value bidder bidding above his value. In this setting the refinement of truthful equilibrium seems compelling.

At about the same time that Bernheim and Whinston (1986b) introduced the notion of menu auctions they also introduced in Bernheim and Whinston (1986a) the idea of a common agency problem a related but rather different model in which there are no constraints on negative payments but the agents have an outside option. Although in the literature the two models are sometimes confused, the common agency model is less well suited to the analysis of political influence and less widely used for this purpose.

1.3. The Themes of the Book

This book is about the political competition between influence groups. It puts together a number of different ideas about how groups behave and about how they interact. The models of political conflict we study are those widely used in the literature on voting and lobbying. While that literature well-captures the strategic aspects of competition between groups for political influence, it does less well capturing the internal organization of groups. This internal organization is not a mystery. We know from the sociology literature that groups maintain internal cohesion and discipline through loyalty, peer pressure and by the punishment of defectors. This is perspective we take in this book. We adopt a sociological model in which social norms are endogenous and enforced through peer punishment - it is a perspective we have developed in our own work: primarily Levine and Modica (2016), Levine and Modica (2015) and Levine and Mattozzi (2016).

The goal of these sociological models is to address a basic issue in the strategic behavior of individuals who are members of large groups: why do people contribute effort towards a victory that represents a public good? Everyone would like their group to win, but every selfish individual would like the rest of the group to bear the cost of that victory. The large literature on voting, lobbying, and conflict has addressed this issue largely through models of altruism: people contribute to the public good because of concern for their fellow group members. While people are surely altruistic the evidence suggests that they are not strongly so and we think it doubtful that the reason, for example, that farmers contribute substantially to farm lobbying effort, is out of altruistic concern for millions of farmers they do not know and have never met.

This does not mean that we reject the existing literature. On the contrary, as we emphasize, the sociological approach is compatible with models of altruism. We can, for example, view altruism as a social norm that is internalized so that there is no enforcement cost. Rather we view the sociological approach as an extension of models of altruism: we accept the many useful results of existing research while adding additional depth and meaning. In these pages we do not only examine sociological models: we also review more traditional models and the results from those models - albeit from our own sociological perspective.

In more detail, our perspective is this.

1. We view groups as social organizations. While groups engage in political activity, they are generally formed for reasons separate from that activity. For example, while farmers engage in farm lobbying they do not become farmers for that purpose. But once they are farmers through mutual interest and location

they form socially interactive networks - they socialize with one another. A quote from Adam Smith nicely captures this idea: "People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices."

2. Social organizations through their social networks enforce social norms. Members who fail to conform to social norms are punished: they are excluded from social activities, they are ostracized, in some cases they may be dealt harsher punishments such as imprisonment, beatings or even death.

3. In order to enforce social norms it is necessary for peers in social networks to monitor one another. Monitoring is imperfect and hence costly. It introduces incentive constraints into the study of groups.

4. Groups collectively choose social norms to achieve group objectives. When considering situations with *ex ante* identical group members, it is natural to focus on the objective of maximizing the common *ex ante* utility of a group member. Groups collectively design mechanisms for their members recognizing that individual incentives may cause members to diverge from group objectives.

5. Political activity is costly and this cost is *ex post* heterogeneous: some group members will find it more costly to engage in a particular activity - voting, lobbing - than others. There are costs of taking time and commuting to a polling place to vote; there is the cost of donating money to a lobbying group, and so forth and so on. In some cases some members may find the activity beneficial: people may feel satisfaction at fulfilling their civic duty of voting; people may enjoy engaging in a protest on a pleasant day. We refer to these activities as *duties*. Other activities may be costly for every group member: few people feel anything but cost at lobbying. We refer to these activities as *chores*.

6. For chores costly monitoring leads naturally to a fixed cost to the group of providing effort for political activity. For duties some degree of participation will occur without cost to the group - in voting there are committed voters who will vote regardless of strategic incentives.

In this context we examine political contests. Two of the highlights:

1. Fixed cost favors a smaller group who must pay the fixed cost for fewer members. Committed members favors a larger group who will have more committed members.

2. We draw in part on auction theory to study a variety of mechanisms for resolving political conflict but for the most part the details of the mechanisms do not much matter: what matters is the structure of group cost. Lobbying because it typically is a chore involving a fixed cost tends to favor smaller groups while voting because it is typically a duty tends to favor larger groups.

These basic ideas have many variations - mechanisms for resolving conflict can be determinate or random; they may involve the winner paying or all groups providing effort but only some succeeding. Agendas and political platforms may be endogenous. We may be interested in voter turnout or the most efficient resolution of conflict and how factors such as group size, the structure of costs, the mechanism for resolving conflict or heterogeneity impact on these outcomes. These issues have been explored by researchers in political economy and form the topics explored in this book.

We are interested not only in theory but in applications. To give a foretaste of these: changes in social networks over time - greater mobility of workers between jobs and locations and decreased interest in organized religious activities - has loosened the ties in social networks. In the "old days" in Britain workers in the labor party socialized in their pubs while the conservatives socialized in their clubs. Today commuters go their separate ways. Looser social ties increases the cost of monitoring: nobody knows their neighbors. What does the theory say about how this impacts on voter turnout and on the competitiveness of elections? Turnout goes down and elections become more competitive. The theory, in other words, captures the idea of political scientists that looser social ties have led to the breakdown of the party system reflected in lower voter turnout and greater success for insurgent and populist parties.

1.4. Voting, Lobbying and Populism

It is certainly true that the subversion of democracy by special interests can lead to economic problems. One of the most astute observers of special interest politics was the late Mancur Olson. Olson (1982) argued that inevitably as time goes on, more and more special interests arise until they strangle the economy entirely. It is true that countries that are notably corrupt such as Italy and India are not great economic powerhouses. But we can hardly describe them as abysmal failures. Italy has maintained a standard of living high by world standards, and India as the graph above shows has had slow - but steady - improvement.

There greatest problem with corruption in our view is that the historical antidote to corruption is populism: a far more dangerous phenomenon than corruption itself. Populism is the popular backlash against successful lobbying and rent-seeking. In the early 21st Century, for example, modern populist movements in the U.S. and Europe grew as the corruption of the banking sector and of experts became painfully clear during the financial crisis that arose late in the first decade of the century. Those who were able to do so used taxpayer funds as life-rafts to escape the consequences of their own bad decisions. In the 2016 US Presidential primary the populist proposals of Donald Trump on the Republican side to a large extent matched those of Bernie Sanders on the Democratic side. Both argued strongly for "America First" especially when it comes to trade. Both demanded for the average voter some of those protections from competition previously reserved only for rich and successful special interests. Populism is political victory through the ballot box and appears as the inevitable consequence of successful rent-seeking by the small special interests groups who dominate lobbying. Yet there is no guarantee that populist movements will succeed. The middle of the second decade of the 21st century has seen mixed electoral success by populists: success in Greece, the UK and the US but failure in Spain, the Netherlands and France.

The key danger of populism is that there is little guarantee that populist movements will achieve their stated goals. Indeed, popular remedies are often as harmful as folk medicine is for the genuinely ill patient - and not just metaphorically: the Italian populist movement M5S is anti-vaccine. Looking around we see that Trump far from draining the swamp has deepened it; Brexit is on track to crush a thriving British economy; in Greece Syriza managed the nearly impossible task of adopting economic policies even more harmful than those of its predecessors; while the oil powerhouse Venezuela, where street protest exploded in the spring of 2017, lacks basic amenities such as toilet paper. Looking farther across time we see that Peronism in Argentina condemned the country to decades of economic stagnation, while Mugabiism in Zimbabwe has condemned an entire nation to poverty. By contrast the US populist Teddy Roosevelt brought antitrust law and other measures against monopoly that mitigated rent-seeking without much harmful side effect.

The point is this: populism - the ballot box - where the large group is favored - is the inevitable consequence of rent-seeking by small groups who dominate lobbying. If you are concerned about the problem of populism - then figure out what to do about lobbying. That is largely what this book is about - and the conclusions we reach are not always the obvious ones. For example, it seems evident that to reduce lobbying we should increase its cost. However: this increases the cost of the general interest lobbyists as well as the special interest lobbyists - and as we shall see this actually increases the ability of the special interest lobbyists to gain favors from government - the opposite of the intended effect.

2. Political Contests

Economics is focused on mutual gains to trade: you have a banana that I want, I have an apple you want, we trade and are both better off. That is what markets are about and they work pretty well. Politics is different. If we raise taxes to pay subsidies to farmers it doesn't make both farmers and urbanites both better off: it is money out of the pocket of the urbanite and in the pocket of the farmer - with a little money falling along the side of the road on the way. That is: a great deal of politics is not about trade but is about conflict.

Political conflict is complicated. Different groups compete - lobbying groups, political parties - and provide effort in the form of money for bribes and advertising, votes, time, demonstrations, strikes and so forth and so on. What they are competing for is complex: a party may lose a national election but increase the number of regional governments it controls; legislation may be passed into law with more or less favorable amendments. On the grounds that it is better to walk before you can run, we are going to start with the simplest case.

To make things concrete, think of country like Greece where the political party that wins the election gets a lot of government jobs to reward its followers. There are just two two parties, the large L and the small S. The government jobs are worth V. The key to understanding elections - as every political scientist knows - is voter turnout. Polls do a good job predicting how people are going to vote. When we see an unexpected outcome like Brexit or Trump it isn't because the polls were wrong in predicting how people were going to vote - it is because *polls do a poor job of predicting whether people are going to vote or not.* A good example of this is the Spanish national election that took place on March 14, 2004. The incumbent People's Party was favored to win by around 6 percent. However, three days before the election 191 people were killed in a terrorist bomb attack on four commuter trains approaching the Atocha station. The People's Party responded to the attack by lying and blaming the attack falsely on Basque terrorists despite the evidence that it was conducted by al-Qaeda. Three days after the attacks the election was held and furious voters voted the People's Party out of office. What happened? Did People's Party supporters vote for other parties? No. What happened is that opposition voters turned out in much greater than expected numbers.

It makes sense then to think of the parties as having a fixed set of members who support the party. Let's say that relative size of the two parties is $\eta_L > \eta_S > 0$ with $\eta_L + \eta_S = 1$. Voter turnout by party $k \in \{L, S\}$ is the fraction of its members it sends to the polls $0 \le \varphi_k \le 1$. Getting a voter to the polls is costly and we normalize this cost so that it costs 1 per unit of voter sent to the polls. The party that sends the most voters to the polls wins the prize: if there is a tie the prize is split. This model of parties that win a prize by sending voters to the polls is basically that proposed by Shachar and Nalebuff (1999).

Seen this way the election is a game between two players - the parties. The payoff to party k is $V - \eta_k\varphi_k$ if it wins, that is if $\varphi_k\eta_k > \varphi_{-k}\eta_{-k}$, it is $-\varphi_k\eta_k$ if it loses and $V/2 - \varphi_k\eta_k$ in case of a tie. This is a model of competition that economists are familiar with - it is an all-pay auction. We can think of the number of voters $b_k = \varphi_k \eta_k$ sent to the polls as a *bid* - and the highest bidder wins the prize. It is, however, not standard winner-pays form of auction - it is called an all-pay auction because you have to pay your bid even if you don't win. In fact, the cost of turning out people is sunk no matter what the outcome of elections is.

Notice - and this is one of the strengths of game theory and of the beauty of simple formal models - that the same game could be a lobbying game. That is, two lobbying groups might compete over a piece of legislation by bribing a politician. Successfully getting your own agenda passed into law is worth V to the winning group, and now each group member is endowed with a unit of money, and φ_k represents the fraction of their budget contributed by a member of group k to bribe the politician. This general conceptual framework in which several groups compete in a game for a prize by providing effort is the workhorse model political economists use to study voting, lobbying and other political conflicts including warfare. In the sequel we will see many variations on this theme.

Notice that while in the case of voting it makes sense that both parties expend effort voting in the case of lobbying it is more likely that the politician collects only from the winning group. That is - in the case of lobbying the auction might be an ordinary winner-pays auction rather than an all-pay auction. So here we have one possible difference between voting and lobbying - perhaps small groups are more effective in winner-pays auctions and large groups more effective in all-pay auctions.

2.1. The All-Pay Auction

As Hillman and Riley (1989) showed the all-pay auction with complete information has a unique Nash equilibrium in which each party chooses an optimal turnout given the turnout of the other party. This equilibrium has two key characteristics:

- The equilibrium is not in pure strategies so the outcome of the election is necessarily unpredictable.
- The large party never does worse than the small party in expected utility and sometimes does better, with higher stakes favoring the large party.

This makes sense - in fact elections are dominated by large parties. The first point is crucial in understanding real elections: there cannot be a pure strategy equilibrium - the outcome of the election cannot be predicted in advance, it must be uncertain. Upsets such as Brexit or Trump are to be expected - and there is nothing any pollster or political scientist can do to make it otherwise. We call this the *uncertainty principle for elections.* Here already the theory tells us something: it tells us why pollsters are often wrong. This theory is pretty good for elections. It also works for wars, strikes, public demonstrations and other conflicts in which both sides pay regardless of whether the win or lose.

How the All-Pay Auction Works. The lack of a pure strategy equilibrium is easy to establish.

Theorem 2.1. *In equilibrium there cannot be a positive probability of a tie and there is no equilibrium in pure strategies.*

Proof. Why can there not be a positive probability of a tie? If tie is at $b < V$ then each party would wish to break the tie by shading its bid a little higher raising its probability of winning by $1/2$ at trivial extra cost. If there is a tie at $b = V$ then both parties receive $-V/2$ so would prefer to bid 0.

We can then make use of this to show that there is no pure strategy equilibrium. With pure strategies and no tie one party must lose with probability 1 and so must be bidding 0. But if one party bids 0 the other party should bid the smallest number bigger than zero and there is no such number. □

To develop a deeper understanding a useful concept is the *willingness to bid.* This is $W_k = \min\{V, \eta_k\}$ and is the highest bid a party is *willing and able* to provide. That is a party is not willing to bid more than V units for a prize worth V and the most it can bid is η_k (when it turns out all of its voters). With the notion of willingness to bid we can distinguish between a *medium stakes election* where $V \leq \eta_S$ in which both parties have the same willingness to bid, $W_S = W_L = V$, and a *high stakes election* where $V > \eta_S$ and the large party has a higher willingness to bid, $W_S = \eta_S < W_L$. With this definition we can be more specific about the equilibrium. Notice that $V - W_S \geq 0$, with the inequality being strict in a large stakes election.

Theorem 2.2. *The large party has an expected payoff of $V - W_S$, while the small party gets 0. In a high stakes election the small party bids 0 with probability $1 - \eta_S/V$ and the large party bids W_S with probability $1 - \eta_S/V$. All remaining probability of either party is a uniform density on $(0, W_S)$ of height $1/V$.*

Proof. To prove the theorem we will start by showing that both parties must bid arbitrarily close both to zero and to W_S. The first fact will imply that one of them will get zero. The second fact will imply that it is the small party. That the large party gets $V - W_S$ then follows easily.

We know that there is no pure strategy equilibrium. How much mixing is there? Quite a bit in the sense that both parties must place positive probability very close to 0 and positive probability very close to W_S. Why?

Consider the situation near 0 first. Suppose $\underline{b}$ is the lowest bid by either party. It cannot be that bidding $\underline{b}$ leads to a tie with positive probability. So one party k must face an opponent who has zero probability of bidding $\underline{b}$. That means that k must be almost certain to lose if it bids near $\underline{b}$ and it would be better to avoid the cost and just bid 0. Hence we must have $\underline{b} = 0$.

Lemma 2.3. *If k's opponent $-k$ has zero chance of playing the lowest bid $\underline{b}$ then k must be almost certain to lose bidding near $\underline{b}$.*

To prove this we must first formally define a mixed strategy. This is a probability distribution represented by a cumulative distribution function over bids, that is, a *mixed strategy* G_k is a non-decreasing function on $(-\infty, \infty)$ with $G_k(b) = 0$ for $b < 0$ and $G_k(1) = 1$. It is right continuous and if it fails to be left continuous at a bid b the height of the jump at b is the probability with which b is bid - it is an *atom* in the probability distribution. At points of continuity of G_k the probability of the bid is zero.
When we speak of a "lowest bid" $\underline{b}$ we mean that for $b < \underline{b}$ we have $G_k(b) = 0$ for both parties while for $b > \underline{b}$ we have $G_k(b) > 0$ for at least one of the parties. (If there is no such $\underline{b}$ then $\underline{b} = 0$.)

Proof of the Lemma. No positive probability of a tie means that both parties cannot have an atom at $\underline{b}$: that is both G_k do not have a discontinuity at $\underline{b}$. So suppose that party $-k$ has a continuous G_{-k} at $\underline{b}$. If it was the case that for some $b > \underline{b}$ we have $G_k(b) = 0$ then it must be that $G_{-k}(b) > 0$ (as we defined $\underline{b}$ they cannot both be zero). Then $-k$ would strictly improve by shifting the positive mass less than or equal to b down to zero since those bids are losing for certain - so having $G_{-k}(b) > 0$ was not optimal.
We may assume, then, that for $b > \underline{b}$ we have $G_k(b) > 0$. Now we reverse the argument. Since G_{-k} is continuous at $\underline{b}$ we have $G_{-k}(b) \to 0$ as $b \to \underline{b}$. That is to say that bids by k in the range $(\underline{b}, b)$, which we know have positive probability, lose with probability near 1. $\square$

Now since one of the parties must bid near zero with positive probability so must the other - otherwise the first should shift that probability to zero in the first place. Since both parties have to bid very near 0 and there can be no tie at zero we see that one party must get an expected payoff of zero.

Lemma 2.4. *If both parties bid very near* 0 *one party must get an expected payoff of zero.*

Proof. Suppose the opponent's strategy G_{-k} is continuous at zero. The probability of winning with a bid b is at most equal to the probability that the opponent bids less than or equal to b: so the payoff of k is at most $G_{-k}(b)V - b$ which goes to zero as $b \to 0$. Hence for any $\epsilon > 0$ there is b_ϵ such that all bids $b \in [0, b_\epsilon]$ by k yield less than ϵ. If k's expected payoff were larger than ϵ he would not place positive probability on this interval hence k's expected payoff is less than ϵ. Since this is true for any ϵ this means the equilibrium payoff of k must be zero. □

Note that we can also conclude this from the fact that bidding near $\underline{b}$ must lose almost for certain for one of the parties k: that party must therefore earn less than or equal zero and since it does not have to bid in fact earns zero. Notice that this argument applies to all auctions not just the all-pay auction. The special feature of the all-pay auction is that to get zero you must actually bid zero, while in a winner pay auction you can get zero by losing with probability one but bidding a positive amount. It is the necessity of bidding near zero that forces all-pay auctions to have mixed strategy equilibria - winner pay auctions have pure equilibria because the loser can bid high an earn zero by losing for certain.

Now consider the situation near W_S. If the highest bid is less than W_S the party getting an expected payoff of zero should bid a shade higher because this would turn a profit. Moreover, one party cannot have a higher highest bid than the other, since the party with the higher highest bid could lower its bids, saving cost and still winning with probability 1. Hence both parties must bid near W_S.

We can conclude that the small party must get 0 in equilibrium. In a low stakes election, as bids approach W_S, the most it can earn approaches 0. In high stakes elections the large party must get at least $V - W_S$ which is positive, so the small party must be the one earning zero. For the same reason the large party cannot get more than $V - W_S$. In a low stakes election it cannot get less than this since it is 0. In a high stakes election the large party also cannot get less than this because it can always bid a shade higher than W_S and win the election for sure.

Turning to the equilibrium strategies: since the large party gets $V - \eta_S$ in a high stakes election and bids close to zero those bids must still get $V - \eta_S$ meaning that the probability it wins must be close to $1 - \eta_S/V$. For this to be the case the small party must bid zero with that probability.

To find rest of the equilibrium strategies we need to know that the probability that party k bids less than or equal to b denoted by $G_k(b)$ is continuous and strictly increasing on the open interval between 0 and W_S.

Lemma 2.5. *$G_k(b)$ is continuous and strictly increasing on $(0, W_S)$.*

Proof. If G_k is not strictly increasing this means there is a gap where party k does not bid. Notice that if there is a gap for one party the other party must have the same gap as there is absolutely no point bidding in a range where the other party does not bid: better to bid at the bottom. At the top b of a hypothetical gap we know one party $-k$ does not have an atom. Hence party k should not bid above but close to b: it would do better to bid at the bottom. Since we are assuming $b < W_S$ this is a contradiction. Hence there are no gaps. If G_k is discontinuous at b then there is an atom there. Suppose this is the case. Then party $-k$ should not bid just below b: it would be better to bid just a bit above, increasing substantially the probability of winning while increasing cost only a shade. So there would have to be a gap below b and we know that is impossible. □

With G^k continuous the utility for party k from the bid b is $G_{-k}(b)V - b$. Since G_k is continuous and strictly increasing, utility must be constant for any b. As we already know the equilibrium payoffs we may directly compute that $G_L(b) = b/V$ and $G_S(b) = 1 - W_S/V + b/V$. □

2.2. The Tripartite Auction Theorem

As Olson (1965) and many others since have argued, the smaller lobbying group often seems to have much greater success than the larger one. Why is lobbying different than voting? The obvious difference is that we do not think that lobbying is an all-pay auction. Think of the bids $b_k = \eta_k \varphi_k$ as bribes offered to a politician who decides which group gets V. Politicians do not generally collect bribes from each group, rather they typically sell themselves to the highest bidder - taking a bribe only from the group that offers the better bribe. That is: lobbying is typically a winner-pays rather than all-pay auction.

There are two important kinds of winner-pays auctions. One is a first-price sealed bid auction: each of the two lobby groups offers a bribe in a sealed envelope, and the politician returns the envelope holding the smaller bribe. The other is an English auction in which the lobby groups compete with each other increasing their offers until one drops out of the bidding. In this case the winner winds up paying just a shade more than the losers last bid - so from a game theoretic point of view it is pretty much the same as if each group put their best offer in an envelope with the high bid winning - but paying only the losing bid. This is called a second-price sealed bid auction and while less descriptively realistic, it captures the right idea and is easier to analyze.

We now have three different kinds of auctions: all-pay, first price sealed bid and second price sealed bid. What difference does it make?

The second price sealed bid auction is a classic illustration of the idea of dominated strategies. The price you pay if you win does not depend on your bid, only on the other bidders bid. That means the only thing your bid does is determine whether you win or lose. As a consequence you can do no better than bidding your willingness to pay - in that case you win whenever it is advantageous to do so (the other bid and the amount you have to pay for winning is less than your willingness to pay) and lose whenever it is advantageous to do so. So: if each group bids their willingness to pay, since $W_L \geq W_S$ the large group always bids at least as much as the small group and gets $V - W_S$, while the small party gets 0. The amount that the groups earn is exactly the same as in the all-pay auction. The description of what happens is rather different, however: while in the all-pay auction it is necessarily uncertain which party wins, in the second price sealed bid auction if the stakes are high enough, $V > \eta_S$ the large party wins for sure.

In the first price sealed bid auction if $V < \eta_S$ - so that both groups have the same willingness to pay - it is pretty obvious that the only pure strategy equilibrium is for both groups to bid their willingness to pay. Indeed, if the winning bid is less than that, it would pay the other group to bid a shade more, while if the winning bid is that amount and the losing group bid less, the winner would want to bid less. This is the same as for the sealed bid second price auction. On the other hand, when $V > \eta_S$ the small group cannot bid more than η_S, so the only equilibrium is for the large group to bid this amount and win for sure. Notice that here the tie-breaking rule must be endogenous: it must be that in case of a tie the large group wins. If we tried to say that the prize is split equally in case of a tie, the large group would always try to bid the smallest number bigger than the tie and there is no such number.[8] The fact that the large group wins in equilibrium reflects the fact that it is the group willing to bid a bit more in order to win. So we see that it does not matter whether the winner pays auction is a first price or second price auction.

This result, the tripartite auction theorem, says that with a certain prize the utility of the bidders in an all-pay, first price sealed bid and second price sealed bid auction is exactly the same. This result is quite robust - it does not require the two bidders to have the same costs for providing effort, nor does it require that they value the prize the same way. Notice that the tripartite auction theorem has nothing to do with the well-known revenue equivalence theorem - the tripartite auction theorem is about the bidders utility in an auction with a commonly known value, while the revenue equivalence theorem is a theorem about the sellers utility in an auction with private values. In fact, while it is possible to show that a first price sealed bid and second price sealed bid auction generates the same revenue to the auctioneer, this is certainly not the case for an all-pay auction.

So our theory can explain why elections are uncertain and lobbying much less so. But it certainly does not explain why small lobbying groups are effective - quite the opposite, it says that they should be ineffective. Whatever it is that explains the difference between elections and lobbying it is not the fact that one is an all-pay auction and the other a winner pays auction.

[8] The endogeneity of the tie-breaking rule follows from the fact that the willingness to bid is a finite number. This is equivalent to having an exogenous cap on admissible bids.

2.3. Costs of Organization

According to our theory the large group should not do worse than the small group in either voting or lobbying. In Table 2.1 we give some data about farm subsidies and the size of the agricultural sector. In these advanced highly urbanized countries, agriculture is a tiny fraction of GDP, less than 3%. Yet the amount of time annually that the average person must work to pay these subsidies is as high as half a week. More importantly, there seems to be a systematic relationship: the less important is agriculture the more time non-farmers have to work in order to support them. It really does seem that smaller groups are more effective.

Table 2.1: Farm Subsidies

country	% agriculture	farm subsidy hours
Switzerland	0.8	23
Japan	1.2	19
U.S.	1.3	11
Norway	1.6	17
EU	1.7	14
Canada	1.7	8
Australia	2.4	2

This is a subset of OECD countries of similar development characteristics and size. Iceland, Israel and New Zealand are excluded as they are much smaller. Mexico and Turkey are excluded because they are much poorer. Korea is excluded because it has stable democratic institutions for a shorter period of time. The % agriculture is the share of agriculture in value added in 2014 from http://data.worldbank.org/indicator/NV.AGR.TOTL.ZS. The farm subsidy hours is total agricultural support as a percent of GDP in 2014 from http://stats.oecd.org/viewhtml.aspx?QueryId=70971&vh=0000&vf=0&l&il=&lang=en multiplied by 2000 working hours per year.

Since the farmers are obviously not winning elections, it must be that they are successful at lobbying. But why? Why does not the 90% plus of the people in the economy who are not farmers form an anti-farm lobby and prevent the farmers from picking their pocket? The question seems to answer itself. Take the United States. Is it worth it to take the time and effort to find, learn about, join and support an anti-farm lobby in hopes of getting an extra 11 hours a year? It is hardly worth it to the lobby to vet me, process my application and so forth if I am only going to contribute the equivalent of a few hours a year. There is a substantial fixed cost in joining an organization: you cannot simply write a check for 32 cents to the "anti-farm lobby" as an effective way to lobby against them - it would cost more than 32 cent to process my check. Considerable cost would be incurred even as I contributed absolutely nothing to the lobbying effort. We will describe this situation where there is a fixed cost of effort provision as a *chore*.

Elections provide a contrast: for example, if there is a ballot referendum against farm subsidies, while it is costly to go to the polling place and take time to vote I view it as my civic duty, so I vote and the satisfaction of having discharged my duty might more than offset the direct cost of participating. While lobbying is a chore voting is a duty. Here we interpret duty in the broad sense of a benefit - for example, a political demonstration or protest might be an enjoyable event - to be outdoors in good weather, meet new people, chant, march and sing - these bring benefits that offset the cost of time and commuting.

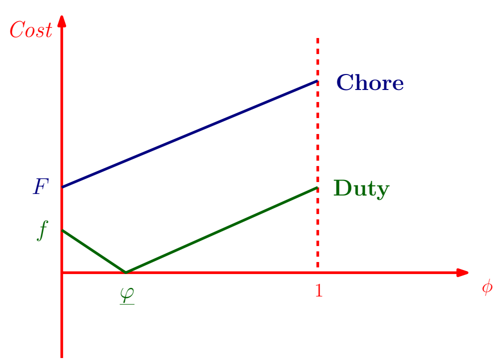
Duties Versus Chores. As the examples may indicate, we tend to think of voting as a duty and lobbying as a chore. This is a fundamental distinction and we shall find that duty favors larger groups and chore favors smaller groups. We modify our model of costly provision of effort $\varphi_k \in [0,1]$ to allow for a per capita fixed cost of $F \geq 0$ of organizing the group and an individual level of duty $1 > \underline{\varphi} \geq 0$. Until the duty level $\underline{\varphi}$ is fulfilled the marginal cost of effort is negative $-f < 0$ since you have the satisfaction of fulfilling your duty. Here f is the benefit of duty. Beyond the duty level $\underline{\varphi}$, as before, additional effort has unitary marginal cost. Define $q(\varphi_k)$ to be 1 if $\varphi_k > \underline{\varphi}$ and 0 otherwise. We can write the per capita cost of effort provision as

$$C(\varphi_k) = q(\varphi_k)F + f \cdot \max\{0, \underline{\varphi} - \varphi_k\} + \max\{0, \varphi_k - \underline{\varphi}\}.$$

We focus on two polar cases: effort is a *duty* meaning $\underline{\varphi} > 0$ and $F = 0$; and effort is a *chore* meaning $\underline{\varphi} = 0$ and $F > 0$.

Figure 2.1 illustrates.

Figure 2.1: Cost of Effort Provision



In the case of chore the cost for $\varphi_k > 0$ is $F + \varphi_k$. In the case of duty $F = 0$ for $\varphi_k = 0$ the cost is f - the cost of not fulfilling the duty. As φ_k increases the cost becomes zero at $\varphi_k = \underline{\varphi}$ when the duty is fulfilled, and it starts increasing thereafter.

As before, the *willingness-to-bid* of a group is the greatest amount of effort the group would be willing to provide to get the prize for certain. Naturally, since it brings a benefit, a group is always willing to provide $\eta_k \underline{\varphi}$ units of effort. If a greater level of effort is provided, the additional total cost is $\eta_k F + \eta_k(\varphi_k - \underline{\varphi})$. If we equate that to V and solve for effort $B_k = \varphi_k \eta_k$ we find the *desire to bid*

$$B_k = \eta_k \underline{\varphi} + V - \eta_k F.$$

If B_k is less than $\eta_k \underline{\varphi}$ then the willingness to bid is $W_k = \eta_k \underline{\varphi}$. If B_k is greater than η_k the the willingness to bid is $W_k = \eta_k$. Otherwise, for $\eta_k \underline{\varphi} \leq B_k \leq \eta_k$, we have $W_k = B_k$. Note that the benefit of duty f does not enter into this calculation because the group can receive that benefit regardless of whether or not it wins the prize.

Consider first the interesting case in which $V > \eta_S F$.[9] A group with the highest willingness to bid is called the *advantaged group* and the other group is called *disadvantaged*. We say the prize is *medium* if $V < F\eta_L + \eta_S$ and the prize is *large* if $V > F\eta_L + \eta_S$. There are three key results

- The level of utility of the two groups is the same regardless of whether the prize is allocated by an all-pay, first-price or second-price auction.
- Only an advantaged group can receive a positive level of utility and always does so.
- The small group is advantaged for a chore with a low to medium prize, the large group is advantaged for a duty, and for a chore with a large prize.

We indicated that our earlier theory with $F = \underline{\varphi} = 0$ worked well for voting. If we think of voting as a duty the result here strengthens that: the large group is advantaged and while with $\underline{\varphi} = 0$ the large party earned no utility with $\underline{\varphi} > 0$ the large party always earns something. For a chore such as lobbying we get a different result: for a small prize it is the small group that is advantaged. Roughly speaking, with a fixed cost per person of organization, a large group faces a greater fixed cost so it is less willing to bid. However, if the prize is big enough they will take advantage of their greater resources to get the prize.

Notice that in Table 2.1 the prize is indeed relatively modest. While farmers are successful at getting subsidies, they are not imposing a very great tax on the non-farmers. If, for example, the numbers for the amount of time spent paying for farm subsidies in Table 2.1corresponded to months rather than hours, it seems likely that the non-farmers would lobby and lobby effectively. Indeed the defeat in the U.S. Congress of the "Stop Online Piracy Act" mentioned earlier

[9] This rules out the case in which, for a chore, we may have $B_k < \eta_k \underline{\varphi} = 0$ for both k in so that neither group submits a bid: neither is willing to pay the fixed cost even for a certainty of getting the prize.

seems to be a case in point. The act was put forward by the pro-copyright lobby. More modest efforts to impose broad internet restrictions on general internet users to protect a few holders of copyrights had passed the U.S. Congress relatively easily. This more ambitious act was sponsored by a majority of the U.S. Congress, but the drastic nature and the non-negligible consequences of the act led to a broad grass roots lobbying effort against it. As a result many of the sponsors dropped out and the act was quietly shelved.

Auctions with Duties and Chores.

Theorem 2.6. *The small group is advantaged in a chore with a low to medium prize. Otherwise, the large group is advantaged.*

Proof. The cost to party k of a bid b is $\eta_k C(b/\eta_k) = b\,(C(b/\eta_k)/(b/\eta_k))$. In other words the party with the lower cost for the bid b will be the one for whom the average cost is lower. The key point to bear in mind is that the small party must always choose a higher value of φ_k to match a bid of the large party, simply because the bid is $b_k = \varphi_k \eta_k$ and $\eta_S < \eta_L$.

If $C(\varphi_k)$ is globally convex, as is true in the case of a duty, then average cost $C(\varphi_k)/\varphi_k$ is increasing and it follows that $\eta_L C(W_S/\eta_L) < \eta_S C(W_S/\eta_S)$. This implies that the large group is willing to pay more than W_S, that is, $W_L > W_S$.

For a chore with a small prize we have $V < F\eta_L + \eta_S$. Hence $W_L < \eta_S$. On the range $[0,1)$, $C(\varphi_S)$ is concave. Since average cost is decreasing, $\eta_S C(W_L/\eta_S) < \eta_L C(W_L/\eta_L)$. This implies that the small group is is willing to pay more than W_L, that is, $W_S > W_L$.

Finally, if $V > F\eta_L + \eta_S$ then $W_L > \eta_S \geq W_S$ so the large party is advantaged. □

The key idea here is that the convexity or concavity of $C(\varphi_k)$ determines whether average costs of bidding are increasing or decreasing. The smaller group must always choose a higher fraction φ_k to match the bid of the larger group. In the convex case this implies a higher average cost disadvantaging the smaller group, and conversely in the concave case, provided that the small group is able to bid that high. Note that the fixed cost plus constant marginal cost is not important here, merely the overall convexity or concavity of the function $C(\varphi_k)$.

A brief overview of the model so that we understand what constitutes an equilibrium is this: a pure strategy for group k is a social norm φ_k. If the group has probability p_k of winning the prize and follows the pure strategy φ_k it receives utility $p_k V - q(\varphi_k)\eta_k F - \eta_k \max\{0, \varphi_k - \underline{\varphi}\} + \eta_k f \max\{0, \underline{\varphi} - \varphi_k\}$.

Let d be the disadvantaged group. If $W_d \geq \eta_{-d}\underline{\varphi}$ it costs the advantaged group $W_d - \eta_{-d}\underline{\varphi} + \eta_{-d}F$ to match the bid of the disadvantaged group, while if $W_d < \eta_{-d}\underline{\varphi}$ it costs nothing to overbid the disadvantaged group. We define the *surplus* as the difference between the value of the prize and the cost of matching the bid of the disadvantaged group if this is positive, zero otherwise.

Theorem 2.7 (Tripartite Auction Theorem)**.** *In the second-price, first-price and all-pay auction, the disadvantaged group gets 0 and the advantaged group* *gets the surplus. It follows that the expected effort provided is the same for all three mechanisms.*

The argument given in Section 2.2 shows that for the winner-pays auctions the disadvantaged group gets 0 and the advantaged group the value of the prize minus the cost of matching the willingness to bid of the disadvantaged group. For the all-pay auction the computation of utilities follows the lines of that for the simple all-pay auction in Theorem 2.2.

Say that a group *opts out* in the case of a duty if it turns out the minimal effort $\eta_k \underline{\varphi}$. If $\eta_L \underline{\varphi} < W_d < \eta_S$ then the disadvantaged group is willing not to opt out and both groups are able to bid the willingness to pay of the disadvantaged group. Also, it is easily verified that in this case the prize is not large. Then we have a more precise description of the equilibrium.

Theorem 2.8. *If $\eta_L \underline{\varphi} < W_d < \eta_S$ the small group is advantaged for chores, the large group for duties and the surplus is $(\eta_L - \eta_S)(F + \underline{\varphi})$. In the all-pay case groups opt out with equal probability and otherwise play uniformly on $[\eta_L \underline{\varphi}, W_d]$.*

The calculation of surplus is just a matter of subtracting desire to bid. For the all-pay auction the proof follows the lines of that for the simple all-pay auction, Theorem 2.2. This result as stated compresses information so may sound misleading: despite the fact that both groups opt out with equal probability and play the same uniform distribution when they do not opt out, the advantaged group has a higher probability of winning. This is because when both groups opt out the advantaged group wins. In the case of a duty when the large group is advantaged if both groups opt out the large group wins for sure since it has more committed members. Hence the large group has a higher probability of winning. In the case of a chore if both groups opt out the definition of opting out requires that the advantaged group pay the fixed cost and the disadvantaged group not to. Hence also in the case of a chore the advantaged group has a higher chance of winning.

2.3.1. Appendix: Types of Equilibria in the All-Pay Auction

In the all-pay auction there are qualitatively different equilibria depending on the size of the prize. We categorize this by the level of stakes, running from high to very low and summarize the situation in the table below.

stakes	condition	advantaged group
high	$W_S = \eta_S, W_L > \eta_S$	large
medium	$\eta_S > W_S, W_L > \eta_L \underline{\varphi}$	duty: large; chore: small
low	$W_S < \eta_L \underline{\varphi}$ or $V < \eta_L F$	duty: large; chore: small
very low	$V < \eta_S F$	none

In describing the equilibrium it is useful to introduce the concept of bidding *at the bottom*. For a duty this means bidding only the committed members $b_k = \eta_k \varphi_k$. For a chore it means bidding zero and for the small group paying the fixed cost $q_S(0) = 1$ and for the large group not paying the fixed cost $q_L(0) = 0$.

High stakes. The constraint that the greatest effort group k can provide is η_k cannot bind on the large group since if the large group is willing to bid η_L and the small group can bid at most η_S if the constraint was binding on the large group it would already bind on the small group. In this case the small group would have to be disadvantaged and the large group would not need to bid more than η_S. We refer to this as the *high stakes* case. It occurs when both groups desire to pay exceeds the ability of the small group to pay and the ability of the small group to pay exceeds the committed bid of the large group. For duties this is the same as the desire to pay of the small group exceeding its ability to pay. For chores this as the willingness to pay of the large group exceeding the ability of the small group to pay. In all cases the large group is advantaged.

Description of Equilibrium: Both groups bid using the same uniform distribution on $[\eta_L \underline{\varphi}, \eta_S]$. Both groups have positive probability of bidding at the bottom. The large group also has a positive probability of bidding η_S winning for sure if there is a tie.

The probability of the large group bidding at the bottom is determined by the small group earning zero. For a duty the probability the small group wins by bidding the large group committed effort times the value of winning for sure must equal the cost of that bid $G_L(\eta_L \underline{\varphi})V = (\eta_L - \eta_S)\underline{\varphi}$. For a chore the probability the small group wins by incurring the fixed cost times the value of winning for sure must equal the fixed cost to the small group $G_L(0)V = \eta_S F$. The remaining probability is determined by computing the probability p_L "left over" from the uniform. The height of the uniform is $1/V$ so the probability of the uniform is

$$1 - p_L = \frac{\eta_S - \eta_L \underline{\varphi}}{V}$$

The small group bids 0 with exactly probability p_L while the large groups bids the top η_S with probability $p_L - G_L(0)$, that is, in the case of a duty

$$\frac{V - (\eta_S - \eta_L \underline{\varphi}) - (\eta_L - \eta_S)\underline{\varphi}}{V} = \frac{V - \eta_S(1 - \underline{\varphi})}{V}$$

and in the case of a chore

$$\frac{V - (\eta_S - \eta_L \underline{\varphi}) - \eta_S F}{V} = \frac{V - \eta_S(1 + F)}{V}$$

Medium stakes. If the willingness to pay of both groups is less than the ability of the smaller group to pay and the willingness to pay of both groups exceeds the committed bid of the large group then the constraint does not bind on the small group and both groups are active in bidding. For duties this is the same as the desire to pay of the small group being less than its ability to pay but greater than the committed bid of the large group. For chores this is the same as the willingness to pay of the large group less than the ability of the small group to pay but positive. This case also be described as the *interior case*. Here the advantage depends on whether cost are a duty or a chore: in the case of a duty the large group is advantaged and in the case of a chore the small group.

Description of equilibrium: Both groups bid using the same uniform distribution on $[\eta_L \underline{\varphi}, W_d]$. Both groups also bid with the same positive probability at the bottom. The probability of a bottom bid is determined by the disadvantaged group earning zero. For a duty the probability the small group wins by bidding the large group committed effort times the value of winning for sure must equal the cost of that bid $G_L(\eta_L \underline{\varphi})V = (\eta_L - \eta_S)\underline{\varphi}$. For a chore the probability the large groups wins by incurring the fixed cost times the value of winning for sure must equal the fixed cost to the large group $G_S(0)V = \eta_L F$

Low stakes: duty. If the willingness to pay of the small group is less than the committed bid of the large group then the small group will bid only its committed members and the large group should do the same. The large group is advantaged and wins for certain.

Description of equilibrium: Both groups bid their committed voters with probability one.

Low stakes: chore. If the willingness to pay of the small group is positive but that of the large group is not the large group will not bid nor pay the fixed cost. The advantage lies with the small group and there are two different types of equilibrium. First the small group may also not bid and win for sure. Second the small group may pay the fixed cost and if it does not do so it loses with sufficiently high probability that it does not gain.

Description of equilibrium 1: Neither group pays the fixed cost, the small group wins with probability p_S high enough it does not wish to enter $p_S V \geq V - \eta_S F$.

Description of equilibrium 2: The large group does not pay the fixed cost, the small group does. If the small group fails to pay the fixed cost it wins with probability no greater than p_S determined by the condition it cannot profit by failing to pay the fixed cost $p_S V \leq V - \eta_S F$.

Very low stakes: chore. In the case of chores only it may be that neither group is willing to pay the fixed cost. In this case neither does and the tie-breaking rule is arbitrary.

Low and very low stakes: chore. In the case of a chore with low or very low stakes there are equilibria in which neither group pays the fixed cost. Rather than applying an arbitrary tie-breaking rule in this case in many applications is may be more interesting to assume that the contest does not take place and both groups get zero. In the case of low stakes this rules out the equilibrium of type 1 leaving on the equilibrium of type 2 in which the small group wins by paying the fixed cost.

2.4. Why Lobbyists Win

There are two main themes of these book: why pollsters are wrong and why lobbyists always win. We are now in a position to explain why lobbyists win. As we observed earlier - they do not. The Disney Corporation is very effective in getting retroactive copyright extensions whenever their Mickey Mouse copyright is due to expire - but large pharmaceutical companies have never managed to get a retroactive patent extension when their blockbuster drug patents are due to expire. The copyright industry does sometimes lose in Congress as it did when it proposed the "Stop Online Piracy Act." If we accept that lobbying is a chore then indeed small groups - "special interests" have an advantage at lobbying - they derive advantage from the fact that they incur - as a group - a lower fixed cost of providing resources for lobbying. On the other hand, they control fewer resources so if the prize is large and both groups are "all in" they will lose. This is the heart of Theorem 2.6: small groups are advantaged for chores provided the prize is not too large. Indeed, the cost to the large group of pharmaceutical consumers - and generic manufacturers - of patent extension is very large so it is difficult for the small group of pharmaceutical producers to succeed. On the other hand, the size of the prize involving copyrights over Mickey Mouse is relatively small so that the Disney corporation is quite successful. In general the stakes are low for copyright and the copyright lobby is quite effective. Once in a while the prize gets too large - as it did with the "Stop Online Piracy Act" and it becomes worthwhile for the large group to suffer the fixed cost and start lobbying. That is a pretty good description of the events surrounding the "Stop Online Piracy Act." Usually copyright laws are fixed in the Congress in the dark of night - and "everyone else" does not find it worthwhile to pay the fixed cost of getting involved. The "Stop Online Piracy Act" was attempted in the same way, but the stakes were higher, organizations such as Wikipedia became involved in coordinating lobbying, and suddenly ordinary people started phoning and emailing their congress members. Rather than passing in the dark of night the "Stop Online Piracy Act" vanished into the dark of night.

3. Groups and the Provision of Public Goods

The problem in the model we have studied is that the groups act - in particular, bid - as single individuals. In reality groups are made up of individuals and often in large number, in the sense that even in the case of the "small" farm lobbying group, there are over two million farms in the United States so that when we speak of the "farm lobby" we are in fact speaking of a great many people. The problematic aspect of this is that when we construct the game based on the individual members of the group we immediately see that there is an enormous public goods problem. In political economy this is called the paradox of voting. The chances of an individual vote changing the outcome of an election is so small that the incentive to vote is negligible - so indeed, why does anybody bother? Similarly why do farmers contribute to lobbying efforts when their individual effort makes little difference? Everybody of course would like their group to win the contest - but of course would much prefer that everyone else contribute to the effort while they do not.

3.1. Individual Effort

So far we have treated effort as being continuously provided by a group. At the individual level voting is indivisible: either a party member member votes or does not vote but does not cast half a vote. In lobbying effort may be more continuous, but often the group asks for a fixed levy of time, effort, or money. We will focus on the case of a simple decision of whether or not to participate in a group effort - that is, to provide a unit of effort. This effort is costly. Even if group members are *ex ante* identical, typically people will face different participation costs at the time the participation decision is made. To take an example: it may be that on election day a party member is in the hospital and so it is very costly for the member to vote that day. A standard model is that group members independently draw types y_k uniformly distributed on $[0, 1]$ and may contribute zero effort at zero cost (not participate) or contribute a single unit of effort (participate). The cost of participation is $c(y_i)$, where we assume that types are ordered so that this is a non-decreasing function: higher types have higher cost. - Furthermore, we assume that cost is linear $c(y_i) = c_0 + y_i$.

Effort for group k is determined by a threshold φ_k for participation. We may regard this as a *social norm*: those types with $y_i < \varphi_k$ are expected to participate and those with $y_i > \varphi_k$ are not. If the social norm is followed, the expected fraction of the group that will participate is φ_k and in a large group we may assume that since we are averaging over many independent draws the realized participation is equal to the expected value.

From an individual point of view the social norm seems meaningless: in a large group the chances of an individual influencing the outcome is negligible so the individual sees only the cost $c(y_i)$ and will naturally want to minimize it in choosing whether to participate or not. Define $\underline{\varphi} = -c_0$ if this is between 0 and 1, to be 0 if $-c_0$ is negative and to be 1 if $-c_0$ is is greater than 1. We call $\underline{\varphi}$ the fraction of *committed voters* - those who find it individually optimal to participate because they have negative voting costs.

> Why do voters vote? Two reasons given by political scientists are civic duty and expressive voting. Both can be interpreted as a negative cost of voting, although for different reasons. Civic duty means voting out of a sense of obligation to society. Expressive voting is more akin to low stakes sport betting on a favored team: a way to show solidarity with or support for a favored candidate cause - not out of expectation of winning but as a symbolic gesture. Both motivations for voting are captured by the theoretical notion of committed voters.

If voters behave purely individualistically this is the end of the story: the committed voters vote, the rest do not and the only meaningful social norm is $\varphi_k = \underline{\varphi}$. Voting in this theory is completely non-strategic and whichever party has the most committed voters wins for sure. Since there is plenty of evidence that this is not the case - for example, turnout in high stakes national elections is higher than in low stakes local elections - nobody believes this story.

3.2. Peer Enforcement

In practice large groups have little difficult in overcoming public goods problems. Often coercion is involved: for example through mandatory voting laws, a military draft or penalties for tax evasion. In the setting of political groups this kind of direct coercion is not relevant - farm lobbies cannot punish non-contributors. Even in settings where coercion is relevant it is rarely the entire reason why people contribute to public goods: consider for example the huge increase in military enlistments to go fight in Afghanistan after September 11. There is, however, another form of coercion: peer pressure.

A crucial reason people vote is because they want to keep the good opinion of their friends and neighbors. The important role of peer pressure as a motivation is well documented, and it is widely discussed in the sociology literature, for example Coleman Coleman (1988). To take a few of many pieces of evidence, Della Vigna et al (2014) show that an important incentive for voters to vote is to show others that they have voted; Gerber, Green and Larimer (2008) shows that social pressure significantly increase voter turnout; Palfrey and Pogorelskiy (2016) provide experimental evidence showing that communication among voters and in particular communication within parties increases turnout. Typically social norms are maintained by various forms of social disapproval and ostracism.[10] This is well documented in Elinor Ostrom's work, especially Ostrom (1990).

Consider a simple model of peer punishment used to sustain a social norm based on Levine and Modica (2016) and Levine and Mattozzi (2016). We assume that group members are organized into a simple social network on the circle. The action of a member, whether she has participated or not, is observable by everyone, but there is only a noisy signal of the type. In particular, for those who did not participate, the signal is $z^i \in \{0, 1\}$ where 0 means "good,

[10] This is in contrast to models of social conformity such as Akerloff and Kranton (2005) which do not explicitly consider punishments or rewards.

followed the social norm" and 1 means "bad, did not follow the social norm." If the social norm was violated, that is $y_i < \varphi_k$ but member i did not participate, the bad signal is generated for sure, while if i did not participate but did follow the social norm so that $y_i > \varphi_k$, there is nevertheless a chance θ of the bad signal where θ is a measure of the noise of the signal. This signal is observed only by adjacent network members, who report it honestly to the group.[11] When the bad signal is reported the group member receives a punishment of size P_k.

A social norm φ_k is *incentive compatible* if and only if $P_k = c(\varphi_k)$. Any member with $y \leq \varphi_k$ would be willing to pay the participation cost $c(y)$ rather than face the certain punishment P_k, while any member with $y > \varphi_k$ prefers to pay the expected cost of punishment θP_k over the participation cost of voting $c(y) > \theta P_k$. The punishment itself, as it is paid by a member, is a cost to the party. For simplicity we assume that the overall cost of a punishment P_k to the party is exactly P_k.[12]

We are now ready to determine how costly it is for the group to induce additional members other than the committed ones to participate. Of course if $\underline{\varphi} = 1$ the problem does not arise, so we assume $\underline{\varphi} < 1$, that is $-c_0 < 1$. We continue to measure all costs per capita. The *total cost* of choosing an incentive compatible social norm $\varphi_k \geq \underline{\varphi}$ is denoted by $C(\varphi_k)$ with the convention that $C(\varphi_k) = 0$ for $\varphi_k \leq \underline{\varphi}$. We can decompose the total cost $C(\varphi_k)$ into two additive components. The first component is the *turnout cost* $T(\varphi_k) = \int_{\underline{\varphi}}^{\varphi_k} c(y)dy$, which is the participation cost of voting to the member types who vote. The second component is the *monitoring cost* $M(\varphi_k) = \int_{\varphi_k}^{1} \theta P_k dy$, which is the (expected) cost of punishing party members who did not vote. Substituting the incentive compatibility condition $P_k = c(\varphi_k)$ we can write $M(\varphi_k) = \theta c(\varphi_k)(1 - \varphi_k)$. For $\varphi_k \geq \underline{\varphi}$ the total cost is $C(\varphi_k) = T(\varphi_k) + M(\varphi_k)$.

How does this cost function relate to our earlier model of either a duty or a chore with constant marginal cost? If we define $F = \max\{0, \theta c_0\}$ $\gamma = [(1/\theta) - 1]F + \theta(1 - \underline{\varphi})$ and then we may explicitly compute the total cost as

$$C(\varphi_k) = F + \gamma(\varphi_k - \underline{\varphi}) + (1/2)(1 - 2\theta)(\varphi_k - \underline{\varphi})^2$$

In the special case where $\theta = 1/2$ this is linear and identical to the earlier model. As we assumed there, either $F > 0$ or $\underline{\varphi} > 0$ but not both. Chores are characterized by $c_0 > 0$ and duties by $c_0 < 0$.

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[11] We are assuming that there is no cost to observing and reporting signals. If there is, additional rounds of monitoring and punishment are needed so that the monitors will behave honestly. We examine this case below.

[12] However, there may be other costs: for example, if the punishment is ostracism this may not only be costly to the member punished, but also to other party members who might otherwise have enjoyed the company of the ostracized member.

Effort as Numeraire. Notice that in the earlier model we normalized $\gamma = 1$. Here we normalized individual cost so that $c(y_i) = c_0 + y_i$. Since the units of cost are arbitrary we can use either normalization: however the size of the prize must be measured in compatible unit. That is, we can divide $C(\varphi_k)$ above by γ to get the same normalization as in the original model, but then we must also divide V by γ.

Why is there a fixed cost when $c_0 > 0$? In this case even the lowest draws of y find it costly to participate. If nobody participates the cost of course is zero. To get anyone to participate, though, incentives must be provided: and a punishment of at least c_0 is needed before anyone will participate. However, when very few participate most bad signals will arise from those who are legitimately excused from participation in order that those few who are supposed to participate will do so. Each of those who are legitimately excused will have an expected punishment of θc_0 and this is the fixed cost of, so to speak, getting thing off the ground. To get anyone to participate we have to monitor everyone and this is costly.

The monitoring costs play a key role. If $\theta = 0$ so there are no monitoring costs, then $F = 0$ and the small group cannot be advantaged. It should be clear now that we can replace duty with the global convexity of $C(\varphi_k)$ and chores with the global concavity of $C(\varphi_k)$. Hence, large party advantage arises when $\underline{\varphi} > 0$ and $\theta < 1/2$. On the other hand, small party advantage arises for a small prize when $F > 0$ and $\theta > 1/2$. High monitoring cost favors the small party because it forces the quadratic component of $C(\varphi_k)$ to have a negative coefficient. We can see this also looking at the turnout cost. The turnout cost can be computed by integration to be $T(\varphi_k) = (F/\theta)\varphi_k + (1/2)(\varphi_k - \underline{\varphi})^2$. For $c_0 = 0$ this is the same quadratic expression used in Coate and Conlin (2004) and it is convex. By contrast the monitoring cost can be shown to be equal to $M(\varphi_k) = \theta(1 - \varphi_k)[(F/\theta) + (\varphi_k - \underline{\varphi})]$ and it is concave above $\underline{\varphi}$.

Why is the turnout cost convex while the monitoring cost is concave? The turnout cost is convex because the individual with lowest cost participate first, hence there is increasing marginal cost of turnout. The monitoring cost must have an element of concavity for two reasons. If $c_0 > 0$ it gives rise to a fixed cost. Furthermore, when $c_0 < 0$ it is zero at both $\underline{\varphi}$ and at 1. At $\underline{\varphi}$ there is no cost of monitoring because there is no need of punishment. At 1 there is no cost of monitoring either because everyone participates and nobody is punished for not having a good excuse.

The Algebra of Participation Cost

We derive the cost as follows. First, notice that the assumption $\underline{\varphi} < 1$ is equivalent to $c_0 > -1$, and in this case $\gamma > 0$ and $c_0 + \underline{\varphi} = \max\{0, c_0\} = F/\theta$. So monitoring cost is

$$M(\varphi_k) = \theta(1-\varphi_k)[c_0 + \underline{\varphi} + (\varphi_k - \underline{\varphi})] = \theta(1-\varphi_k)[F/\theta + (\varphi_k - \underline{\varphi})].$$

It is convenient to rewrite this measuring φ relative to $\underline{\varphi}$ as

$$M(\varphi_k) = \theta[(1-\underline{\varphi}) - (\varphi_k - \underline{\varphi})][F/\theta + (\varphi_k - \underline{\varphi})]$$

and express this more clearly as a quadratic

$$M(\varphi_k) = [(1-\underline{\varphi})][F + \theta(\varphi_k - \underline{\varphi})] - [(\varphi_k - \underline{\varphi})][F + \theta(\varphi_k - \underline{\varphi})]$$

$$M(\varphi_k) = (1-\underline{\varphi})F + \theta(1-\underline{\varphi})(\varphi_k - \underline{\varphi}) - F(\varphi_k - \underline{\varphi}) - \theta(\varphi_k - \underline{\varphi})^2$$

$$M(\varphi_k) = (1-\underline{\varphi})F + [\theta(1-\underline{\varphi}) - F](\varphi_k - \underline{\varphi}) - \theta(\varphi_k - \underline{\varphi})^2$$

and we may observe that $(1-\underline{\varphi})F = F$ so that we have

$$M(\varphi_k) = F + [\theta(1-\underline{\varphi}) - F](\varphi_k - \underline{\varphi}) - \theta(\varphi_k - \underline{\varphi})^2,$$

which is clearly concave. Next we compute the total cost

$$C(\varphi_k) = (F/\theta)\varphi_k + (1/2)(\varphi_k - \underline{\varphi})^2 + F + [\theta(1-\underline{\varphi}) - F](\varphi_k - \underline{\varphi}) - \theta(\varphi_k - \underline{\varphi})^2.$$

Again noticing that when $F > 0$ it must be that $\underline{\varphi} = 0$ we can rewrite this as

$$C(\varphi_k) = (F/\theta)(\varphi_k - \underline{\varphi}) + (1/2)(\varphi_k - \underline{\varphi})^2 + F + [\theta(1-\underline{\varphi}) - F](\varphi_k - \underline{\varphi}) - \theta(\varphi_k - \underline{\varphi})^2.$$

Collecting terms this becomes

$$C(\varphi_k) = F + \left((1/\theta) - 1)F + \theta(1-\underline{\varphi})\right)(\varphi_k - \underline{\varphi}) + (1/2)(1-2\theta)(\varphi_k - \underline{\varphi})^2$$

as given in the text.

3.3. Endogenous Social Norms

How is the φ_k threshold determined? Our key assumption is that the social norms enforced by peer pressure are endogenous, and maximize a group objective. The endogeneity is hardly questionable: we know that turnout, for example, in U.S. national elections is considerably higher in presidential election years than off-years, and in general participation rates and the social norms that lead to them adjust strategically to reflect the stakes in the elections. A simple starting point for modelling endogenous social norms that reflect party interests is to assume that they are optimal with respect to a group objective function. Indeed, Coleman (1988) and Ostrom (1990) as well as Olson (1965) argue that - within the limits of available monitoring and punishment - peer pressure mechanisms do a good job of solving public goods problems. That is our assumption as well. We would prefer that this was not the case - it would be well if lobbying groups were not effective in looking out for their best interests. Unfortunately it rarely seems so.

Given the strategy of the other group let $p_k(b_k)$ denote the probability of winning as a function of the bid. Group utility as a function of the social norm is $p_k(\eta_k\varphi_k)V - \eta_k C(\varphi_k)$. This is the same type of model we have been studying: now the cost to the group of turnout includes the monitoring cost needed to establish the social norm as incentive compatible.

The results of Theorem 2.6 for the small group advantaged holds for C concave and for the large group advantaged holds for C convex. Hence our main conclusions

- Costly monitoring favors the small group and cheap monitoring the large group.
- Effort can only be a chore if monitoring costs are positive.

3.4. Rule Consequentialism and Altruism

Suppose each group member asks what would be in the best interest of the group, that is, what social norm is most advantageous for the group? Then having determined this hopefully unique social norm each member "does their part" by implementing $\varphi^i = \varphi_k$. This is called the *rule-consequentialism* and has been studied by Harsanyi (1977), Coate and Conlin (2004), Roemer (2010), Hooker (2011). Conceptually the individuals choose the rule that is the best for the group and implements their part of it. The same model is called the ethical voter model in the voting context and has been studied by Riker and Ordeshook (1968), Feddersen and Sandroni (2006), Li and Majumdar (2010), Ali and Lin (2013). Conceptually this is supposed to capture the idea that it is unethical to free ride. Implicitly this is the model used in most of the lobbying literature where it is assumed that the group acts to maximize a particular objective. In principle rule consequentialism can be decentralized so that each group member independently calculates what they are supposed to contribute. Roemer (2010) points out this requires that members know each others utility functions and proposes and alternative Kantian notion of equilibrium which can be more completely decentralized.

With ethical voters or rule consequentialism the group maximizes $p_k(\eta_k\varphi_k)V - \eta_k T(\varphi_k)$ - it should be apparent that this is equivalent to $\theta = 0$ that is, there are no monitoring costs. Indeed, social norms may be internalized - in effect people punish themselves for violating the social norm. In this case the signal is perfect since we have no trouble observing ourselves and this is an alternative interpretation of rule consequentialism - a model of monitoring where because monitoring is internal it has no error and hence no cost.

Rule consequentialism as with peer enforcement was developed to explain why people contribute effort in the face of a severe public goods problem. An alternative explanation is altruism: people contribute because they care about their fellow group members.

Take a pure altruist who cares only about the total utility of the group. Suppose the fraction of the population represented by a single person is $1/N$. If no effort is contributed by the altruist, total group effort is $\eta_k\varphi_k - (1/N)\varphi_k$ while if the altruist contributes a unit of effort the total group effort is $\eta_k\varphi_k + (1/N)(1-\varphi_k)$. The benefit of contributing for an altruist of type y is the difference in group utility with and without the individual effort:

$$\begin{aligned} & [p_k(\eta_k\varphi_k + (1/N)(1-\varphi_k))V - ((N-1)/N)\eta_k T(\varphi_k) - (1/N)c(y)] \\ - \; & [p_k(\eta_k\varphi_k - (1/N)\varphi_k)V - ((N-1)/N)\eta_k T(\varphi_k)] \,. \end{aligned}$$

For very large N and differentiable $p_k(\cdot)$, the expression above is approximately equal to $(1/N)\,(p_k'(\eta_k\varphi_k)V - c(y))$. Hence, in an altruistic group, each member contributes provided that $p_k'(\eta_k\varphi_k)V > c(y)$.

The model of pure altruism is quite extreme, since people care about their own utility as well. In a model of partial altruism, a weight λ is given to own utility that, for large N, is effectively equal to the negative of the cost of contributing $-c(y)$. Partial altruists have a benefit of contributing approximately equal to $p_k'(\eta_k\varphi_k)V - (1+\lambda)c(y)$. The partial altruist should then choose a social norm ϕ_i so that $p_k'(\eta_k\varphi_k)V = (1+\lambda)c(\phi_i)$ and if everyone does this we have $p_k'(\eta_k\phi_i)V = (1+\lambda)c(\phi_i) = (1+\lambda)T'(\phi_i)$. When $\lambda = 0$, that is in the case of pure altruism, this is also the necessary condition for maximizing $p_k(\eta_k\varphi_k)V - \eta_k T(\varphi_k)$ with respect to φ_k, the problem solved by ethical voters.[13] There is a lot of research in voting that uses the altruism model: for example Schram and Sonnemans (1996), Fowler (2006), Fowler and Kam (2007), Edlin, Gelman and Kaplan (2007), Faravelli and Walsh (2011), Evren (2012), and Jankowski (2007).

The bottom line is that if we are free to choose the cost function and the population is large all three approaches - peer enforcement with low monitoring costs, rule consequentialism and partial altruism give rise to the same mathematical model: group utility net of a convex effort cost is maximized. Some of those who have used the partial altruism model have viewed it as kind of a reduced form of a model arising from underlying peer pressure: to quote Esteban and Ray (2011)

> An equivalent (but somewhat looser) view is that α [λ in our notation] is some reduced-form measure of the extent to which within-group monitoring, along with promises and threats, manages overcome the free-rider problem of individual contribution.

[13] Divide the first order necessary condition for a maximum of $p_k(\eta_k\varphi_k)V - \eta_k T(\varphi_k)$ by the positive constant η_k.

Here we see in a formal sense that this is true. It is important because it means that whichever point of view we take there is no need to disregard the results of studies using one of the other points of view. So if we were to redo Esteban, Ray and Mayoral (2012) replacing their model of altruism with, say, a model of peer pressure, we would not learn anything new because we would get the same result. It is reassuring to know that their results are robust to the particular model they use of group participation.

The peer enforcement model is a rich model that has several advantages over the other two models. First, while altruism towards people we know well may be relatively strong, in the anonymous setting of the laboratory the forces of altruism are relatively weak (give some data/references here) and, for example, the altruism of farmers towards millions of other farmers who live in other states does not seem likely to be terribly strong. Hence measures of λ estimated from voting models would likely be much larger than in other domains. By contrast, rule consequentialism presupposes a remarkable degree of public spiritedness that also is unlikely to hold in other domains. While in a sense it does not matter whether people vote because they are altruistic, or ethical, or because they are punished for not being so, a model of monitoring has richer implications since it relates the degree of altruism or ethicality to the objective circumstances of monitoring. Moreover it provides a model of forces - peer pressure in social networks - that we know are important in explaining contributions to group effort.

Second, models of altruism or ethicality predict that the large group is always advantaged if the value of the prize to the smaller group is no greater than that to the larger group and both groups have similar cost structures. While there is reason to think this is true in many electoral settings there are also cases where small groups are very successful at rent-seeking in elections - and they are certainly so in lobbying. The peer enforcement model is able to account for this as chores arise naturally: with peer enforcement to give even a few people with positive participation costs an incentive to participate it is necessary to "accidentally" punish the many with higher participation costs who will not participate.

Coordination and Altruism

There is one important difference between models such as peer enforcement and rule consequentialism and models of altruism: a group may face a coordination problem and peer enforcement and ethical voting models assume that groups can solve this coordination problem. Take a group consisting of two people who play a coordination game choosing either participate or not. If both participate both get 1, if neither participates both get 0. However if one participates and the other not both get -1. This could be the case if the activity is a costless demonstration against a repressive government. Doing nothing - neither participates - is the status quo and nothing happens. If both participate the government is overthrown bringing both a benefit of 1. However, if one participates and the other does not the government is not overthrown but suppresses the group throwing both members in prison resulting in a negative benefit to both.

For simplicity in this example we have assume away any individual costs - the interests of both group members are perfectly aligned, so their payoffs are unchanged regardless of how altruistic they are and incentive compatibility requires no monitoring cost. This game has three equilibria: two pure equilibria, one where both participate, one where neither participates and a mixed equilibrium. The model of equilibrium of a game with altruistic players allows all three of these possibilities. Peer enforcement and rule consequentialism by contrast assume that group members can coordinate their effort to pick the most preferred alternative: that is they both participate since they both agree this is the best. For groups that can communicate this seems the most sensible assumption.

3.5. Why not Split a Large Group?

In the case where effort is a chore it is intuitive that the smaller group has an advantage: it must pay the fixed cost for a smaller number of members. A natural question is why the larger group does not just “act like a smaller group” by appointing a smaller subgroup to act on its behalf. The problem is that the prize is evenly split among the entire group. For example, for the non-farmers the benefit of eliminating farm subsidies is lower taxes and lower prices for food. This is shared by all non-farmers regardless of who bears the cost of lobbying. If, for example, the urbanites of Manhattan were appointed to do the anti-farm lobbying they would care only about the reduction of their own taxes and food prices, not the reduction in Los Angeles.

To see how this works, suppose a subgroup of size $\mu_k < \eta_k$ is appointed to act on behalf of the group. The prize is only worth $(\mu_k/\eta_k)V$ to the subgroup. Recall that willingness to bid is a non-decreasing function of the desire to bid

$$B_k = \eta_k \underline{\varphi} + V - \eta_k F.$$

For the subgroup this is

$$B_k^{\mu_k} = \mu_k \underline{\varphi} + (\mu_k/\eta_k)V - \mu_k F = \frac{\mu_k}{\eta_k}\left(\eta_k \underline{\varphi} + V - \nu_k F\right) = \frac{\mu_k}{\eta_k} B_k.$$

The desire of the subgroup to bid is always a fraction μ_k/η_k of the raw willingness of the entire group to pay. Hence if the entire group is disadvantaged - the subgroup is even more so.

3.6. Who Will Guard the Guardians?

We have assumed that peers monitor and punish each other. But why? If monitoring and reporting fellow peers to the group is costly - as it may often be in practice - why do it? An effective social norm requires not only incentives to do one's part but also incentives to monitor. For this to be the case the guardians must guard each other - the monitors must monitor other monitors to make sure they are doing their duty - and indeed this requires an indefinite number of rounds of monitoring and punishment. As the experiment in Fu et al (2017) shows these additional rounds are actually relevant in practice.

Suppose that the group agrees on a social norm φ_k. They also must choose a monitoring scheme to enforce this norm. We suppose that they have access to a *peer discipline* technology - based on Kandori (1992)'s information systems approach - in which members may audit each others behavior, the audit possibly resulting in a punishment for the auditee. We account for the self-referential nature of punishment equilibria by supposing that the group plays a potentially unlimited number of audit rounds $t = 1, 2, \ldots$. As before peer monitoring takes place over a simple circular network in which each group member is connected with the member in the clockwise direction. Each auditor chooses whether or not to conduct the audit. The group wishes to minimize the cost of incentive compatible monitoring. How does it do so?

3.6.1. A Model of Peer Auditing

Following Levine and Modica (2016), we assume that the group has access to a punishment of size $\overline{P}$ and chooses probabilities δ_t, $t = 0, 1, \ldots$ that an audit round will take place at $t+1$ (with probability $1-\delta_t$ the game ends after round t). We assume that auditing rounds take place sufficiently quickly so that there is no discounting beyond that induced by δ_t. If an audit round takes place members are matched in pairs with *auditee* i matched *auditor* $j = i - 1$ the person in the counter-clockwise direction.

Recall the monitoring technology in the first round. Participation is observed. For those who did not participate the signal is $z_0^i \in \{0, 1\}$ where 0 means "good, followed the social norm" and 1 means "bad, did not follow the social norm." Note that we now subscript the signal by 0 to indicate that the signal refers to the participation decision round, not one of the subsequent auditing rounds. If the social norm was violated, that is $y_i < \varphi_k$ but member i did not participate, the bad signal is generated for sure, while if i did not participate but did follow the social norm so that $y_i > \varphi_k$ there is never-the-less a chance θ of the bad signal where θ is a measure of the noise of the signal. This signal is observed only by adjacent network members. Previously we assumed that this was reported honestly to the group. Now we instead assume that the auditor may choose whether or not to report honestly. If bad behavior

- non-participation and a bad signal - is reported the auditee is punished with the fixed punishment $\overline{P}$.

In effect the signal observed by the auditor j has two parts: $a_0^i \in \{0,1\}$ whether or not the auditee participated and the signal z_0^i. The participation part of the signal is commonly observed, the signal only by the auditor. We assume that if $a_0^i = 1$ the auditor reports honestly and bears no cost. If $a_0^i = 0$ the auditor may choose not to observe the signal, report falsely (what report is made does not matter), and to bear no cost. Otherwise, if the signal is observed the auditor bears a cost which for convenience we measure in units of the maximum punishment: the cost is $\lambda_1\overline{P}$.

The subsequent rounds $t = 2, 3, \ldots$ are different from the first round in that it is now the auditors themselves who are being audited. Again, we assume that auditor who is now auditee i has two components to the signal observed by auditor j and the group. Let $h = i - 1$ denote the person audited by i at time $t - 1$. The first component $a_t^i = a_{t-1}^h$ is commonly observed. That is: it is known if the auditor audited someone who participated, or audited someone who audited someone who participated and so forth and so on. In case $a_t^i = 0$ so that i is part of an audit trail that began with non-participation, the signal $z_t^i \in \{0,1\}$ where 0 means "good, observed the signal and reported honestly" and 1 means "bad, did not observe the signal." In other words, when an auditor is audited the question asked is "did the auditor follow the rules?" If the audit was honest then the bad signal is generated with probability $\pi < 1/2$; if the audit was not conducted the bad signal occurs with probability 1. As in the initial round we assume that if $a_t^i = 1$ the auditor reports honestly and bears no cost. If $a_0^i = 0$ the auditor may choose not to observe the signal, report falsely (what report is made does not matter), and to bear no cost. Otherwise, if the signal is observed the auditor bears a cost which for convenience we measure in units of the maximum punishment: the cost is $\lambda\overline{P}$. We do not assume that $\lambda = \lambda_1$ since auditing participation might have rather different cost than auditing an audit.

Since the group is bound by incentive constraints only incentive compatible plans can be chosen.

Definition 3.1. A plan $\varphi_k, \delta_t|_{t=0}^{\infty}$ is *peer feasible* if the individual strategies of following the social norm φ_k in the initial round and auditors reporting honestly in the audit rounds is a Nash equilibrium[14] of the super-game induced by the continuation probabilities δ_t.

[14]We use Nash equilibrium because this is an infinite horizon game with private information, where refinements such as subgame perfection have no bite and the definition and analysis of more suitable refinements such as sequentiality is complicated. However for this class of games refinements do not matter in the generic case in which both signals have positive probability. In this case every information set is reached with positive probability. Hence the Nash equilibrium problem of "off the equilibrium path play" does not arise. It follows that sequential equilibrium or even stronger refinements such as extensive form trembling hand perfect equilibrium are identical to Nash equilibrium.

3.6.2. Optimal Punishments

Set $\lambda_t = \lambda$ for $t > 1$. The next result says that optimal implementation requires minimizing the probabilities δ_t while preserving incentive compatibility.

Theorem 3.2. *The social norm φ_k is peer feasible for some $\delta_t|_{t=0}^{\infty}$ if and only $\overline{P} \geq c(\varphi_k)$, $\lambda_1 \leq 1-\pi$ and $\lambda < 1-\pi$, in which case to minimize the monitoring cost the group optimally chooses the termination probabilities*

$$\delta_0 = c(\varphi_k)/\overline{P}, \delta_t = \lambda/(1-\pi) \ \textit{for} \ t \geq 1$$

The corresponding monitoring cost of punishment is

$$M(\varphi_k) = (1-\varphi_k)\left(\theta + \lambda_1 + \frac{\lambda_1(\lambda+\pi)}{1-\pi-\lambda}\right) c(\varphi_k).$$

Proof. Let $\pi_t = \theta$ if $t = 0$ and $\pi_t = \pi$ otherwise. Accounting for the fraction of φ_k of audit trails that have no cost because they begin with participation the cost of monitoring is then

$$M = (1-\varphi_k)\sum_{t=1}^{\infty}\left(\prod_{\tau=0}^{t-1}\delta_\tau\right)(\lambda_t + \pi_{t-1})\overline{P},$$

which is strictly decreasing in δ_t for each t.

We know that incentive compatibility in the initial round requires $\delta_0\overline{P} = c(\varphi_k)$ so that $\delta_0 = c(\varphi_k)/\overline{P}$. This must be no larger than 1 so the first requirement is the obvious one that the punishment be large enough to give compliance with the social norm φ_k.

Consider next the decision by auditor j not to audit in round t. The only consequences of this decision are the saving of the cost $\lambda_t\overline{P}$ and the increased probability of punishment in the subsequent round $\delta_t(1-\pi)\overline{P}$. The incentive constraint is therefore $\delta_t(1-\pi) \geq \lambda_t$. Since $\delta_t \leq 1$ this gives the condition $\lambda_t \leq 1-\pi$. If the incentive constraint holds with strict inequality, then we should lower δ_t as this lowers cost, so the optimum requires this constraint hold with exact equality, that is, $(1-\pi)\delta_t = \lambda_t$.[15] Hence $\delta_t = \lambda_t/(1-\pi)$If the cost of auditing is to be finite, this must be than one for $t > 1$, giving the condition $\lambda < 1-\pi$. Substituting in we find

$$M = (1-\varphi_k)\delta_0\overline{P}\left((\theta+\lambda_1) + (\lambda_1/(1-\pi))\left[\sum_{t=1}^{\infty}(\lambda/(1-\pi))^{t-1}(\pi+\lambda)\right]\right),$$

adding up the geometric series, and substituting for $\delta_0\overline{P}$ using $\delta_0 = c(\varphi_k)/\overline{P}$ from above we get the result. □

[15] As is usual in the optimal punishment literature the key step is identifying the incentive constraint that binds.

The key new point here is that $\lambda_1 \leq 1 - \pi$ and $\lambda/(1-\pi) < 1$ must hold if $\varphi_k > \underline{\varphi}$ is to be incentive compatible. If the auditing technology is too costly or inefficient it is not possible to induce participation beyond the committed voters. It is also the case that if we hold fixed the auditing cost and increase the largest possible punishment the λ_1, λ become small: the constraint will be satisfied and the cost of auditing will be small. If very large punishment is available then infrequent auditing is possible, so the cost is low.

The other finding is that while it remains true that the monitoring cost has the form $M = \Theta(1-\varphi_k)c(\varphi_k)$ where the coefficient Θ now depends on the audit cost and efficiency as well as on θ. Notice that it is no longer true that $\Theta \leq 1$: if the noise in auditing is large enough that $1 - \pi$ is very close to λ then Θ will be very large.

Observe that we can always increase δ slightly during the audit stage and obtain an equilibrium that is strict in all the audit rounds - the price is a small reduction in group welfare. Such an equilibrium can be more robust as it does not require individuals to "make the right choice" when indifferent. Notice that in the initial round except in the zero probability event that $y = \varphi_k$ it is also the case that members strictly prefer to follow the social norm.

3.7. The Cost of Punishment

We have assumed that the cost of punishment is borne only by the "guilty" party. In practice, however, the cost of punishment may spill over to other group members. The most common forms of punishment - some sort of exclusion, ranging from being denied the opportunity to participate in group events to imprisonment - will generally harm group members as well as the designated target of the punishment. For example, if Tim is punished by being excluded from joining the group at the bar after work then David suffers the loss of Tim's companionship. Or it may be that David feels sorry for Tim. We refer to this a *punishment cost spillover.*

We first formally introduce the notion of spillover cost. Obviously the consequence of spillover to the auditor is more costly to the group than the cost of spillover to other members, since it increases the incentive of the auditor to not conduct the audit so as to avoid the spillover cost. Consequently if different group members suffer different levels of spillover costs, the group will want to appoint as auditor the member who least suffers these costs. For simplicity we assume that it is possible to appoint an auditor who suffers no spillover cost and that the spillover costs are equally divided among the remaining group members. Hence when punishment is imposed on i there is a spillover cost of $\psi\overline{P}$ divided equally among group members other than i and his auditor $j = i - 1$. Since the punishment occurs only with probability π_t the expected cost is $\psi\overline{P}\pi_t$. Supposing the group has N_k members, since each member pays a share $1/(N_k - 2)$ of the cost of the $N_k - 2$ matches in which he is neither auditee nor auditor this is also the per capita expected spillover cost, leading to a simple change in the computation of the optimal utility:

$$M = (1-\varphi_k)\left(\lambda_1 + \theta(1+\psi) + \frac{\lambda_1(\lambda + \pi(1+\psi))}{1-\pi-\lambda}\right)c(\varphi_k).$$

Note that the spillover costs do not necessarily have to be positive, although we would generally want to assume that $\psi > -1$ so that the punishment does not bring a net benefit to the group. Negative spillover cost corresponds to punishments that involve a transfer payment. The most obvious example is the payment of a fine, but there are other possibilities. For example, punishment might involve a demotion, in which case another member of the group might benefit from being promoted to fill the vacant spot. Since lower spillover costs are better, punishments involving transfer payments are highly desirable if they are feasible - in the repeated game setting with imperfect public information as in Fudenberg, Levine and Maskin (1994) it is the use of transfer payment punishments that gives rise to near efficiency as the discount factor approaches one.

The bottom line in this is: $M = \Theta(1-\varphi_k)c(\varphi_k)$ where the coefficient Θ is a composite that depends on the monitoring efficiency both in the initial round and in the audit round and on the punishment cost spillover. Depending on these parameters Θ can be as low as 0, for example if $\theta = \lambda_1 = 0$ so that there is no noise in the initial round signal and auditing in the first round is costless. It may also be arbitrarily large if the spillover cost ψ is large.

3.8. Learning Leadership Equilibrium in a Group

Our basic theory is one of political groups that can - somehow - agree on carrying out a plan that is advantageous for the group. In doing so they are constrained by individual rationality: they cannot and do not expect that individual group member will act against their own self-interest. In the applications we have considered group members are *ex ante* identical so there is no ambiguity about what it means to be "advantageous for the group." A theory that only considers groups composed of *ex ante* identical members is not the type of broad theory making wide-ranging predictions that is generally useful. So we now wish to extend that idea by assuming that the group - somehow - has a well defined objective function. Conceptually our basic idea is that the group chooses an incentive compatible plan that is best with respect to that objective.

Such a theory is compatible with the theory of Nash equilibrium - that only incentive compatible plans will be chosen - but refines Nash equilibrium by proposing that a particular plan is chosen according to a well defined objective. We may well ask: how does the group do that? An example can help focus thinking. Consider a group of two players playing a simple coordination game

	A	B
A	5,3(4)	0,0(0)
B	1,1(1)	1,3(2)

The numbers in parentheses are the average utility for the two players, which for the moment we take as the group objective. For simplicity of exposition we restrict attention to pure strategies, so all Nash equilibrium has to say is that the group will choose either AA or BB. Our theory says that the group will in fact choose AA which is the Nash equilibrium that best fulfills the group objective, yielding 4 instead of 2.

How does the group come to such a decision? Does a political party meet and discuss and reach a consensus? Is there a single large meeting or many small meetings? How are these meetings organized? Or is there a leader who instructs the group members on what to do? In a sense the answer does not matter: the situation is similar to competitive equilibrium. In competitive equilibrium individuals - somehow - reveal their preferences to the "market" and - somehow - markets are cleared. At one time this was viewed as a great mystery and many models - tatonnement adjustment, trading posts, and so forth - were examined to see if they lead to competitive equilibrium. The ultimate point is that far from there being no method by which markets are cleared there are many methods - methods that work both in theory and in practice - ranging from double oral auctions to the elaborate methods used by stock exchanges. Yet at the end of the day the particular implementation of the market mechanism does not matter - what matters is that the market clears. Similarly with respect to groups - at the end of the day it does not matter how many meetings are held or how discussion takes place - what matters is that an incentive compatible plan satisfying group objectives is implemented.

Since the theory of groups that act to choose an incentive compatible plan to achieve group objectives does not have behind it the deep theoretical and empirical support of the theory of competitive markets we think it useful to present a concrete implementation of this theory. Specifically we will consider the possibility that the group has a leader who instructs the group members what to do. Hence the group objective is simply the utility function of the leader: for simplicity we may regard this leader - player 0 - as not having an action but simply issuing orders to group members. In the example the leader can order AA, AB, BA or BB. To make things a interesting we might consider a leader with preferences

	A	B
A	5,3(4)	0,0(0)
B	1,1(5)	1,3(2)

so that the leaders preferences are not consonant with the group: the leader preferring BA to all other alternatives.

To proceed formally we will consider a group with N followers $i = 1, 2, \ldots, N$ and a leader. Each follower chooses an action from a finite set of actions $a_i \in A_i$. Followers utility functions are $u_i(a)$ and the leader has a utility function $v_i(a)$. As we are restricting attention to pure strategies we assume that the game between the followers has at least one Nash equilibrium. The leader moves first and issues orders $\overline{a}$ to the followers. The followers then choose actions a_i. How do the followers act on the orders of the leader? In the bible a leader says "go, and he goeth; and to another, come, and he cometh" but such a leader has substantial authority: these orders are followed either because disobedience is expected to be punished or - what from a game theoretic point of view amounts to the same thing - out of respect for the authority. Such a theory would have the leader ordering the incentive incompatible BA. Our theory is one in which the leader can expect orders to be followed but only if they are incentive compatible. We may here recall the words of Sophocles "What you cannot enforce, do not command." So we want theory in which if the leader orders AA or BB the orders will be followed, but AB or BA will not.

A Nash equilibrium of the leadership game consists of an order issued by the leader and a Nash equilibrium of the followers in the subgame that follows after this order, together with play by the followers in the subgames following after all other sets of orders. The off the equilibrium path play by the followers must have the property that - since in Nash equilibrium the leader's orders must be optimal - the leader receives no greater utility than from the on the equilibrium path order. From this we can conclude only that the outcome must be a Nash equilibrium of the underlying game - and any such equilibrium is the outcome of some Nash equilibrium of the leadership game - for example, if the followers choose that equilibrium regardless of orders.

We now propose a refinement of Nash equilibrium. We may say that orders are *credible* if they are a Nash equilibrium: that is if everyone followed the orders nobody would have reason to do otherwise. The refinement we propose - called *leadership equilibrium* - is that credible orders are followed. This has the virtue that it is based on a reasonable theory of individual behavior. In this case since every Nash equilibrium is available to the leader the leader chooses a Nash equilibrium that yields the greatest utility - exactly the refinement we have in mind.

How does a leadership equilibrium come about? We introduce a simple Markov model of learning based on Levine-Modica and Block-Fudenberg-Levine. Specifically, for each subgame in which a player plays that player may be in one of two states: *content* with a particular choice or *discontent*. That is, the followers have a contentment state corresponding to each set of orders and the leader has a state of contentment for the game as a whole. A discontent player chooses an action at random. A content player plays the action with which she is content. The state of contentment for a subgame is updated only when that subgame is played. A player believes that all others will continue to play as they did the last time the particular subgame was reached. If that belief is correct - that is, if opponents play as they did the last time that subgame was played - and the player herself played the same best response both times there is a fixed positive probability less than one that the player becomes content with that action in that subgame. If that belief is wrong - that is, if some opponent changed their play since the last time that subgame was played - or the player failed to play a best response then the player becomes discontent in that subgame. Notice that because of the extensive form this works differently for the leader and the followers: following the choice by the leader of an order if a follower changes their play in the subsequent subgame all the followers become discontent in that subgame only, while the leader becomes discontent. Notice also that changes in the play of the leader never directly causes discontent on the part of the followers.

In this model play by the followers in each subgame follows a separate

Markov process. If the play of the followers is a Nash equilibrium and all are content this is absorbing: all will continue to play the same best response and nobody has reason to be discontent. Any other state is transitory: if anybody failed to play a best response they become discontent. If there is one or more discontent players, they play randomly, hence with positive probability change their action. This makes all followers discontent in that subgame. Once all are discontent there is a positive probability that they all randomly choose a Nash equilibrium two times in a row and all become content - and become absorbed.

Now regard this from the point of view of the leader. He believes that in each subgame the followers will play as they did the last time that subgame was played. If he is content with giving orders for a subgame which yields the highest utility given this belief and the followers are content and playing Nash equilibrium in that subgame then such a state is absorbing. Notice that in such an absorbing state it may well be that some orders if given will not lead to an absorbing state on the part of the followers. What must be true is that the last time the followers played in that state they gave the leader no greater utility than that in the subgame that is actually chosen. This is consistent with the process being absorbed because the state will never change in subgames which are not chosen.

All other states are transient. As long as the subgames which yield the highest utility are populated by followers who are not in an absorbing state there is a positive probability the leader becomes discontent. When discontent he can choose randomly the same subgame several times in a row and with positive probability the followers get absorbed in a Nash equilibrium with which they are content. With positive probability he can continue to do this until all subgames have followers absorbed in a Nash equilibrium in which they are content. Once this is the case, with positive probability he chooses the best subgame several times in a row and becomes content and this state is absorbing.

How does the game start? The initial state of the leader does not matter. However, we give orders meaning in a very simple way: initially all followers are content to follow orders. That's all folks! Every subgame corresponding to orders that form a Nash equilibrium start in an absorbing state and stay there forever. Hence when the final absorbing state of the Markov process is reached the Nash equilibrium in that subgame must give the leader at least as much utility as in the best Nash equilibrium - the final absorbing state must be a leadership equilibrium.

4. Lobbying and the Agenda

In Becker (1983)'s classical work on political influence two groups compete over the size of a transfer from one group to the other. How do lobbying groups with control over an agenda determine the size of a transfer payment? From what we have learned so far we expect that a small group will choose an unambitious agenda so that it will be advantaged, while a large group will choose an ambitious agenda which works in its favor. However, we expect also that the nature of the prize may play a role. Consider the case of civil rights or other rights such as the right to bear arms, to have an abortion, to marry, or to sit at the front of the bus. Here as we change the size of the group the value of the rights increase - the prize is *non-rival*. As we shall discover this strongly favors the large group which will indeed choose a large prize. By contrast consider farm subsidies or another prize involving money, goods or services. Here the size of the prize is independent of group size - but the prize is *fungible* in the sense that, unlike civil rights, it can be used to pay for the lobbying effort. As we shall discover this strongly favors the small group.

The model we analyze is based on Levine and Modica (2015). As we are interested in lobbying, we will consider only the case of a chore and not that of a duty. Moreover we are going to consider the case where one group is an agenda setter that proposes a transfer from the other group to itself. In this analysis we shall make the natural assumption that if neither group makes a bid the status quo is maintained - that is, the non-agenda setter effectively wins. To make this model operational we then need to allow the agenda setter to make a "tiny bid." We do this by assuming that each group has the option to organize without providing effort, that is each can choose to pay the fixed cost while never-the-less providing an effort level of 0. Hence, instead of Define $q(\varphi_k)$ we let $q_k \in \{0, 1\}$ denote the decision of whether to incur the fixed cost but require that if $\varphi_k > 0$ then $q_k = 1$, allowing unlike in the previous case $q_k = 1$ also with $\varphi_k = 0$. This issue was not terribly important in our previous analysis: here with an endogenous agenda it is more relevant because in some circumstances the agenda setter will wish to bring the non-agenda setter to the point of indifference between bidding and not, and then outbid it by a tiny amount.

4.1. Unequal Prizes and the Tripartite Auction Theorem

We are going to deal now for the first time with prizes that are worth a different amount to the two groups. This does not change the tripartite auction theorem: willingness to bid is now measured for each group by equating the cost of the bid with the value of the prize to that group. The group with the lower willingness to bid still gets nothing and the group with the higher willingness to bid gets the value of the prize to that group minus the cost of matching the willingness to bid of the other group. This is true for all three auctions: the all-pay, and the first and second price sealed bid winner pays auction.

In addition to dealing with prizes that are worth a different amount to the two groups there is now a politician who collects the bribe money. From the politician's point of view all three auctions are not the same. Suppose d is the disadvantaged group. In the winner pays auctions the politician gets at least W_d. In the all-pay auction we know that the advantaged group plays uniformly on $(0, W_d]$, the disadvantaged group does not bid with probability $(W_{-d} - W_d)/W_{-d}$ and places the remaining probability uniformly on $(0, W_d]$. From this we can compute the expected payoff to the politician as the sum of the expected bids of the two group

$$\frac{W_d}{2} + \frac{W_d}{W_{-d}}\frac{W_d}{2} = \left(\frac{W_{-d} + W_d}{W_{-d} + W_{-d}}\right) W_d.$$

Since $W_d < W_{-d}$ this is less than W_d: the politician gets less from the all-pay auction than the winner pays auctions. This likely why bribery rarely involves all-pay auctions, and from this point on we restrict attention to the winner-pay auctions in which the group with the higher willingness to bid wins and pays the smaller willingness to bid of the opposing group.

4.2. Non-Rival Prize

So far we have assumed that the prize was a fixed amount V to be divided equally among the two groups. We are interested now in examining more closely how the prize is determined. Let us suppose that group a controls the agenda, that is it can choose the size of the prize. Each member of the other group $-a$ has ν utility units of a resource we shall call *rights*. Suppose $v \leq \nu$ units of rights are taken away $-a$. Each member of group a benefits from this loss of rights by group $-a$. We assume that this is worth γv where $\gamma < 1$: the benefit of taking someone else's rights is assumed to be less than the cost to the person losing those rights. In particular if a feasible agenda $v \leq \nu$ is selected then the value of the prize to $-a$ is $V = \eta_{-a} v$ and the value of the prize to a is $\eta_a \gamma v = (\gamma \eta_a / \eta_{-a}) V \equiv \beta V$. Notice that if the agenda setter is the small group then $\beta < 1$ meaning the transfer is inefficient, while if the agenda is the large group it is possible that $\beta > 1$. [16]

The formal setting then is that the agenda setter chooses a feasible prize $V \leq \eta_{-a} \nu$ then the two groups compete to provide effort costing $F + \varphi_k$ in a winner pays auction. We say that the agenda setter has a *winning agenda* if there is a choice of V for which the agenda setting group is advantaged, in which case we can speak of the optimal agenda as the one that maximizes the surplus the agenda setter gets from winning the auction.

The main result is this

[16] There is a problematic aspect of welfare analysis in this case. It might be that each member of a minority loses two units of utility by being deprived of their rights, while each member of a majority receives one unit of utility by seeing the minority deprived of their rights. If the majority is more than twice the size of the minority then it is apparently efficient to deprive the minority of their rights. From a moral point of view this seems absurd.

Theorem 4.1. *Only the large group may have a winning agenda and has one if and only if $\gamma\nu > F + (\eta_S/\eta_L)$. If it has a winning agenda it is $V = \nu\eta_S$: it asks for and gets the most possible from the small group.*

Proof. The desire to bid for the agenda setter a is $\beta V - F\eta_a$ and for the non-agenda setter it is $V - F\eta_{-a}$. Both are increasing in V and the desire of the non-agenda setter increases more rapidly if $\beta < 1$. Define the *crossover point* $\hat{V} \equiv F(\eta_{-a} - \eta_a)/(1-\beta)$ as the point where the two desires are equal. We also define the *payoff point* $\tilde{V} \equiv F\eta_a/\beta$ as the point where the desire of the agenda setter is zero. To the right of this point the agenda setter may possibly wish to set an agenda, to the left of this point never. Recall that a winning agenda is a $V \le \nu\eta_{-a}$ such that willingness to bid $W_a > W_{-a}$.

Case 1: $\beta < 1$.

To the right of the crossover point the non-agenda setter has a higher desire to pay. This means that if the constraints on his ability to pay do not bind he is at least as willing to pay as the agenda-setter. To the left of the crossover point the same is true of the agenda setter.

We first analyze the right of the crossover point, that is, $V_{-a} > \hat{V}_{-a}$. Here group a has a winning agenda only if the constraint binds on the non-agenda setter, that is $W_{-a} = \eta_{-a}$. Moreover since the bid of the non-agenda setter cannot increase once the constraint binds the agenda setter should propose the highest possible agenda, that is $V = \nu\eta_{-a}$. For this to be a winning bid it must be $W_a > \eta_{-a}$ which is impossible for $a = S$ (since $W_S \le \eta_S$) and true for $a = L$ if and only if $\beta\nu\eta_{-a} - F\eta_a > \eta_{-a}$ which is equivalent to $\beta\nu > 1 + F\eta_a/\eta_{-a}$. This is the same as the condition in the Theorem: $\gamma\nu > F + (\eta_S/\eta_L)$.

In case $a = L$ the crossover point $\hat{V} < 0$ so necessarily the optimum satisfies $V = \nu\eta_S > \hat{V}$ hence the large group has a winning agenda if and only if $\gamma\nu > F + (\eta_S/\eta_L)$ in which case it sets the agenda $V = \nu\eta_S$.

In case $a = S$ the crossover point $\hat{V}$ is positive, so we must also analyze what happens for $V < \hat{V}$. The small group will not propose any agenda below the payoff point $\tilde{V} = F\eta_S/\beta$. Since $\beta = \gamma\eta_S/\eta_L$ we must have $\hat{V} < \tilde{V}$. Since this is the case there is no winning agenda for the small group below the crossover point, and we already saw that to the right of the crossover point the small group has no winning agenda.

Case 2: $\beta > 1$.

For this to be true it must be that $a = L$. Now there is more rather than less desire to pay to the right of the crossover point. Hence to the *left* of $\hat{V}$ group L may have a winning agenda only if S is constrained that is $\eta_S \le V - F\eta_S$ or $\eta_S \le V/(1+F) \le V$. Since bigger V is better for L when S is constrained, it follows that if the optimal agenda for L is on the left of $\hat{V}$ it must be at $V = \nu\eta_S$. Similarly to the right of the crossover point group L's desire to pay rises faster than that of group S so it wants as large a transfer as it can, that is again $V = \nu\eta_S$. The maximal agenda is winning if $W_L \ge W_S$, which is to say that $\beta\nu\eta_S - F\eta_L \ge \eta_S$, that the same condition as in Case 1. $\square$

4.3. Fungible Prize

We turn now to lobbying over a fungible prize: by this we mean that the proceeds of the prize can also be used to pay the politician. We assume as before that the resources available to a group are enough to pay the fixed cost and provide a unit of effort. That is, each group k has an endowment of $\eta_k(1+F)$. Hence the non-agenda setter $-a$ can supply a prize at most equal to $V \leq \eta_{-a}(1+F)$ and can bid up to η_{-a}. If the prize is V the agenda setter gets βV where we focus on the case of inefficient transfers[17] so that $\beta < 1$. The agenda setter can bid up to $\eta_a + \beta V$. The cost of effort provision remains as in the non-rival case $F + \varphi_k$.

Theorem 4.2. *Only the small group may have a winning agenda and has one if and only if $\beta > \eta_S/\eta_L$. If it has a winning agenda it is $V = F\eta_L$: it asks for and gets just enough to keep the large group from bidding.*

If the transfer is too inefficient (β small) the status quo is maintained. This is the effect pointed out by Becker (1983): inefficient transfers are less likely to take place. In addition the small group is not too "greedy" in the sense that it asks only for $F\eta_L$ while it could ask for as much as $(1+F)\eta_L$. Moreover, the amount that the small group wins $F\eta_L$ is increasing in the fixed cost F.

Proof. As in the proof of Theorem 4.1 define the *crossover point* $\hat{V}_{-a} \equiv F(\eta_{-a} - \eta_a)/(1-\beta)$ as the point where the two desires to bid are equal. We also define the *payoff point* $\tilde{V}_{-a} \equiv F\eta_a/\beta$ as the point where the desire of the agenda setter is zero. To the right of this point the agenda setter may possibly wish to set an agenda, to the left of this point never. Recall that a winning agenda is a $V_{-a} \leq (1+F)\eta_{-a}$ such that $W_a > W_{-a}$.

To the right of the crossover point the non-agenda setter has a higher desire to pay. This means that if the constraints on his ability to pay do not bind he is at least as willing to pay as the agenda-setter. To the left of the crossover point the same is true of the agenda setter.

We first analyze the right of the crossover point, that is, $V_{-a} > \hat{V}_{-a}$. Here group a has a winning agenda only if the constraint binds on the non-agenda setter, that is $W_{-a} = \eta_{-a}$. Moreover since the bid of the non-agenda setter cannot increase once the constraint binds the agenda setter should propose the highest possible agenda, that is $V_{-a} = (1+F)\eta_{-a}$. For this to be a winning bid it must be $W_a > \eta_{-a}$ which is true if and only if $\beta(1+F)\eta_{-a} - F\eta_a > \eta_{-a}$ which is equivalent to $\beta(1+F) > 1 + F\eta_a/\eta_{-a}$.

In case $a = L$ the crossover point $\hat{V}_S < 0$ so necessarily $V = (1+F)\eta_S > \hat{V}_S$ and the large group has a winning agenda if and only if $\beta(1+F) > 1 + F\eta_L/\eta_S$; but this is inconsistent with $\beta < 1$.

In case $a = S$ the crossover point $\hat{V}_L$ is positive, so we must also analyze what happens for $V_L < \hat{V}_L$. The small group will not propose any agenda below

[17] For an efficient transfer government intervention may be unnecessary.

the payoff point $\tilde{V}_L = F\eta_S/\beta$. There are two cases depending on which of $\hat{V}_L$or $\tilde{V}_L$ is larger. Notice that $\hat{V}_L \leq \tilde{V}_L$ may be written as $\beta \leq \eta_S/\eta_L$.

If $\hat{V}_L \leq \tilde{V}_L$ then there is no winning agenda for the small group below the crossover point, so the small group is in the same boat as the large group: it has a winning agenda if and only if $\beta(1+F) > 1 + F\eta_S/\eta_L$. However this is inconsistent with $\beta \leq \eta_S/\eta_L$ so the small group has no winning agenda in this case.

We now analyze the remaining case $a = S$ for $\beta > \eta_S/\eta_L$ that is $\tilde{V}_L < \hat{V}_L$. Observe that $\beta > \eta_S/\eta_L$ implies $\beta(1+F) > F\eta_S/\eta_L$ so that the highest feasible bid $(1+F)\eta_L$ lies above $\tilde{V} = F\eta_S/\beta$, therefore the small group is willing to propose an agenda to the right of the payoff point. We claim that its optimal agenda is $F\eta_L$. Observe that the large group bids zero if and only if $V \leq F\eta_L$ and note that $F\eta_L > \tilde{V}_L = F\eta_S/\beta$. So there there is no point in proposing an agenda less than $F\eta_L$. Larger agendas up to $(1+F)\eta_L$ are feasible. On the other hand the willingness to pay of the large group rises faster than the small group as long as the large group is not constrained; hence either the small group should propose $F\eta_L$ or should propose enough that the constraint binds, in which case it proposes $(1+F)\eta_L$. Proposing $F\eta_L$ gives $W_S - W_L = \beta F\eta_L - F\eta_S$ (since $W_L = 0$) and proposing $(1+F)\eta_L$ gives $W_S - W_L = \beta(1+F)\eta_L - F\eta_S - \eta_L$. But the former is always larger than the latter, so that in all cases the optimal winning agenda for the small group is $F\eta_L$. □

4.4. Subsidies versus Civil Rights

Do decisions favoring a group have substantial public support or limited public support? Our agenda setting model suggests that for fungible prizes widespread public support is not so important while for non-rival prizes it is. Two significant non-rival issues have been civil rights for blacks and civil rights for gays. In both cases significant advances have occurred when public support has become widespread. That is, when we talk about the group lobbying for rights we do not mean just those who directly receive the rights but all those who support those rights: while the fraction of blacks or gays may not change much over time those who support them does.

Long term polling by Gallup[18] asks about willingness to vote for a black person for President, which may be taken as an indicator of general attitudes towards civil rights. In 1958 only 38% responded positively, By 1959 this rose to about 50% where it remained until about 1963 when it rose to 60%, dipped briefly in 1967 and then rose steadily to about 95% by the year 2000. Civil rights have been largely reflective of these public attitudes towards blacks. The "separate but equal" doctrine permitting racial discrimination in a variety of domains, but most significantly in education was established in 1896 in Plessy v. Ferguson, and although it was repudiated in law in 1954 in Brown v. Board of Education, desegregation was not immediately implemented: George Wallace's stand in the school house door took place in 1963 - well after turn of public

[18] `www.gallup.com/poll/3400/longterm-gallup-poll-trends-portrait-american-public-opinion.aspx`

opinion, and the landmark legislation was the 1964 Civil Rights Act. Political action occurred only when the size of the group supporting civil rights became large. We find a similar story with respect to gay civil rights. The Pew Research center finds that in 2003 only 32% of Americans favored same-sex legal marriage - this increased steadily, reaching parity by 2011.[19] From 1975 to 2000 various states and the Federal government passed a series of laws banning gay marriage. By 2009 only seven states had recognized gay marriage. This rose to thirteen by 2013 and to fifty with the Supreme court decision in 2015. Again the recognition of rights - non-fungible as it is - seems to have followed public opinion and indeed, majority public opinion.

By contrast if we look at an important fungible issue - farm subsidies - we see that support for large farms which receive the bulk of subsidies has only 15% popular support.[20] While there are only about 2 million farms in the US it is not just farmers that benefit from farm subsidies. An upper bound should be the rural population of the US of about 60 million people or roughly 20 million households out of the 120 million U.S. households - which is also about 15%. So we see that a minority of roughly 15% is effective at getting a fungible prize from the remaining 85%. This number 15% is similar to the fraction of the population that is either black or gay - yet those groups have been ineffectual in realizing the non-fungible prize of civil rights until they achieved the support of roughly a majority.

It is the presence of a fixed cost per member that prevents a large group from being effective. But is the level of fixed cost needed to explain the data plausible? As we observed, η_L is about 85% and η_S is about 15% of households, so that η_L is indeed much larger than η_S. In Table 2.1 the median level of farm subsidies measured in hours per person per year to the large group is 14, or about two working days. According the theory this should be equal to F the fixed cost of joining a lobbying organization - it seems a plausible number.

We should note also that a puzzle remains about the data in Table 2.1. In the theory smaller groups extract from each member of the large group a fixed amount F. In the data smaller groups actually extract more from each member of the large group the smaller they are. The theory does point us in a useful direction: the condition for the small group to have a winning agenda is $\beta > \eta_S/\eta_L$. If β is randomly drawn then smaller groups are more likely to succeed. Since farm subsidies are composed of many different pieces of legislation involving different lobbying efforts, the overall level of subsides is a composite of success and failures, so that countries with a smaller farm groups will on average have more successes and hence a higher level of subsidies.

The difference in the effectiveness of lobbying between fungible prizes - largely monetary transfers - and non-rival prizes - largely rights - has important consequences. For fungible prizes lobbying may be an effective alternative to overcoming a voting majority. For rights it is not. However, there are two

[19] pewforum.org/2015/07/29/graphics-slideshow-changing-attitudes-on-gay-marriage

[20] www.worldpublicopinion.org/pipa/articles/brunitedstatescanadara/602.php

effective strategies for minorities to establish rights. One is direct action: since more is at stake they may have a greater willingness to engage in protests, for example, so that they resources they are willing to spend on direct action may overcome the resources the majority is willing to spend. While protests occur over many things, the most powerful and prolonged protests have been over issues of civil rights. Second is conversion: the relevant group is not just those that lack rights but those that agree that they should have them. By converting people to the point of view that rights are right a minority may effectively become a majority. As the public opinion polls indicate - this was an effective strategy for both blacks and gays.

4.5. Why Not a Cartel?

A basic fact that we capture in our theory is that lobbying groups are very effective at overcoming the public goods problem through peer enforcement. Take the case of farmers: Table 2.1 shows that despite the fact these groups are large in absolute size they are very effective at lobbying. Of course in addition to lobbying government for subsidies, like every industry, farmers would like to form a cartel, reduce output, and get monopoly profits. Also like lobbying, forming a cartel poses a public good problem for the group - and the conventional wisdom in industrial organization is that in an industry with many producers such as farming this is difficult. As we will discuss subsequently when we examine the anti-folk theorem - if the punishment for cheating on a cartel is a reduction in future industry output this is indeed theoretically correct. But if peer enforcement can be used to overcome the public goods problem in contributing to the common good of lobbying, why is it not equally effective in overcoming the public goods problem of reducing industry output?

Before examining this more closely we should make three points. First, it could be argued that public policy and anti-trust law play a role in inhibiting cartel formation. This seems unlikely to be the case: peer enforcement is subtle. No doubt if farmers got together and talked about colluding to reduce output this would be legally problematic. But if they get together - as they do - to discuss best farming practices and agree that a number of fields should be left fallow, that less fertilizers and less intensive farming is a better practice - and if this were a social norm enforced by peer sanctions - it seems unlikely it would run afoul of anti-trust policy. Moreover, most governments encourage farmers to discuss and adapt best farming practices. Second, it could be argued that monitoring is more difficult in a cartel setting than in a public goods setting. However, it is not immediately obvious that farmers living in a farm community are less able to observe how many fields their neighbors plant than to observe their neighbors contribution to farm lobbying efforts. Third it may be argued that the cost to a farmer of reducing output is much greater than that of contributing to a lobbying effort. With an upper bound on the size of punishment a large effort or output reduction cannot be sustained by peer enforcement. This, however, does not explain why farmers to not engage in a "minor cartel" reducing output a modest amount.

Finally: it must be observed that some industries with a large number of "firms" do indeed have peer enforced social norms of restricting output. This is a common way in which workers exploit their monopsony power, especially in a labor union setting. That is, a social norm of "do not work too hard" with social sanctions against those who are overly energetic are very common one in many blue-color settings. Since the demand for effort is downward sloping workers as a group can take advantage of their monopsony power by reducing effort - and indeed they often do exactly that.

To understand what is going on it often pays to write down a simple model, and we propose to do exactly that. We will consider a competitive industry with many identical firms and denote per firm output by x. Suppose that is produced at constant marginal cost up to a capacity constraint $\overline{x}$ and that the margin between price and cost as a function of average firm output $\mu(x)$ is smooth and strictly decreasing. Assume that $\mu(0) > 0$ and for sufficiently large x we have $\mu(x) < 0$. Denote by x^C the unique per firm competitive output where either $\mu(x^C) = 0$ or $\mu(\overline{x}) > 0$ and $x^C = \overline{x}$. Assume in addition that the monopoly problem of maximizing $\mu(x)x$ subject to $x \leq \overline{x}$ has a unique solution x^M.

Now we wish to regard an x with $\mu(x) > 0$ as a quota set by a cartel of colluding firms - the social norm of the cartel. Cartel members observe a noisy signal of whether each individual firm adheres to the quota - in a social network, say. Specifically we suppose that a firm that violates the quota is caught for sure, but that there is a probability π that a firm that adheres to the quota is never-the-less believed guilty of cheating. There is a fixed cost $F > 0$ of observing the signal.

The competitive assumption is that individual firms are too small to have any important effect on the price. Hence if the quota is x the optimal way to cheat is to produce $\overline{x}$, which - given no price effect - gives an extra profit of $\mu(x)(\overline{x} - x)$. If you do not cheat you suffer the punishment P with probability π. If you do cheat you suffer the punishment P with probability 1. Hence the incentive constraint is $\mu(x)(\overline{x} - x) - P \leq -\pi P$. So the optimal size of the punishment needed to enforce the social norm is $P = \mu(x)(\overline{x} - x)/(1 - \pi)$. When everyone follows the social norm the fraction of the population that suffers this punishment is π.

Defining $\theta = \pi/(1-\pi)$ we may write the per firm cartel profit when the signal is observed and accounting for monitoring costs as $\mu(x)x - \theta\mu(x)(\overline{x} - x) - F$. The optimal cartel quota is the maximum of this subject to $x \leq \overline{x}$ if the profit is non-negative or to ignore the signal and set $x = x^C$ if the maximum is negative. In the latter case we say that the *cartel does not form* and we are interested in establishing when this might be the case. To this end define the function $\mu(x)x - \theta\mu(x)(\overline{x} - x)$ which for $\mu(x) > 0$ and $x \leq \overline{x}$ represents cartel profit net of the fixed cost of monitoring. Assume that for all x this is single-peaked and has a unique argmax denoted by $\hat{x}$. We can identify three circumstances in which the cartel will not form.

Theorem 4.3. *The cartel will not form if any of the three following conditions are met*

1. $F > \mu(\hat{x})\hat{x} - \theta\mu(\hat{x})(\overline{x} - \hat{x})$
2. $\overline{x} \leq \min\{x^C, \hat{x}\}$
3. $\overline{x} > x^C/\pi$

The first condition - that the fixed cost F or the monitoring inefficiency θ is large is obvious, unsurprising and would equally inhibit lobbying. This case is, however, relevant: there are industries where firms do not have close social ties, for which, as a consequence, monitoring costs are high - and we should not expect and do not see in such cases either lobbying or cartel formation. The second case where the capacity constraint is small also unsurprising: here competitive rents are already equal to what the cartel can hope to achieve so forming a cartel is pointless. Notice that in this case the industry will still want to lobby against those who are not part of the social network - against entry, against foreign competition and so forth. From an applied point of view it seems unlikely that the reason farm cartels do not form is because capacity is already so limited that a cartel is pointless.

The third case is the most interesting because it says that, regardless of the demand and marginal cost and of the corresponding competitive equilibrium, monopoly solution and potential monopoly profit, if the capacity constraint is sufficiently large the cartel will not form. This reveals the key difference between cartel formation and other public goods problems. In a standard public good problem - such as lobbying - the incentive to cheat is the amount that is saved by reducing effort to zero. In a cartel setting the corresponding incentive to cheat would be to increase output to the competitive level - the default level without a cartel. In a cartel setting, however, a cheating firm should not limit its output to x^C - it should produce as much as it can, that is, to capacity $\overline{x}$. So if capacity $\overline{x}$ is much bigger than x^C the benefit of cheating is very great and the corresponding cost of enforcement very high. If the capacity constraint does not exceed the competitive output, that is, if $\overline{x} \leq x^C$ then this third reason for cartel formation does not exist and the situation is indeed very similar to lobbying.

From an empirical point of view the theory that peer enforced cartels are much less likely to be form when capacity is large relative to the size of the market matches well with what trade union and industrial organization economists observe. If there are many firms and each can easily replace the output of another firm by hiring additional inputs we should not expect to see peer enforced cartels. When the "firms" are individual workers they are capacity constrained by the hours and intensity with which they work - they cannot simply increase output by going out and hiring additional inputs to increase their output. Hence it seems that capacity constraints are more significant in the setting of workers and less binding in the case of firms - which coincides with the observation that we do not see peer enforced cartel behavior with firms, but we do with workers.

Proof of Theorem. The condition in part 1 implies that cartel profit must be negative if the cartel is formed, so it is better not to form the cartel. Since profits are assumed single-peaked, the condition in part 2 implies that the cartel solution and competitive equilibrium are at $\overline{x}$ hence there is no point in paying to observe the signal.

Turning to part 3 since $x^C < \overline{x}$ we can compute the derivative of cartel profit with respect to x at the competitive equilibrium (where $\mu(x^C) = 0$) as $\mu'(x^C)(x^C - \theta(\overline{x} - x^C))$. The condition $\overline{x} > x^C/\pi$ implies that this is positive. Hence $\hat{x} > x^C$ implying cartel profit at $\hat{x}$ is negative and the cartel should not form. $\square$

5. More Than Two Groups

What if several different social networks have the same objective? Let us specifically think of a number of interest groups or political parties: $k = 1, 2, \ldots K$ each of size η_k. In auction theory we think of these "players" or "bidders" bidding for a particular prize. Often in a political setting it is an alliance of groups that wins and not the individual groups. For example: if the issue is farm subsidies it may well be the urbanites in New York and San Diego belong to separate social networks but share a common interest in ending farm subsidies. Similarly on the other side with mid-western wheat growers and California alfalfa growers.

We are going to study a setting where there are two alternatives, which we will label $\kappa = S, L$ and each bidder k is interested in one or the other of these alternatives. That is, if $k \in \kappa$ and κ wins then group k gets $V_k > 0$ and if it loses they get nothing. We say that the groups $k \in \kappa$ belong to the *coalition* κ. For each group, we will continue with the linear model of a common cost of providing effort

$$C(q_k, \varphi_k) = q_k F + f \max\{0, \underline{\varphi} - \varphi_k\} + \max\{0, \varphi_k - \underline{\varphi}_k\}.$$

How is the outcome determined? There is little point in speaking of standard auctions where everyone bids and someone wins: here lots of people may win. This is exactly the situation contemplated in the theory of menu auctions - so the two mechanisms we will consider are those of the menu auction and that of the all-pay auction where the outcome is decided, for example, by voting. What we are going to discover is that while with two groups it makes little difference whether a winner pays or all pay mechanism is used - with alliances of groups it makes a big difference: the tripartite auction theorem no longer holds.

In this setting of multiple groups and two alternatives we can still define willingness to bid W_k as before: it is the greatest amount of effort the group would be willing to provide to get the prize for certain. The desire to bid is still

$$B_k = \eta_k \underline{\varphi} + V_k - \eta_k F.$$

If B_k is less than $\eta_k \underline{\varphi}$ then the willingness to bid is $W_k = \eta_k \underline{\varphi}$; if B_k is greater than η_k the the willingness to bid is $W_k = \eta_k$; otherwise where $\eta_k \underline{\varphi} \leq B_k \leq \eta_k$ we have $W_k = B_k$.

5.1. The Menu Auction

Recall from the Appendix to Section 1.2 Bernheim and Whinston (1986b)'s notion of menu auctions when there are several bidders bidding on a list of choices. Here each bidder $k \in \kappa$ makes a bid b_k for the alternative κ. The auctioneer chooses the action that maximizes income $\sum_{k \in \kappa} b_k$. Implicitly there is an endogenous tie-breaking rule: in case of a tie the auctioneer is free to choose the alternative that supports a particular equilibrium.

A *truthful equilibrium* consists of a choice κ by the auctioneer and bids such that the bid differentials reflect the utility differential between alternatives and such that no bidder can improve their utility by an alternative bid. For two alternatives the requirement that bid differential reflect the utility differentials between alternatives has meaning only for the losers $k \notin \kappa$: it requires their bid be equal to their willingness to bid $b_k = W_k$.

Since the losers bid their willingness to bid, to win the winning coalition must in aggregate bid at least this amount, which means that their aggregate bid must be at least equal to the aggregate willingness to bid of the losers: $\sum_{k \notin \kappa} W_k$. Moreover, the aggregate bid of the winning coalition must equal the aggregate willingness to bid of the losing coalition: otherwise each group in the winning coalition should reduce their bid. We see then that the winner is the coalition with the highest aggregate willingness to bid.

The division of bids among the winners is indeterminate: no group can bid more than their willingness to bid, but subject to that constraint any allocation of bids among the winners is an equilibrium - to bid less loses and there is no reason to bid more.

5.2. The All-Pay Auction

We now examine the all-pay auction. Here each bidder $k \in \kappa$ makes a bid b_k for the alternative κ. The alternative that receives in aggregate the highest bid wins and everyone - winners and losers alike - pay their bid.

We are not going to analyze every possibility here: simply work out one equilibrium to show how different it is from the menu auction. Call group $\hat{k} \in \kappa$ the *leading* group in κ if it has the highest per capita value of the prize V_k/η_k. Let $\eta_\kappa = \sum_{k \in \kappa} \eta_k$ be the aggregate size of the groups who prefer κ.

Theorem 5.1. *Suppose that each coalition has a unique leading group and that for $\hat{k} \in \kappa$ we have $\underline{\varphi}[\eta_{-\kappa} - \eta_\kappa + \eta_{\hat{k}}] < W_{\hat{k}} < \eta_{\hat{k}}$. Then there is an equilibrium in which only the leading groups submit bids above the committed level and they do so as if they were the only bidders. The leading group with the lowest willingness to bid gets zero; all other groups in both coalitions get a positive surplus. If $\underline{\varphi}[\eta_{-\kappa} - \eta_\kappa + \eta_k] > W_k$ for all group k in coalition κ then all those groups get zero surplus, and coalition $-\kappa$ wins the prize at zero cost.*

Notice that each group bids at least $\underline{\varphi}\eta_k$, so group $\hat{k} \in \kappa$ must put up the difference $\underline{\varphi}[\eta_{-\kappa} - \eta_\kappa]$ - a disadvantage for them. We refer to the coalition whose leading group is disadvantaged as the disadvantaged coalition. This is a rather remarkable result. It says that only the leading groups provide any effort and all the other groups simply free-ride on the leading groups. This is most starkly seen in the case of a chore (lobbying): here $\underline{\varphi} = 0$ so the only requirement is that the leading group not value winning so much that it is willing to commit all its resources to victory - and all the groups in the disadvantaged coalition except the leading one get surplus. In the case of a duty (voting) if there are many groups in each coalition and the aggregate size of one group is much larger than the other, then we fall back into the trivial case in which only committed members provide effort.

Proof. Given that all the committed members of all the groups will participate, the hypothesis that $\underline{\varphi}[\eta_{-\kappa} - \eta_{\kappa} + \eta_{\hat{k}}] < W_{\hat{k}} < \eta_{\hat{k}}$ puts the auction between the two leading groups into the interior: leading group $\hat{k}$ find the marginal benefit of increasing a bid slightly to be equal to the marginal cost of the effort. That marginal benefit is the increased chance of winning time the per capita value of the prize. Since for every other group sharing the same goal as $\hat{k}$ the per capita value of the prize is less than the marginal cost of the effort, so they all bid as little as possible. In the second case no group in κ is willing to bid enough to overcome the committed members of the other coalition provided that no other group is their coalition is doing so, so it is an equilibrium for them all to stay out and the other coalition to win on committed voters alone. □

5.3. Pivotality and Uncertainty

Why is the outcome so different in the menu and all-pay auction? Take the case $\underline{\varphi}[\eta_{-\kappa} - \eta_{\kappa} + \eta_{\hat{k}}] < W_{\hat{k}} < \eta_{\hat{k}}$. In the menu auction the coalition with the least aggregate willingness to bid gets nothing; in the all-pay auction every group except the leading group in the disadvantaged coalition get a positive surplus. Moreover, supposing that the winning coalition in the menu auction is the same as the advantaged coalition in the all-pay auction: the aggregate bid of the winning coalition in the menu auction is much higher than any bid made with positive probability by the advantaged coalition in the all-pay auction.

The key to understanding the difference is the concept of pivotality - a notion that played a key role in our analysis of common punishment. In the menu auction each group in the winning coalition is pivotal: if any group lowers its bid slightly they lose. Hence each group bears the entire burden if they fail to contribute their "fair" share to the acquisition of the public good of winning the prize. Such a group (or individual) whose decision is decisive in determining the result is called *pivotal.* Pivotality has historically played an important role in voting theory as we shall see: we will argue that in the case of large mass elections the pivotality of individual voters is not likely to be important. This is different in contests between coalitions of a small number of groups where the significance of the effort of a single group may be great.

Why then is the all-pay auction so different? Because pivotality depends on uncertainty. The menu auction promises a group a certain loss for a failure to contribute. If the outcome is random and the loss depends more continuously on the effort provided, the group has less incentive to contribute. In the case of the all-pay auction we know the equilibrium has to be mixed: that is, the outcome is uncertain. Hence the public goods problem of getting groups in a coalition to "do their share" is much greater in the all-pay auction than the menu auction. With uncertainty about the outcome the public goods problem faced by a coalition is much greater - and indeed, as we shall see, it is greater the larger the coalition - it is, indeed, the reason why peer punishment mechanisms are needed to overcome the public goods problem in large groups. With certainty about the outcome the public goods problem can be overcome entirely - as we see in the menu auction.

The relationship between the certainty of the outcome and pivotality is a crucial one that we shall explore. In the next section we examine conflicts where the outcome is an uncertain function of effort. In the section after we examine the implications for pivotality.

6. Uncertain Outcomes in Conflict

So far we have assumed that the highest bidder wins: the group that provides the most effort wins the prize. This is not always realistic - especially not in the case of all-pay auctions and certainly not so in the case of voting and even more in the case of political conflicts such as street demonstrations or civil war. When both sides put forth effort, quite often there is an element of uncertainty about who will win. Let us focus on voting.

- There is uncertainty about the outcome because with a particular social norm participation by individuals is not certain. In the model in which individuals independently draw participation costs the total effort of each group is random: it is the sum of independent random decisions on whether or not to vote and so total participation follows a binomial distribution as in Palfrey and Rosenthal (1985) or Levine and Palfrey (2007). Moreover, individual draws of participation costs may be correlated: for example, bad weather may raise participation costs for all members in regions where a party is heavily concentrated.
- The size of the two parties may be uncertain - nobody knows for sure how many people support their candidate or cause. This is the approach taken, for example, in Shachar and Nalebuff (1999) and Coate and Conlin (2004).
- There can be random errors in the counting of votes, in the way that votes are validated, or courts may intervene in the vote counting - as happened, for example, in the 2000 U.S. Presidential election between Bush and Gore.

We are going to focus in this chapter on the case of the all-pay auction. A convenient device for expressing the outcome of a contest the result of which is uncertain is that of the *conflict resolution function* in which the probability of winning the election is a continuous function of the effort - the bid - of each group. Specifically, we denote by $p_k(b_k, b_{-k})$ the probability that group k wins the prize given their own bid b_k and the bid of the other group b_{-k}. So far we have assumed that $p_k(b_k, b_{-k}) = 1$ if $b_k > b_{-k}$ and is discontinuous when the bids are the same - either undetermined or 0.5 in the case of a tie. Now we wish to assume that $p_k(b_k, b_{-k})$ is a continuous function: it should satisfy two basic properties. First, it should be increasing in b_k so that higher bids result in a higher probability of winning. Second, it should be the case that one party wins the prize for certain so that $p_k(b_k, b_{-k}) + p_{-k}(b_{-k}, b_k) = 1$. A convenient function that satisfies this property is that introduced by Tullock and analyzed in the case of voting by Herrera, Morelli and Nunnari (2015):

$$p_k(b_k, b_{-k}) = \frac{b_k^\alpha}{b_k^\alpha + b_{-k}^\alpha}.$$

As $\alpha \to \infty$ this approaches the ordinary all-pay auction in which the highest bidder has probability 1 of winning.

Recall the basic finding for the all-pay auction: there is no pure strategy equilibrium - the groups must mix to make the outcome sufficiently uncertain that neither can be sure of winning. As we shall see, if there is enough exogenous uncertainty about the outcome - for example in the Tullock model if α is sufficiently small - then there can be pure strategy equilibria. This does not undercut the main insight into the all pay auction that the outcome must be uncertain. If there is little exogenous uncertainty then the outcome is uncertain because the groups must mix. If there is a lot of exogenous uncertainty then the groups need not mix - but the outcome is uncertain on account of the exogenous uncertainty. Either way there must be substantial doubt about the outcome.

We first explore the all-pay auction with the Tullock conflict resolution function and examine what difference it makes to assume $\alpha < \infty$. We then explore alternative models of uncertainty that have appeared in the literature, and finally give a broader overview of the implications of assuming uncertainty about the outcome of bidding.

6.1. Interior Pure Strategy Equilibrium in the Tullock Model

We now adapt our workhorse model for conflict resolution. This means that we write the objective function of a group as

$$U_k(b_k, q_k, b_{-k}, q_{-k}) = p_k(b_k, b_{-k})V - \eta_k \left(q_k F - f \max\{0, \underline{\varphi} - b_k/\eta_k\} + \max\{0, b_k/\eta_k - \underline{\varphi}\}\right)$$

where we now assume

$$p_k(b_k, b_{-k}) = \frac{b_k^\alpha}{b_k^\alpha + b_{-k}^\alpha}.$$

The two groups are still S, L with sizes $\eta_S < \eta_L$. Our goal is to study the simplest type of equilibria in this model: *interior pure strategy equilibria*. This are bids for each group (B_L, B_S) that form a Nash equilibrium and such that $\eta_k \underline{\varphi} < B_k < \eta_k$. These equilibria do not exist for all values of the parameters. We first examine what they are like when they exist and subsequently examine the parameter values for which there are equilibria of this type.

In analyzing Tullock contests it is useful to introduce a new notion of advantage. Before we introduced the notion of advantage as a greater willingness to bid. We now say that a group is *utility advantaged* if it receives more utility than the other group in equilibrium and that the other group is *utility disadvantaged.* Theorem 2.7 implies that for an auction - a certain outcome - advantaged and disadvantaged are the same as utility advantaged and disadvantaged.

Theorem 6.1. *If there is an interior pure strategy equilibrium it is unique and each group choose the common bid $\hat{b}_k = \alpha V/4$ and consequently has an equal chance of winning. In the case of a chore the small group is utility advantaged and in the case of a duty the large group is utility advantaged. The utility advantaged group receives a utility advantage of $(\eta_L - \eta_S)(F + \underline{\varphi})$. The other group, however, receives a positive level of utility equal to $(\frac{1}{2} - \frac{1}{4}\alpha)V - F\eta_L + \eta_S\underline{\varphi}$.*

The most significant difference with the all-pay auction is that the utility disadvantaged group receives a positive level of utility. The reason for this should be obvious in the case of a duty: if a group puts forth no effort at all and so incurs no cost it still has a positive probability of winning because of "good luck." The amount earned by the utility disadvantaged group increases as the noise increases.

Proof. Since the group always provides at least $\underline{\varphi}$ of effort the objective function for group k is

$$\frac{b_k^\alpha}{b_k^\alpha + b_{-k}^\alpha} V - \eta_k \left(q_k F + (b_k/\eta_k) - \underline{\varphi} \right).$$

Differentiating with respect to b_k we find

$$\left(\frac{\alpha b_k^\alpha b_{-k}^\alpha}{\left(b_k^\alpha + b_{-k}^\alpha\right)^2} \right) V b_k^{-1} - 1$$

and the necessary first order condition for a maximum is that this should be equal to 0. The expression in large brackets is the same for both groups: hence the solution of the first order condition is the same for both groups: as asserted they make a common bid. Substituting $b_{-k} = b_k$ we then find the unique common equilibrium bid $\hat{b}_k = \alpha V/4$. The utility of each group is found by substituting back into the objective function.

Calculation of the Surplus

Plugging the equilibrium into the objective function group we find the surplus for group k

$$\frac{V}{2} - \eta_k \left[\frac{\alpha(V/\eta_k)}{4} - \underline{\varphi} + F \right] = (\frac{1}{2} - \frac{1}{4}\alpha)V - F\eta_k + \eta_k \underline{\varphi}.$$

We see immediately that when $F = 0, \underline{\varphi} > 0$ the large group is utility advantaged, and its advantage is given by the difference in the saving in bidding due to having a larger number of committed members: $(\eta_L - \eta_S)\underline{\varphi}$. Similarly when $F > 0, \underline{\varphi} = 0$ the small group is utility advantaged and its advantage is given by the difference in fixed costs $(\eta_L - \eta_S)F$. Since only one of $F, \underline{\varphi}$ can be non-zero we can add these to together as done in the statement of the result.

□

6.2. Existence of Interior Pure Strategy Equilibria

The cases of a duty and a chore are different: as the most usual type of contest is voting which in most cases we think is a duty, we examine that case first.

Theorem 6.2. *In the case of a duty, where $F = 0$, an interior pure strategy equilibrium exists if and only if $(4/V)\eta_L \underline{\varphi} \le \alpha \le (4/V)\eta_S$ and either $\alpha \le 1$ or* *for both groups*

$$\left(\frac{1}{2}-\frac{1}{4}\alpha-\frac{1}{1+\left(\frac{1}{4}\alpha(V/\eta_k)\right)^{\alpha}}\right)V+\eta_k\underline{\varphi}\geq 0.$$

Proof. With a common bid interiority means that the large group turns out more than just committed members $\hat{b}_L \geq \eta_L\underline{\varphi}$ and the small group turns out fewer than all its members $\hat{b}_S \leq \eta_S$. Plugging in and rearranging in the form of a bound on α these conditions can be written $4\eta_L\underline{\varphi}/V < \alpha < 4\eta_S/V$.

We now have to examine the boundaries and second order condition. The sign of the second derivative of the objective function is negative if $\alpha \leq 1$ which is sufficient for the interior pure strategy to be optimal. If $\alpha > 1$ the second derivative of the objective function is determined by another function that is zero at the interior pure strategy equilibrium and is decreasing in b_k so it has at most one 0 and must be convex to the left and concave to the right. This implies that the optimum must either be at the interior pure strategy equilibrium or on the left boundary.

Analysis of the Second Derivative

The derivative of the objective function with respect to the bid is

$$\left(\frac{\alpha b_k^{\alpha}b_{-k}^{\alpha}}{\left(b_k^{\alpha}+b_{-k}^{\alpha}\right)^2}\right)Vb_k^{-1}-c$$

and this has the same sign as$\left(\alpha b_{-k}^{\alpha}\right)V - cb_k^{1-\alpha}\left(b_k^{\alpha}+b_{-k}^{\alpha}\right)^2$ If $\alpha \leq 1$ this is decreasing, so the second derivative is negative.

When $\alpha > 1$ we look at the second derivative:$(\alpha-1)cb_k^{-\alpha}\left(b_k^{\alpha}+b_{-k}^{\alpha}\right)^2 - 2\alpha c\left(b_k^{\alpha}+b_{-k}^{\alpha}\right)$ divide by $c\left(b_k^{\alpha}+b_{-k}^{\alpha}\right)$ without changing the sign to get $(\alpha-1)\left(1+b_k^{-\alpha}b_{-k}^{\alpha}\right)-2\alpha$. This is decreasing in b_k so it has at most one 0 and must be convex to the left and concave to the right. Moreover in the equilibrium where $b_k = b_{-k}$ it is negative, so there cannot be an optimum on the upper boundary. Hence in all cases it suffices to compare the utility from the first order solution to the left boundary.

Utility on the left boundary, that is, from turning out only committed members is

$$\frac{\eta_k^{\alpha}}{\eta_k^{\alpha}+\left(\frac{1}{4}\alpha V\right)^{\alpha}}V$$

so the remaining condition is that this be smaller than the surplus at the first order solution for both groups. □

The key point here is that α cannot be too large. Not only is it bounded above by $(4/V)\eta_S$ in order to remain in the interior, but we must have

$$\left(\frac{1}{2}-\frac{1}{4}\alpha-\frac{1}{1+\left(\frac{1}{4}\alpha(V/\eta_k)\right)^{\alpha}}\right)V+\eta_k\underline{\varphi}\geq 0$$

in order that it not be optimal to concede the election. If $\alpha > 3$ this condition necessarily fails.

The case of a chore is also of interest: not only may this be the case in some elections - but in more direct conflicts such as civil wars it is likely to be the case.

Theorem 6.3. *In the case of a chore $F > 0$ an interior pure strategy equilibrium exists if and only if $\alpha \leq 2$ and $4F\eta_L/(2-\alpha) \leq V \leq 4\eta_S/\alpha$.*

Proof. The analysis of interiority is similar to the case of a duty, except that now $\hat{b}_L$ is certainly positive and we rewrite the condition $\hat{b}_S \leq \eta_S$ as $V \leq 4\eta_S/\alpha$.

The analysis of the second order condition is as before: we need only check the left boundary, and we need to do this anyway there is a fixed cost. We need check only for the large group since if the large group is willing to make a positive bid, the small group is as well. Moreover, in the case of a chore dropping out means a zero bid, so no chance of victory. Hence the relevant condition is that the utility for the large group at the interior equilibrium is non-negative, in other words $(\frac{1}{2} - \frac{1}{4}\alpha)V \geq F\eta_L$. This requires that $\alpha \leq 2$ and if it is we may rewrite it in the form given: $4F\eta_L/(2-\alpha) \leq V$. □

6.3. Comparison of the Auction and the Contest

To evaluate the impact of exogenous uncertainty we compare the Tullock contest with substantial uncertainty - in the sense that $\alpha = 1$ - with the auction, for the latter referring to the result in Theorem 2.8 of section 2.3. We want to focus on the interior case.

Theorem 6.4. *Suppose in the case of a duty that $4\eta_L\underline{\varphi} < V < \eta_S(1-\underline{\varphi})$ or that in case of a chore $4F\eta_L \leq V < \eta_S + \eta_L F$. Then with $\alpha = 1$ an interior pure strategy equilibrium exists in the Tullock contest and in the auction the willingness to pay of the disadvantaged group satisfies $\eta_L\underline{\varphi} < W_d < \eta_S$. We refer to this as the case of the* common interior.

Proof. For the case of a duty from Theorem 6.2 the condition for an interior pure strategy equilibrium with $\alpha = 1$ for the Tullock contest is $4\eta_L\underline{\varphi} < V < 4\eta_S$. The conditions in the auction for $\eta_L\underline{\varphi} < W_d < \eta_S$ is that the same be true of the desire to pay $\eta_L\underline{\varphi} < B_d < \eta_S$ where we know that $B_d = V - \eta_L F$. Hence the condition is $\eta_L\underline{\varphi} - \eta_S\underline{\varphi} < V < \eta_S - \eta_S\underline{\varphi}$. At the bottom the Tullock condition is stronger, so we require $4\eta_L\underline{\varphi} < V$. At the top the auction condition is stronger so we require $V < \eta_S - \eta_S\underline{\varphi}$.

For the case of a chore from Theorem 6.3 the condition for an interior pure strategy equilibrium with $\alpha = 1$ for the Tullock contest is $4F\eta_L < V < 4\eta_S$. The conditions in the auction for $0 < W_d < \eta_S$ is that the same be true of the desire to pay $0 < B_d < \eta_S$ where we know that $B_d = V - \eta_L F$. Hence the condition is $\eta_L F < V < \eta_S + \eta_L F$. At the bottom the Tullock condition is stronger, so we require $V > 4F\eta_L$. At the top if we impose the condition required for the auction $V < \eta_S + \eta_L F$ we must then have $\eta_S + \eta_L F > 4F\eta_L$ or $\eta_S > 3F\eta_L$. Hence $V < \eta_S + \eta_L F$ implies $V < \eta_S + \eta_S/3 < 4\eta_S$ so that the Tullock upper condition holds as well. □

In the common interior case - where the prize is of intermediate size and the committed voters or fixed costs are not too large - we have a clean comparison between the Tullock contest and the auction.

First in the case of a duty the large group is both advantaged and utility advantaged; in the case of a chore the small group is both advantaged and utility advantaged - so the group getting the higher utility is the same in both cases.

Next we examine the utilities. From Theorem 2.8 in the auction the disadvantaged group gets 0 and the advantaged group gets $(\eta_L - \eta_S)(F + \underline{\varphi})$. From Theorem 6.1 in the Tullock contest the disadvantaged group gets $V/4 - F\eta_L + \eta_S\underline{\varphi} > 0$ while the advantaged group gets $V/4 - F\eta_L + \eta_S\underline{\varphi} + (\eta_L - \eta_S)(F + \underline{\varphi})$. The utility advantage of the advantaged group is the same in both cases: $(\eta_L - \eta_S)(F + \underline{\varphi})$ - but while the disadvantaged group gets 0 in the auction it gets $V/2 - F\eta_L + \eta_S\underline{\varphi}$ in the Tullock contest. An immediate implication is that total surplus is higher in the Tullock contest by twice the amount the disadvantaged group gets: $V/2 - 2F\eta_L + 2\eta_S\underline{\varphi}$. Since in both cases a prize of value V is awarded for certain, this must be because less effort is provided in the Tullock contest. The uncertainty surrounding the outcome reduces equilibrium effort provision - and so lowers costs. The conclusion is therefore the following:

- The Tullock contest Pareto dominates the auction and is the preferred mechanism. So - for example - calls to eliminate the electoral college in the U.S. might be misguided.
- On the other hand if the effort is a payment to a third party, the third party prefers a less noisy mechanism. So - for example - we should expect corrupt politicians to prefer the auction.

Finally we examine the strategies. This will make it clear that the higher surplus of the advantaged group is due to lower bidding costs in the Tullock context while partly to higher winning probability in the auction. In the Tullock case both groups make the same bid $V/4$ and have equal chance of winning. Bids are independent of either the fraction of committed members or the size of the fixed cost. The advantaged group gets its utility advantage not through a greater chance of winning but through a lower cost of making the bid. By contrast in the auction case both groups bid the minimum with equal probability of $\underline{G} = (\eta_L - \eta_S)(1/V)\underline{\varphi} + \eta_L(F/V)$ and otherwise play uniformly on $[\eta_L\underline{\varphi}, W_d]$ where[21]

$$W_d = B_d = \eta_S\underline{\varphi} + V - \eta_L F.$$

Since in the interior Tullock equilibrium the equilibrium bid must be strictly less than the willingness to pay of both groups and strictly larger than the number of committed members of larger group, we see that the support of the equilibrium bidding distribution for the auction contains the Tullock bid in its interior: there is a positive probability of both higher and lower bids. In the

[21] Here we use the fact that if $F > 0$ then $d = L$ and if $\underline{\varphi} > 0$ then $d = S$.

interior Tullock equilibrium the two groups have exactly the same chance of winning - the additional surplus of the advantaged group comes entirely from it having lower costs - while in the interior auction equilibrium the advantaged group has a $\underline{G}^2$ higher chance of winning due to a tie at the lowest bid - the higher surplus of the advantaged group comes in part due to a higher probability of winning.

6.3.1. Appendix: Power Sharing, Efficiency and Federalism

There is an alternative interpretation of the conflict resolution function: rather than viewing $p_k(b_k, b_{-k})$ as a probability of winning a prize of size V it could equally well be a share of a prize of size V. That is: it could be that each group gets a deterministic share of the prize. For example, think of of a number of regions. In a federal system each region would be separately governed: an election would determine how many districts each group controls. By contrast there could be a central system in which the winner controls all the districts. A reasonable model of power sharing is the Tullock model with $\alpha = 1$. Another interest in comparing this with the auction case is to understand the consequences of power sharing: which system yields the greatest welfare? We continue to focus on the case of a duty since that is most relevant.

Define W to be the difference between the surplus in the Tullock $\alpha = 1$ interior equilibrium and the auction. How does this depend upon the stakes V? For interiority of the Tullock model we know that we need $V \geq 4\eta_L \underline{\varphi}$ and that this gives interiority in the auction model. Starting then at $V = 4\eta_L \underline{\varphi}$ as we increase the stakes up to $V = c\eta_S(1-\underline{\varphi})$ we continue to have interiority for both models and we know that so $W = V/2+2\eta_S\underline{\varphi}$. That is: power sharing generates higher welfare and the greater the stakes the greater the welfare benefit of power sharing. This is an argument in favor of federalism.

Above $V = \eta_S(1-\underline{\varphi})$ up until $V = 4\eta_S$ we remain in the interior for the Tullock model but enter the constrained case for the auction. When the stakes grow high enough that $V = 4\eta_S$ the Tullock bids approach η_S: by contrast in the auction they never exceed this and there is a substantial probability they are below it, so we see that for higher stakes federalism leads to a welfare loss.

We can do a more precise computation. The power sharing (Tullock, $\alpha = 1$) continues to yield a surplus of $V/2 + 2\eta_S\underline{\varphi} + (\eta_L - \eta_S)\underline{\varphi} = V/2 + \underline{\varphi}$ while in the auction the surplus is the value of the prize to the large group minus the cost of bidding η_S , that is $V - (\eta_S - \eta_L\underline{\varphi})$. This gives the Tullock welfare advantage as $W = -V/2 + \underline{\varphi} + (\eta_S - \eta_L\underline{\varphi}) = -V/2 + \eta_S\underline{\varphi} + \eta_S$. Once the stakes are high enough the Tullock advantage declines with the stakes and indeed at the upper bound is equal to $W = -\eta_S(1-\underline{\varphi})$ -this is, as we expect negative.

So the overall conclusion is that federalism is a good system when the stakes are low but not so good when they are high.

6.4. Sources of Uncertainty

As we indicated there are many sources of exogenous uncertainty that may impact the outcome of an all-pay contest between two groups. An important one that has been widely used is uncertainty about how many adherents each group has. That is: we may imagine that $\tilde{\eta}_S$ is drawn from a probability distribution with mean η_S with the large group size $\tilde{\eta}_L = 1 - \tilde{\eta}_S$. The conflict resolution function is then determined by $p_k(b_k, b_{-k}) = \Pr\{b_k \tilde{\eta}_k / \eta_k > b_{-k} \tilde{\eta}_{-k} / \eta_{-k}\}$. Shachar and Nalebuff (1999), for example, assume η_S is normally distributed (although this does not respect the fact that it must lie between 0 and 1). Coate and Conlin (2004) assume that η_S is drawn from the beta distribution of which the uniform is a special case.

The Case of a Uniform Distribution. We assume that $\tilde{\eta}_S$ is drawn from a uniform distribution on $[0, 1]$. Note that in this case the name small party is a misnomer since the parties are equally likely to be large or small, but we keep the labels as S, L. Notice that the expected number of members in each party is $\eta_S = \eta_L = 1/2$ so that the bid as measure by the expected effort is half the social norm: $b_k = (1/2)\varphi_k$.

We see immediately that $p_S(b_S, b_L) = \Pr\{2b_S \tilde{\eta}_S > 2b_L(1 - \tilde{\eta}_S)\}$. We may rewrite this as $\Pr\{(2(b_S + b_L))\tilde{\eta}_S > 2b_L\} = b_S/(b_S + b_L)$, which is in fact the Tullock contest success function with $\alpha = 1$. In general if we replace the uniform with any symmetric distribution on $[0, 1]$ with cumulative distribution function G the contest success function will be $p_S(b_S, b_L) = G(b_S/(b_S + b_L))$.

Another possibility, for example, used, for example, by Herrera, Levine and Martinelli (2008) is to assume that there is a negatively correlated shock to the objective function of the two groups: when participation decisions are made the difference in valuation of the prize between the two groups is random. In our setting it is natural to think of this exogenous shifts as taking place in the cost rather than the value of the prize: for example, whether may be bad in a location that is inhabited predominately by one group.

Correlated Costs. Suppose that the types y_k have both a common and an idiosyncratic component where the common component may be correlated between the two parties - weather would be one such example. We have indexed types by a uniform distribution on $[0, 1]$. It is convenient in developing an example with a common component to index types by z_k drawn from continous strictly increasing cumulative distribution function $G_k(z)$ on $[0, \infty)$. The original index y_k can then be recovered from the formula $y_k = G_k(z_k)$. With the index z_k the party chooses a type threshold ζ_k. We assume that the population is large so that the idiosyncratic component of the shock matters only in expected value.

The specific example is defined by a parameter $0 < \alpha$. We assume that costs are sufficiently high relative to the prize so that $W_k < \eta_k \alpha/(1+\alpha)$. Each member i in group k takes an iid draw u_i from a uniform distribution on $[0, 1]$. A single independent common draw ν is taken also from a uniform on $[0, 1]$. We set $\nu_S = \nu^{1/\alpha}$ and $\nu_L = (1-\nu)^{1/\alpha}$ so that the common shock pushes the groups in different directions. A group member's type is $z_{ik} = \beta u_i/(1+\beta)\nu_k$.

Conditional on the common shock ν_k the expected fraction of the group that turns out is $\Pr(z_k \leq \zeta_k|\nu_k) = \Pr(u_i \leq ((1+\alpha)/\alpha)\zeta_k\nu_k|\nu_k)$. For $\zeta_k \leq \alpha/(1+\alpha)$ this is $\Pr(z_k \leq \zeta_k|\nu_k) = ((1+\alpha)/\alpha)\zeta_k\nu_k$ (since the RHS is no greater than 1). Observe first that $\Pr(z_k \leq \zeta_k) = \int((1+\alpha)/\alpha)\zeta_k\nu^{1/\alpha}d\nu = \zeta_k$ from which we can conclude that for $\zeta_k \leq \alpha/(1+\alpha)$ we have $y_k = z_k$. Since it cannot be optimal to choose $b_k > W_k$ and $W_k \leq \eta_k\alpha/(1+\alpha)$ we see that for $b_k \leq W_k$ the expected fraction of the group who turn out conditional on the common shock ν_k is $((1+\alpha)/\alpha)b_k\nu_k$.

Because we are assuming a large population we suppose that the actual fraction of group who turn out conditional on ν_k is exactly $((1+\alpha)/\alpha)\varphi_k\nu_k$. Hence party k wins the election if $\eta_k\varphi_k\nu_k > \eta_{-k}\varphi_{-k}\nu_{-k}$. Taking logs, this reads $\log(b/(\eta_{-k}\varphi_{-k})) + (1/\alpha)(\log(\nu) - \log(1-\nu) > 0$. Since for a uniform ν on $[0, 1]$ the random variable $\log(\nu) - \log(1-\nu)$ follows a logistic distribution the probability of winning is the Tullock contest success function

$$\frac{b_k^\alpha}{b_k^\alpha + b_{-k}^\alpha}.$$

The bottom line is: there are a variety of models - which may or may not be convenient for a particular application - all of which capture the same basic idea: given effort provision there is uncertainty about the outcome captured in a conflict resolution function. The functional form may vary from model to model, but all reflect the idea that greater effort leads in a continuous way to greater probability of winning. There is no evidence that any significant comparative static or empirical result is sensitive to the exact functional form chosen - so indeed we may view the specific choice of functional form or model underlying that functional form as largely a matter of convenience.

6.5. Exogenous versus Endogenous Uncertainty

We are studying equilibria with respect to conflict resolution functions indexed by α. As α increases there is less exogenous uncertainty about the outcome - that is, for a given pair of bids we can be reasonably confident that the larger bid will win. Formally, we can talk about conflict resolution functions $p_k(b_k, b_{-k}, \alpha)$ converging to the all-pay auction as $\alpha \to \infty$ if for all $\epsilon > 0$ we have $p_k(b_k, b_{-k}, \alpha) \to 1$ uniformly on $b_k \geq b_{-k} + \epsilon$.

There are two key facts discussed in the Appendix. First, for fixed α, there must be equilibria. Second, in the limit those equilibria must approach the unique equilibrium of the all-pay auction. We are interested primarily in the consequences of these facts. First: there must be substantial uncertainty. If the exogenous uncertainty measured by α is too low then the equilibrium must be similar to that of the all-pay auction, which is to say involve a great deal of endogenous uncertainty - in other words *endogenous uncertainty must replace exogenous uncertainty.* Second, when outcomes are determined by independent random draws of individual costs the outcome is uncertain, the number of votes cast by each party following a binomial distribution. However, as the population grows the law of large numbers implies - as shown, for example in Levine and Mattozzi (2016) - that the corresponding conflict resolution function converges to the all-pay auction. Consequently with a large population the equilibrium is much like that of the all-pay auction: this is a formal justification for ignoring the small degree of uncertainty caused by independent random cost draws.

6.5.1. Appendix: Upper Hemi-Continuity of the Equilibrium Correspondence

A key fact is that as we vary the parameter α decreasing the amount of exogenous noise the corresponding equilibria converge to an equilibrium in which there is no noise: to an equilibrium of the all-pay auction model. This implies that if the noise becomes sufficiently low then the groups must mix and endogenous noise replaces the exogenous noise. The reason is simple: the all-pay auction has a unique equilibrium with mixed strategies and pure strategy equilibria cannot converge to mixed strategy equilibria.

The underlying fact - that equilibria indexed by a parameter $\alpha \to \infty$ converge to an equilibrium - is a more general property known as *upper hemi-continuity* of the equilibrium correspondence. Let us focus on the case of a duty. Strategies are then just cumulative distribution functions G_k over bids and the objective function of group k is $U_k(G_k, G_{-k}, \alpha)$ where $\alpha = \infty$ corresponds to the all-pay auction. To fix ideas suppose for the moment that G_k are in a compact subset of a finite dimensional space and that U_k is continuous. Suppose that $\hat{G}_k(\alpha)$ are equilibria for finite α. What can we say about the limit? Since the strategies are in a compact space there must be a limit point $\hat{G}_k(\infty)$ and by choosing a subsequence we may write $\lim_{\alpha \to \infty} \hat{G}_k(\alpha) = \hat{G}_k(\infty)$. We would like to know that the $\hat{G}_k(\infty)$ are equilibria of the all-pay auction. Suppose in fact that they are not - we will reach a contradiction. One group would have to have a deviation G_k that represents an improvement: $U_k(G_k, \hat{G}_{-k}(\infty), \infty) > U_k(\hat{G}_k(\infty), \hat{G}_{-k}(\infty), \infty)$. The idea underlying upper hemi-continuity is that if there is a deviation that works in the limit it ought to work also before the limit is reached: that is, for α sufficiently large it ought to be the case that $U_k(G_k, \hat{G}_{-k}(\alpha), \alpha) > U_k(\hat{G}_k(\alpha), \hat{G}_{-k}(\alpha), \alpha)$. To show this observe that continuity implies $\lim_{\alpha \to \infty} U_k(G_k, \hat{G}_{-k}(\alpha), \alpha) = U_k(G_k, \hat{G}_{-k}(\infty), \infty)$ and $\lim_{\alpha \to \infty} U_k(\hat{G}_k(\alpha), \hat{G}_{-k}(\alpha), \alpha) = U_k(\hat{G}_k(\infty), \hat{G}_{-k}(\infty), \infty)$ so the strict inequality must hold before the limit is reached. But the existence of a profitable deviation before the limit is reached contradicts the hypothesis that the $\hat{G}_k(\alpha)$ are equilibria for finite α. This line of argument is not specific to this model - upper hemi-continuity results are ubiquitous in the literature and regarded as "easy" since the proof is simple, easy to understand, and requires only the modest assumption of continuity.

In our setting there are technical complications. First, strategies are not in a finite dimensional space, second continuity is tricky because the continuous conflict resolution functions are continuous for finite α but converge to a the all-pay auction for which the conflict resolution function is discontinuous.

The Weak Topology. In the study of mixed strategies over a continuum of pure strategies it is necessary to introduce a topology - a notion of convergence - on the space of probability measures. The useful topology has two names: it is called the weak topology in the probability theory literature and the weak* topology in the literature on functional analysis. In either case it has several equivalent definitions or characterizations of convergence $G_k^\alpha \to G_k^\infty$. One is that for any continuous random variable the expectation with respect to G_k^α converges to the expectation with respect to G_k^∞. Another is that for any open set of bids $\mathcal{B}$ the probability $\Pr(\mathcal{B}|\alpha)$ has limit values that are not smaller than the limit probability. Roughly what this says is that in the limit probability can escape to the boundary of an open set, but probability cannot enter an open set. A third is that for any closed set of bids $\mathcal{B}$ the probability $\Pr(\mathcal{B}|\alpha)$ has limit values that are not larger than the limit probability. Roughly what this says is that limit probabilities remain trapped within a closed set, but the closed set may pick up some extra probability from nearby points just the other side of the boundary. Yet another characterization involves the idea of continuity sets, which are sets that in the limit probability distribution have boundaries of measure zero (for example a closed interval of bids for which the endpoints are not atoms in G_k^∞): for such sets $\Pr(\mathcal{B}|\alpha)$ must converge exactly to the limit probability.

There are two crucial properties of the weak topology. First: the space of probability measures is compact in the weak topology. Hence, just as with a compact set in a finite dimensional space, we know that sequences have convergent subsequences. That is: for $\hat{G}_k(\alpha)$ we know that there exist some $\hat{G}_k(\infty)$ and a subsequences for which $\lim_{\alpha \to \infty} \hat{G}_k(\alpha) = \hat{G}_k(\infty)$. This gives us candidates for equilibria in the limit - what we will need to do is to show that these candidate equilibria are actually equilibria.

Second: for finite α the utility function in the case of a duty is

$$U_k(G_k, G_{-k}, \alpha) = \int_0^1 \left[p_k(b_k, b_{-k}, \alpha)V - \eta_k \max\{0, b_k/\eta_k - \underline{\varphi}\}\right] dG_k(b_k) dG_{-k}(b_{-k})$$

which is the expectation of a continuous random variable hence by one of the equivalent definitions of weak convergence must converge whenever the G_k do so weakly.

Existence of Nash Equilibrium. We now have the tools to prove the existence of Nash equilibrium for finite α. Since $U_k(G_k, G_{-k}, \alpha)$ is continuous and since it is also concave in G_k (linear in fact) the set of best responses to G_{-k} is convex-valued and upper hemi-continuous. Ordinarily existence would follows from the Kakutani fixed point theorem asserting the existence of a fixed point for such a correspondence: a fixed point meaning that each distribution is the best response to the other, that is, a Nash equilibrium. Here the result follows from the Glicksberg fixed point theorem which asserts the same in infinite dimensional spaces. Glicksberg (1952) who proved the theorem gave exactly the application to Nash equilibria. Roughly speaking we wave our hands and instead of muttering the magic incantation "Kakutani" we mutter the magic incantation "Glicksberg."

It is useful to know that for any finite α there are equilibrium distributions $\hat{G}_k(\alpha)$, however we have very little practical information about what they are like. We generally think of probability distributions as having continuous parts given by a density function along with a discrete part corresponding to atoms. Unfortunately there can also be "singular" parts corresponding to Cantor functions - functions which are continuous, increasing, climb from 0 to 1 are differentiable almost everywhere - and yet the derivative is always equal to zero. Fortunately we know for small α there are equilibria in pure strategies and for large α however bizarre the probability distribution it must be approximately the simple solution to the all-pay auction.

Convergence. At this point we are missing one ingredient to show that the limit of equilibria is in fact an equilibrium of the all-pay auction. To do this we need to know what happens to $U_k(G_k, G_{-k}, \alpha)$ as $\alpha \to \infty$. To be more precise we need to know that the equilibrium utility converges, that $\lim_{\alpha\to\infty} U_k(\hat{G}_k(\alpha), \hat{G}_{-k}(\alpha), \alpha) = U_k(\hat{G}_k(\infty), \hat{G}_{-k}(\infty), \infty)$, and the the utility from a deviation converges, that $\lim_{\alpha\to\infty} U_k(G_k, \hat{G}_{-k}(\alpha), \alpha) = U_k(G_k, \hat{G}_{-k}(\infty), \infty)$. The latter problem is greatly simplified by noticing that if there is a profitable deviation there must be a profitable deviation to a pure strategy, so for deviations we need only show $\lim_{\alpha\to\infty} U_k(b_k, \hat{G}_{-k}(\alpha), \alpha) = U_k(b_k, \hat{G}_{-k}(\infty), \infty)$ for pure strategies b_k.

We will not give a complete proof - that can be found in the Appendix to Levine and Mattozzi (2016), but give the main idea. The only real problem involves ties. That is, we can divide up the space of bids into the set where $|b_k - b_{-k}| \geq \epsilon^2$ and the set where $|b_k - b_{-k}| < \epsilon^2$ which we refer to as the *diagonal* - the set where there are approximate ties. On the off-diagonal there is no problem: p_k is converging uniformly to a continuous function, all the U_k's converge nicely. The trick is to show that before the limit is reached there is not in fact much chance of a tie so it does not effect (much) the computation of utility.

In the all-pay auction the diagonal does not matter because there is zero probability of ties. The idea is use the same argument to prove that near an all-pay auction there cannot be a very high probability of approximate ties because exactly the same deviations that work in the all-pay auction would work. Leaving aside the atoms that we know might exist at the top or the bottom, in the interior instead of showing that the groups strategies have a continuous density function, we show instead that there is a uniform bound $\underline{\Pi}$ such that over intervals of length ϵ parties place probability no more than $\underline{\Pi}\epsilon$ - if they try to lump too much weight on a short interval their opponent would have an incentive to "jump over them." This then means that the squares along the diagonal of width and height ϵ have probability of no more than $\underline{\Pi}\epsilon^2$ and as there are $1/\epsilon$ such squares the probability of the diagonal is only $\underline{\Pi}\epsilon$. Hence as we pass to the limit we take ϵ smaller and can omit the diagonal from our computation of utility.

6.6. An Auction with Exogenous Uncertainty

The key elements of a conflict resolution function for voting is that there should be a positive probability of winning that is strictly increasing in b_k with a jump at b_{-k}. The Tullock conflict resolution function delivers the first of these and moreover is convenient in that it has a single parameter such as α indexing the degree of extrinsic uncertainty. It is also consistent with a jump, or near jump, at b_{-k} provided α is sufficiently large. The drawback is that the convenience of working with a pure strategy equilibrium requires $\alpha \leq 2$ so that the conflict resolution function is concave. Unfortunately concavity - ruling out as it does any kind of jump at b_{-k} is not terribly plausible and for larger - and more realistic - α we know the equilibrium is mixed, but we do not know what it is like except in the limit where it converges to the all-pay auction.

By contrast, the all-pay auction model delivers a jump at b_{-k} and is easy and tractable to work with, but it has the rather implausible property that there is no noise so that increasing b_k does not raise the chances of winning at all until b_{-k} is reached. Here we sketch a simple model that combines some of the desirable features of both models. The basic idea is to combine the Tullock model with $\alpha = 1$ and the all-pay auction model by assuming that with with some fixed probability p_0 the outcome is decided by the Tullock model with the remaining probability $1 - p_0$ the outcome is decided on the basis of greatest effort - the all-pay model. Unfortunately the Tullock model even with $\alpha = 1$ does not combine well with the all-pay model so we adopt a slight variant on the Tullock model.

In the Tullock model with $\alpha = 1$ the probability of winning is given by

$$p_k(b_k, b_{-k}) = \frac{b_k}{b_k + b_{-k}}$$

This can also be written as the differential of the probability of winning

$$p_k(b_k, b_{-k}) - p_{-k}(b_{-k}, b_k) = \frac{b_k - b_{-k}}{b_k + b_{-k}}$$

that is, the difference in the number of votes divided the total number of votes cast. An alternative formulation is the linear differential where the differential in the probability of winning is equal to the difference in the number of votes divided by the number of possible votes rather than the number of votes cast. Since the number of total possible votes has been normalized to 1 (recall that $b_k + b_{-k} = \varphi_k \eta_k + \varphi_{-k} \eta_{-k}$ which is 1 when $\varphi_k = \varphi_{-k} = 1$ because $\eta_k + \eta_{-k} = 1$) this is

$$p_k(b_k, b_{-k}) - p_{-k}(b_{-k}, b_k) = b_k - b_{-k}$$

and comes from the linear conflict resolution function

$$p_k(b_k, b_{-k}) = \frac{1 + b_k - b_{-k}}{2}.$$

Deriving the linear conflict resolution function from a random turnout model. Suppose a fraction of voters $0 \leq \iota \leq 1$ are independents drawn randomly from the two parties. That is to say that fraction of voters lost to the independents for each party is $1 - \iota$ and the total loss of voters is proportional to the size of the party: the size of a party is given by $(1-\iota)\eta_k$. In particular party intends to bid $b_k = \eta_k \varphi_k$ then taking account of the independents the actual bid is $(1-\iota)b_k = (1-\iota)\eta_k\varphi_k$.
Suppose that the fraction of independent voters that support party k is u_k uniform on $[0,1]$. Then given bids b_k the votes of party k are $(1-\iota)b_k + \iota u_k$. The probability that party k wins is the probability that $(1-\iota)b_k + \iota(1-u_{-k}) = (1-\iota)b_k + \iota u_k \geq (1-\iota)b_{-k} + \iota u_{-k}$ or

$$u_{-k} \leq \frac{\iota + (1-\iota)(b_k - b_{-k})}{2\iota}$$

which is to say

$$\max\{0, \min\{1, \frac{\iota + (1-\iota)(b_k - b_{-k})}{2\iota}\}\}$$

or if $\iota \geq 1/2$

$$\frac{\iota + (1-\iota)(b_k - b_{-k})}{2\iota}.$$

The case in the text corresponds to $\iota = 1/2$. Note that there is no particular problem analyzing the case $\iota > 1/2$ while the case $\iota < 1/2$ seems intractable. Basically $\iota \geq 1/2$ is like $\alpha \leq 2$ in Tullock and $\iota < 1/2$ is like $\alpha > 2$. (A picture helps here.) The proposal is to finess this by assuming $\iota \geq 1/2$ but only some probability p_0 that there are independents at all.

The idea then is that with probability $1 - p_0$ the election is decided by the greatest effort, that is, the all-pay auction model; with probability p_0 the election is decided by the linear conflict resolution model, that is by the vote differential.

Suppose the opponent bidding schedule is G_{-k} we let $\tilde{G}_{-k}$ denote the probability of winning schedule derived from G_{-k} and the tie-breaking rule. This is the same as G_{-k} at points of continuity of G_{-k} and in general lies between the left and right limit of G_{-k} inclusive where the value in that range is determined by the tie-breaking rule. For a given bid by the opposing group b_{-k} the group objective function is[22]

$$\left((1-p_0)\tilde{G}(b_k|b_{-k}) + p_0\frac{1 + b_k - b_{-k}}{2}\right) V - \max\{0, b_k - \eta_k\underline{\varphi}\}.$$

[22] If we use the random turnout model in the box we probably want to adjust the costs to take account of the fact that when with probability p_0 some of the parties become independent the size of the party and hence the total cost of turning out voters is reduced. The cost would be $((1-p_0) + p_0(1-\iota))\max\{0, b_k - \eta_k\underline{\varphi}\}$ and when $\iota = 1/2$ it is $(1-p_0/2)\max\{0, b_k - \eta_k\underline{\varphi}\}$.

In the relevant range $b_k \geq \eta_k \underline{\varphi}$ this may be written as

$$\tilde{G}(b_k|b_{-k})(1-p_0)V - (b_k - \eta_k\underline{\varphi})(1-p_0V/2) + p_0(1+\eta_k\underline{\varphi} - b_{-k})V/2.$$

The constant term $p_0(1+\eta_k\underline{\varphi} - b_{-k})V/2$ matter for computing the probability of winning and welfare but since it does not depend upon b_k is irrelevant for decision making. Hence this model is exactly equivalent to an all-pay auction model where the prize is worth $(1-p_0)V$ and the marginal cost of effort is $1-p_0V/2$ which in case the latter is positive is the same as equilibrium in the standard all-pay auction model with unit marginal cost and prize $\overline{V} = (1-p_0)V/(1-p_0V/2)$. In case $1-p_0V/2 < 0$ the unique equilibrium is for both parties to turn out all of their members.

6.7. Why Pollsters are Wrong: The Uncertainty Principle in the Social Sciences

Physicists cannot predict the movement of a particle. Economists cannot predict market crashes. Political scientists cannot predict the outcome of elections. The failure of physicists has a name "Heisenberg's uncertainty principle" and as far as we know nobody criticizes physicists or obsesses over their failure. Economists and political scientists are much criticized for failing to forecast market crashes and elections. This is odd: the uncertainty principle is the foundation of quantum mechanics in which spooky particles seem to anticipate what other particles will do. The failure of economists and political scientists is for the much less spooky reason that people can and do anticipate what other people will do. There is no name for the failure of economists and political scientists: perhaps it will be more acceptable if we make it a principle? The "Lucas critique?" The "Neumann principle?"

To understand why social scientists are necessarily unable to predict certain things let's start with something simple - the familiar game of rock-paper-scissors. As we know rock breaks scissors, paper wraps the rock and scissors cuts the paper. Suppose Jan and Dean are playing rock-paper-scissors and Nate interviews each of them. Jan tells Nate she is going to play rock and Dean tells Nate he is going to play scissors. Nate publishes his prediction on his website: Jan is going to beat Dean by playing rock to his scissors. They play the game: Jan plays rock and Dean - no fool he - plays paper and beats Jan. Oops...looks like Nate was wrong. As John Von Neumann showed in 1928 there is only one solution to this paradox: Jan and Dean cannot know how the other is going to play - they must be uncertain. That uncertainty can be quantified: each must believe the other has one chance in three of playing rock, paper or scissors - or one of them is either stupid or wrong. There is no pure strategy equilibrium. Only if Nate announces that there is a 1/3rd chance of Jan and Dean each playing rock, paper or scissors will Jan and Dean be content to play as he forecasts. Empirical research shows that in real contests - soccer matches, tennis matches - the good players play randomly and with the right probabilities.

No doubt some investors and voters are stupid and wrong - but most are not. Suppose that clever Nate discovers from his big data analysis that the stock market will crash next week. He announces his discovery to the world.

Are you going to wait until next week to sell your stocks? Well nobody else is, so the market is going to crash today. Oops...looks like Nate was wrong again. Just like rock-paper-scissors the only prediction Nate can make that is correct and widely believed is a probabilistic one: For example, he can tell you that every day there is an .01% chance of a stock market crash - but he cannot tell you when the crash will take place. Just as the uncertainty principle underlies quantum mechanics so the fact that people react to forecasts is the basis of rational expectations theory in economics. And just as in the simple rock, paper scissors example this theory enables us to quantify our uncertainty.

So elections. As we have argued people vote for lots of reasons: out of civic duty, to register their opinion - and to help their side win. In 2012 voter turnout in swing states was 7.4% higher than in other states. Any analysis of elections must take into account that there are marginal voters who behave strategically - who only vote if they think there is a chance they might contribute to victory. If you are certain your party is going to lose are you more or less inclined to vote? If you are certain it is going to win? Many people - like those in the states that are not swing states - are less inclined to vote when they are confident of the outcome. So when Nate comes along and tells us that the Democrats are definitely going to win, what does the marginal Republican voter Dean do? Skips the vote. But Jan is no dummy, she realizes since Dean is not going to vote, she needn't bother either: her Democrats can win without her. But...Dean should anticipate Jan and vote and so bring his own party to victory. This is exactly the argument we gave proving that the all-pay auction has no pure strategy equilibrium. As we have shown in this chapter there is no solution to to the problem of strategic voter turnout that does not involve uncertainty about the outcome.

Why are polls wrong? Because people lie to pollsters? Because people change their minds at the last minute? By and large this is not the case - even in upset victories polls do a pretty good job of predicting how people are going to vote. What they do not do is do a good job of predicting who is going to vote - they do not predict turnout well. You read this all the time "this year turnout among Hispanic voters was unusually low" and so forth. You get the idea? We may know how many Democrats and Republicans there are and we may know that they are all going to vote for their own candidate: but if we don't know who is going to turn up at the polls we do not know who is going to win the election. And whether voters expect their party to win or lose changes whether they will bother to vote - so that voter turnout is subject to the Neumann uncertainty principle.

Pollsters argue about their mistakes. Some understand that they do not do a good job of predicting turnout. Some - Sam Wang and his Princeton Election Consortium - made the ludicrous claim - based on "deep math" - that there was a 99% probability that Hillary Clinton would win the 2016 Presidential election. Nate Silver was more conservative giving her only a 73% chance of winning. But as far as we can tell, neither one realizes that there is not something wrong with their models - that the reason that they do not predict the election is because they cannot predict the election. Any forecasts of elections that do not take account of the Neumann uncertainty principle are bound to fail.

That said: the Neumann uncertainty principle is no more a statement that “life is uncertain” than the Heisenberg uncertainty says that we are unsure where cannon balls are going to land. For example: we know that if the stakes are very low in an election the large party will almost certainly win. We can make specific probability predictions about the chances of one side or the other winning. Moreover, the indivisibility of political prizes plays a key role. If, for example, the conflict resolution function represents a deterministic sharing rule rather than a probability of winning an indivisible prize of political power then we can predict the outcome.

7. Pivotality, the Anti-folk Theorem and the Paradox of Voting

To begin it is useful to reflect on the basic model of a political contest with group members organized into two parties with the outcome determined by majority voting. Our basic model of individual behavior is that introduced by Palfrey and Rosenthal (1985) to study voting: each voter has a independent and randomly determined cost of participation which is negative for at least some committed voters. We can describe the model of behavior used by Palfrey and Rosenthal (1985) as rational selfish behavior: voters are rational and care only about their own utility. In this case the incentive to vote is the chance to shift the outcome of the election from unfavorable to favorable and by doing so claim a share of the prize. The key factor in determining individual behavior, then, is the probability that the voter will be *pivotal* meaning that the election is decided by a single vote - otherwise the decision of the voter to participate does not matter. This is a special case of the more general problem of contributing to a public good where the punishment for failing to contribute is a common punishment - either all group members are punished, or none at all. In the case of pivotal voting the punishment is the loss of the election, a cost borne by all group members.

Pivotality is controversial because both computations of equilibrium and empirical studies of the probability of being pivotal indicate that in large elections there is so little incentive to vote that to a good approximation only committed voters will turn out. This, however, has the consequence that turnout should be independent of strategic considerations such as the importance of the election and there is overwhelming evidence that this is not the case. This apparent contradiction in the context of voting has been termed "the paradox of voter turnout." It has motivated the large literature of which this book is part investigating models that are, at least superficially, not selfish rational behavior. Models of ethical or altruistic voters study rational voters who are not selfish - either for ethical or other reasons their preferences are other-regarding and they care about the consequences of their actions for other voters. Our model of peer punishment by group members who collude with each other studies voters who are both selfish and rational - we recognize, however, that groups and political parties are not blank slates, but rather are based on social networks which have the ability to provide incentive to group members through punishments and rewards.

This does not mean we should reject pivotality or models that study pivotality. As Levine and Palfrey (2007) show the simple Palfrey and Rosenthal (1985) model finds strong support in the laboratory. That does not make it relevant to large elections - but does make it relevant to smaller elections - including elections that take place within juries, committees, and legislative bodies. However, just because pivotality is important in these smaller elections does not mean that peer incentives are not important as well.

Here we are going to examine the closely connected issues of pivotality and common punishment. Roughly speaking we will find that when groups are large common punishment does not work well in either theory or practice -

and we will also find common ground for models that incorporate both pivotal considerations and individual punishments by peers.

7.1. How Relevant is Pivotality in Large Elections?

Not surprisingly, the probability of being pivotal in large elections is very low as documented by Mulligan and Hunger (2003) and Shachar and Nalebuff (1999). Based on Shachar and Nalebuff (1999)'s calculations on the probability of casting a pivotal vote in presidential elections, in all but a few states a rational voter for whom it cost \$1 to vote would have to win a prize larger than the wealth of the wealthiest person in the world for it to be worth voting. We can also point to more refined modelling and calculations: Coate, Conlin and Moro (2008) show that in a sample of Texas liquor referenda, elections are much less close than what would be predicted by the pivotal voter model, and Coate and Conlin (2004) show that the ethical voter model better fits the data than the model of pivotal voters.

A less discussed but also important issue is the scaling of elections. As the electorate grows the probability of being pivotal declines, and so should voter turnout. Observe that turnout is highly sensitive to the importance of the election - for example turnout in U.S. Presidential elections is much greater than in election for local issues only. This implies that the cost of voting distribution must be relatively flat. Hence, following the example Feddersen and Sandroni (2006) and Coate and Conlin (2004), we assume that cost of voting is non-negative and uniformly distributed. The probability of being pivotal with a large electorate is approximately proportional to the standard error - and this should decline roughly as the square root of the number of voters. More specifically in a two candidate election with an even number of voters N each casting their vote randomly Penrose (1945) and Chamberlain and Rothschild (1981) show that for large N the probability of a tie is approximately $\sqrt{2/N\pi}$. These same considerations apply to participation rates if voting costs are non-negative and uniformly distributed: participation should decline in inverse to the square root of the size of the electorate.

How do elections scale? If we focus on post-war national elections in consolidated democracies with per capita income above the world average and voluntary voting, and examine how voter turnout depends on the size of the country, we find that there is a group of small countries with population ranging from 300,000 to 10 million with high voter turnout of 78% to 88% and a group of large countries with population ranging from 35 million to 319 million with lower voter turnout ranging of 55% to 71%. Within these groups of countries there is very little variation or evidence of negative correlation between size and turnout.[23] While it is true that the group of smaller countries generally

[23]Turnout data are averages in the post-war period of OECD countries with voluntary voting and Freedom House Index of political freedom below 3. We included UK and excluded the rest of the EU since in the latter substantial power has passed to the EU itself, so that the significance of "national" elections is different than in fully sovereign nations - in particular

have higher turnout than the larger countries, within groups turnout is quite homogeneous while population varies by a factor of nearly 10 - this data is in no way consistent with scaling by the square root of the population. In fact, it is not even consistent with a monotone relation between turnout and population, which is the main prediction of the pivotal voter model. A similar picture emerges if we turn attention to the dynamics of voter turnout in advanced democracies: turnout declined on average by a mere 10% in the past 50 years in the face of a voting age population which more than doubled.[24]

7.2. The Anti-Folk Theorem and Common Punishment

The empirical failure of the pivotal voting model in large elections reflects a deep theoretical problem sometimes called the anti-folk theorem. Empirically it seems that incentives for individuals to contribute do not come from the possibility of failure. Our premise is that groups overcome public goods problems by punishing individuals for failing to adhere to social norms. The game that takes place after a decision about adhering to a social norm - to vote, to lobby and so forth - is a different game than the underlying public goods contribution game: it consists of one or more rounds in which social sanctions are imposed on individual group members based on signals about their adherence to the norm.

It is undeniable that in addition to the individual incentive of facing peer punishment there are incentives to participate due to common punishments. There can be an incentive to contribute for fear of failure. Moreover, real groups face repeated public goods problems and an alternative and widely used model to that of peer punishment is one in which incentives for public goods contributions are due to the possibility of retaliation over future public goods. For example, one reason to adhere to the social norm of voting might be that individual group members understand that if they do not vote the social norm will break down and in the future elections will be lost due to low turnout. Leaving aside the fact that in practice groups generally punish individuals for failing to adhere to social norms, an important reason that we do not examine these types of schemes is they depend on common punishments and because in large groups common punishments do not work. This is an old observation in the literature on oligopoly and the prisoner's dilemma - dating back at least to Radner (1980)'s work on the subject. Radner shows in a repeated oligopoly game cooperation breaks down as the number of competing firms grows large. This is the conclusion of an extensive literature including results by Green (1980), Sabourian (1990), and Levine and Pesendorfer (1995). We base our presentation on Fudenberg, Levine, and Pesendorfer (1998).

To understand the problem with common punishments, let us examine a group composed of identical members $i = 1, 2, \ldots, N$ who can either take a default action $a^i = 0$ at a cost of $c(0) = 0$ or adhere to a social norm $a^i = 1$

for the smaller EU nations. However, including the rest of the EU does not alter the overall picture. Data is taken from http://www.idea.int.

[24] In fact in Denmark and Sweden turnout increased by 3% and 6%, respectively.

at a cost of $c(1) = 1$. There is a noisy signal of whether or not a member has contributed. The signal is $z^i \in \{0, 1\}$ where 0 means "good, followed the social norm" and 1 means "bad, did not follow the social norm." If the social norm was violated, that is $a^i = 0$ the bad signal is generated for sure, while if the social norm was followed so that $a^i = 1$ there is never-the-less a chance θ of the bad signal where $\theta < 1$ is a measure of the noise of the signal.

We now want to explore what happens when the only available punishment is a common punishment P - that is, individuals cannot be punished, either the entire group must be punished or nobody at all. This corresponds to a situation where social sanctions are not available and, for example, the only punishment for not voting is that the party loses or that other group members withhold their votes in a future election. More broadly it corresponds to a situation where the punishment for failing to adhere to a social norm is the breakdown of the social norm resulting in a common punishment for the entire group. This is the type of mechanism studied by Wolitzky (2013), by Acemoglu and Wolitzky (2015) and by Ellison (1994). Ellison recognizes that - as we are about to show - such schemes do not work well when the group is large - and indeed the same observation motivated the Kandori (1992) model of social norms from which our own peer punishment model is derived.

We suppose that the group must pick a mechanism consisting of a punishment size $\overline{P} \geq P \geq 0$ and a rule for determining punishment based on signal profiles in an effort to enforce compliance with the social norm $a^1 = 1$ for all members. We assume that $\overline{P} \geq 1$ so that it is possible to give punishments at least as great as the cost saving in switching from the social norm to the default action. Let us first consider schemes that determine whether or not to punish based on the number of bad signals. Let Q^1 denote the probability of punishment if all group members adhere to the social norm and let Q^0 denote the probability of punishment if all group members except one adhere to the social norm.

First consider the case where $\theta = 0$ so that there is no noise. In this case we can punish if any bad signals are received: then $Q^1 = 0$ and $Q^0 = 1$. If the punishment $P \geq 1$ the social norm is incentive compatible: if everyone adheres to the social norm all pay the cost of 1. If any single member deviates they save the cost of adhering to the social norm but certainly receive a punishment at least equal to this. Hence each individual member is pivotal: if any one violates the social norm the agreement breaks done, so none do so.

So far so good - this simple solution has motivated many successful efforts in theory that unfortunately fail in practice: this mechanism breaks down badly if $\theta > 0$ so that there is some noise. Since the probability of no signal being received if the social norm is adhered to is $(1-\theta)^N$ we have $Q^0 - Q^1 = (1-\theta)^N$. Hence the benefit of deviating is at least $1-(1-\theta)^N\overline{P}$ which is certainly positive if N is large. The problem is that trying to punish on a single bad signal means that with noise and a large population the common punishment is triggered almost for certain, and since you are going to be punished anyway, you might as well cheat.

Despite many efforts in practice - one of the authors was briefly involved with a startup internet firm that believed otherwise - this problem cannot be fixed by a more clever choice of punishment rules. The natural thought is that the problem can be fixed by being more tolerant - recognizing that bad signals will be generated when everyone adheres to the social norm perhaps we should punish only if a threshold $\beta > 0$ is exceeded. For example, punish when twice the expected number of bad signals is observed, or something like that. We might suspect that this does not work since it will not work even in the case that $\theta = 0$: in that case everyone will cheat! To see if this can work in the case that $\theta > 0$, observe that for large N to a good approximation the distribution of the fraction of signals is normally distributed. Let

$$n^1 = \frac{\sqrt{N}(\beta - \theta)}{\sqrt{\theta(1-\theta)}}, n^0 = \frac{\sqrt{N}\left(\beta - ((N-1)\theta + 1)/N\right)}{\sqrt{\theta(1-\theta)}}$$

then $Q^1 \approx \Phi\left(n^1\right), Q^0 \approx \Phi\left(n^0\right)$ where Φ is the standard normal distribution function. So to a good approximation

$$Q^0 - Q^1 \approx (n^0 - n^1)\frac{1}{\sqrt{2\pi}} e^{-(1/2)\left(n^0 - n^1\right)^2} \leq (n^0 - n^1)\frac{1}{\sqrt{2\pi}} = \frac{1-\theta}{\sqrt{N}\sqrt{2\pi}\sqrt{\theta(1-\theta)}}.$$

This is again the inverse square root of N rule for being pivotal. We see immediately the problem: no matter what the choice of β as $N \to \infty$ we have $Q^0 - Q^1 \to 0$ and since the benefit of deviating remains at least $1 - (Q^0 - Q^1)\overline{P}$ once again for large N the social norm fails to be incentive compatible. By choosing a $\beta > 0$ we solve the problem of punishing too frequently - but at the expense of assuring that a deviation by a single individual has very little effect on the outcome. This failure is more general. For any mechanism where a common punishment of P is determined only by the total number of bad signals Fudenberg, Levine, and Pesendorfer (1998) in their Lemma A give the generally valid bound $Q^0 - Q^1 \leq 2/(\sqrt{N}\min\{\theta, 1-\theta\})$, so incentive compatibility must fail for N sufficiently large.

If we drop the assumption that the punishment must be a constant and that it depend only on the number of bad signals and allow general punishment schemes $\overline{P} \geq P(z) \geq 0$, or indeed a fixed[25] finite number of different kinds of punishments that have a different effect on different individuals, then we could, for example, base the punishment only on the behavior of a single member and give that member incentives to follow the social norm. Unfortunately with a common punishment we cannot do this simultaneously for any substantial fraction of the group: Fudenberg, Levine, and Pesendorfer (1998) prove in their Proposition $1'$ that regardless of the punishment scheme the fraction of members adhering to the social norm must fall to zero as $N \to \infty$. This result depends only on the fact that there is a minimal amount of noise in observing individual behavior and that the group is limited to common punishments - it

[25] That is the number of different types of punishments does not grow with N.

does not rest on symmetry assumptions or specific details of the game.

Notice that it is possible to use common punishment to punish a few key individuals - and if they have the ability to punish the larger number of other group members this can be bootstrapped into an effective incentive scheme. To take a simple example, we can imagine the CEO of a business firm who can punish individual workers, for example, by firing them or cutting their pay. The CEO may be tempted to shirk by not monitoring the employees - but then the firm will fail and everyone including the CEO will be punished. This common punishment gives the CEO the needed incentive to monitor and punish workers. Even with limited supervisorial capacity - so that a supervisor can monitor only a few employees - a hierarchical organization of the type studied by Williamson is possible. However, while political organizations are hierarchical in practice and these types of incentives may be relevant for the upper echelon, in political organizations the rank and file - individual voters, or individual farmers in a farm lobby - cannot easily be punished or rewarded by the hierarchy, so that for these types of organizations some form of peer discipline must be at work.

7.3. Incentive Constraints with Pivotality

Just as peer punishment is important in large elections, pivotality is important in small elections and it would be useful to have a model that incorporates both features - the more so as the importance of pivotality will increase as monitoring costs grow large. Here we sketch out how to formulate such a model.

We start by giving a formulation of a contest model that enables us to compute the probability of being pivotal. Recall the setting: group members independently draw types y_k uniformly distributed on $[0,1]$ and may contribute zero effort at zero cost or contribute a single unit of effort at a cost of $c(y_i)$ where we assume the types are ordered so that this is a non-decreasing function - higher types have higher cost. Effort for group k is determined by a social norm in the form of a threshold φ_k for participation. Recall also that the bid of a group $b_k = \eta_k \varphi_k$.

We define two partial conflict resolution functions: $P_k^0(b_k, b_{-k})$, the probability of winning conditional on all voters except one following the social norm b_k/η_k and *the remaining voter not voting*, and $P_k^1(b_k, b_{-k})$, the probability of winning conditional on all voters except one following the social norm b_k/η_k and *the remaining voter voting*. These should be differentiable and non-decreasing in b_k and satisfy $P_S^i + P_L^i = 1$, where $i \in \{0,1\}$. This two functions enable us to compute an overall conflict resolution function and the probability of being pivotal: the overall conflict resolution function is $p_k(b_k, b_{-k}) = (b_k/\eta_k)P_k^1(b_k, b_{-k}) + (1-(b_k/\eta_k))P_k^0(b_k, b_{-k})$ and the probability of being *pivotal* is $Q_k(b_k, b_{-k}) = P_k^1(b_k, b_{-k}) - P_k^0(b_k, b_{-k})$. It is convenient in what follows to view the strategies G_k as measures rather than cumulative distribution functions.

To analyze incentives with pivotality we start by identifying what and individual voter would like to do in the absence of punishment. This depends on what voters from both parties are doing. For any given social norm φ_k and mixed strategy of the other party G_{-k} we may define the pivotal cutoff $\gamma_k(\varphi_k, G_{-k})$

by the solution to $c_k(\gamma_k) = \int Q_k(\eta_k \varphi_k, b_{-k}) v_k dG_{-k}(b_{-k}) = \mathscr{Q}_k(\varphi_k, G_{-k}) v_k$. This represents the type of voter who is indifferent between bearing the cost of voting in order to improve the party's chance of victory and abstaining. Since $c_k(y)$ is differentiable and has a strictly positive derivative the solution is unique and continuous. We can now determine the incentive constraint when there is punishment for not voting. For voters who would not otherwise vote, that is, $y \geq \gamma_k(\varphi_k, F_{-k})$ the incentive constraint is $c_k(y) - \mathscr{Q}_k(\varphi_k, G_{-k}) v_k \leq P_k$. This says that the net cost of voting, which is the direct cost c_k minus the benefit because of pivotality $\mathscr{Q}_k(\varphi_k, G_{-k}) v_k$, must be less than or equal to the punishment for not voting. Notice that the mixed strategy of the other party G_{-k} appears in the incentive constraint since P_k must be chosen before the realization φ_{-k} is known.

From the incentive constraint we can derive the monitoring cost for $\varphi_k \geq \gamma_k(\varphi_k, G_{-k})$ as the cost of punishing the innocent with bad signals $M_k(\varphi_k, G_{-k}) = (1-\pi)(1-\varphi_k)\left(c_k(\varphi_k) - \mathscr{Q}_k(\varphi_k, G_{-k}) v_k\right)$. Notice that if φ_k is pivotal in the sense that $\varphi_k = \gamma_k(\varphi_k, G_{-k})$ then $M_k(\varphi_k, G_{-k}) = 0$ and the function M_k is continuous.

There remains the issue of what happens if the social norm calls for less participation than would be individually optimal in the presence of the pivotality incentive $\varphi_k < \gamma_k(\varphi_k, G_{-k})$. For voters with $\varphi_k < y < \gamma_k(\varphi_k, G_{-k})$ the social norm calls on y to not to vote, but in fact y would like to. This case is not covered by the basic model and there is more than one modeling possibility. One is to assume that there is no cost of getting a voter not to vote, in which case $\varphi_k < \gamma_k(\varphi_k, G_{-k})$ and $M_k(\varphi_k, G_{-k}) = 0$. In this case we may write $M_k(\varphi_k, G_{-k}) = (1-\pi)(1-\varphi_k) \max\{0, (c_k(\varphi_k) - \mathscr{Q}_k(\varphi_k, G_{-k}) v_k)\}$ which is obviously continuous, although scarcely linear in F_{-k}. However, all that is required for the results that follow is that $M_k(\varphi_k, G_{-k})$ is non-negative for $\varphi_k < \gamma_k(\varphi_k, G_{-k})$.

The goal of the party is to maximize per capita utility $p_k(b_k, b_{-k}) v_k - C_k(b_k/\eta_k) - M_k(b_k/\eta_k, G_{-k})$. We summarize the results from Levine and Mattozzi (2016). First, equilibrium distributions G_S, G_L exist are independent of the model chosen of monitoring costs for $\varphi_k < \gamma_k(\varphi_k, G_{-k})$. Second, as the partial conflict resolution functions approach the all-pay auction so the equilibrium distributions approach the unique equilibrium of that model. In particular accounting for pivotality and uncertainty of outcome due to independent draws, as the sizes of population grows the equilibrium approaches that of the all-pay auction in which we ignore pivotality and assume that the greatest expected number of votes wins.

Finally, if we introduce a scaling factor for cost as discussed above so that $M_k(\varphi_k, G_{-k}) = \psi(1-\pi)(1-\varphi_k)\left(c_k(\varphi_k) - \mathscr{Q}_k(\varphi_k, G_{-k}) v_k\right)$ and analyze what happens as the monitoring cost grows $\psi \to \infty$, we find that in the small election case where $C_k(1) > v_k$ we have $G_k^{\psi}(|\varphi_k - \gamma_k(\varphi_k, G_{-k}^{\psi})| \leq \epsilon) \to 1$. This says that the probability that the threshold used by the group differs more than trivially from the pivotal cutoff is very small.

Notice that this does not necessarily imply that the limit is an equilibrium in the sense of Palfrey and Rosenthal (1985) since we allow correlation devices within parties, but rather a correlated equilibrium with pivotality of the type studied by Pogorelskiy (2015).

7.4. The Holdup Problem and the Tragedy of the Anticommons

A nice illustration of pivotality and uncertainty and one quite relevant to political economy, lobbying and public policy is the classical holdup problem. This can be formulated as the problem that was faced prior to the formation of the German customs union in the 19th Century. Along the Rhine river ships carrying cargoes of varying values pass. The value of a cargo ρ is uniformly distributed over $[0, 1]$ and is know to the shipper. Along the rivers are a number N of castles with each castle i charging a fee p_i for passage. The castles do not know the value of ρ but only that they are drawn uniformly on $[0, 1]$. If all other castles set the price p and a deviant castle charges the price p_i then the total cost of passage faced by the shipper is $(N-1)p+p_i$ and the shipper will operate only if this is less than or equal to ρ. Hence the expected revenue of a deviant castle is $(1-(N-1)p-p_i)p_i$. The optimal price to set is therefore determined by the first order condition $(1-(N-1)p-2p_i) = 0$. Hence the symmetric Nash equilibrium of this game is at $p = 1/(N+1)$. The total revenue received by all castles is then $N/(N+1)^2$- and as $N \to \infty$ not only does shipping shrink away to zero, but so does the revenue of the castles.

Here the problem is that each castle by setting a high price imposes an externality on its neighbors by reducing shipping. This is like a public good problem - all castles would benefit if they colluded to set a single price: optimally they would all agree to charge $(1/N)$ of the monopoly price $p = 1/(2N)$ - this is of course what the establishment of the customs union did.

Notice that if there were no uncertainty then the monopoly price is ρ and each castle could charge ρ/N - if any castle tried to raise the price the sale would be lost - without uncertainty each castle would be pivotal.

The bottom line is that many small monopolies producing complementary goods are much worse than a single monopoly controlling all production. This idea has many applications. The presentation here is based on Boldrin and Levine (2005)'s analysis of patent systems. If many different ideas are required to innovate then a strong patent system strangles innovation. The problem is that many independent patent holders each separately license all the ingredients needed to innovate. A similar problem can occur in construction or the opening of a new business. If permits from many different corrupt agencies are required then development will come to a halt: each corrupt official demands too high a bribe. Even if mere paperwork is required - if each agency benefits by a high paperwork requirement so as to get more resources from a central authority - this also can bring development to a halt. Yet another example can be found in the Chari and Jones (2000) analysis of pollution rights: if each individual property owner in a city owned air rights and any polluter had to get a permit from each property owner then there would be no pollution - and also no output. The broad problem of too many owners of complementary resources is called by Heller (2008) the tragedy of the anticommons, and his book documents numerous examples of gridlock brought about by the holdup problem.

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