Voting by Collusive Parties

David K. Levine and Andrea Mattozzi
Overview

Woman who ran over husband for not voting pleads guilty

- Palfrey-Rosenthal setting
- rational voter participation
- two collusive parties: similar to “ethical voters”
- parties can enforce social norms through peer punishment
- results in unique mixed strategy equilibrium of all-pay auction
- enforcement costless and equal prize: large party advantaged
- costly enforcement and equal prize of intermediate size: small party advantaged
Mixing

- ethical voter models of Federson/Sandroni and Coate/Conlin use "sufficiently large" aggregate shocks to avoid mixed equilibria
- we stick to the original Palfrey/Rosenthal model
- we observe that GOTV (get out the vote) effort by parties is a carefully guided secret which makes sense only if the party is engaging in a mixed strategy
Cost of Voting

identical party members privately draw a type $y$ from a uniform
distribution on $[0, 1]$
determines a cost of voting $c(y)$, possibly negative and continuously
differentiable, has $c'(y) > 0$ and $c(y) = 0$ (committed voters)

participation cost of voting

$C'(y) = 0$ for $y < \underline{y}$

$C(y) = \int_{\underline{y}}^{y} c(y) \, dy$ for $y \geq \underline{y}$

(quadratic in Coate/Conlin)
Peer Monitoring Model

simplified version of Levine/Modica, based on Kandori
social norm a threshold $\hat{y}$ and rule to vote if $y \leq \hat{y}$

- each member of the party audited by another party member
- auditor observes whether or not auditee voted
- auditee did not vote and norm not violated probability $\pi$ that auditor will learn this.

$\pi = 0$ then the auditor learns nothing

$\pi = 1$ the auditor perfectly observes whether $y$ is above or below the threshold $\hat{y}$

(auditing costless so unlike Levine/Modica only one round needed)
Peer Punishment

party can impose punishments $0 \leq P \leq \overline{P}$ on members.

- auditee voted or is discovered not to have violated the policy: not punished
- auditee did not vote and the auditor cannot determine whether or not the auditee violated the policy, the auditee is punished with a loss of utility $P$

social norm is \textit{incentive compatible} if and only if $P = c(\hat{y})$
Cost of Monitoring

$\phi$ participation rate of the party (probability of voting)
total cost of inducing participation $D(\phi) = C(\phi) + M(\phi)$

participation cost: $C(\phi) = \int_{y}^{\phi} c(y) dy$ is the total cost

$C'(\phi) = c(\phi)$ so $C(\phi)$ is increasing and convex

monitoring cost: $M(\phi) = \int_{y}^{1} (1 - \pi) P dy$

incentive compatibility requires $P = c(\hat{y}) = c(\phi) = C''(\phi)$

so write $M(\phi) = (1 - \pi)(1 - \phi)C''(\phi)$.

$c(y) = \overline{P}$ most possible turnout
Pivotality and Ethical Voters

modification 1: allow for social costs of punishment to the group as well as the punishee (in the paper)

modification 2: account for chance of being pivotal in the incentive constraint (not in the paper)

then:
  • as monitoring costs go to zero we get ethical voters model
  • as monitoring costs go to infinity we get the pivotal voting model
Convexity and Concavity

\( C(\phi) \) is necessarily convex
\( M(\phi) \) is not

and so \( D(\phi) \) may or may not be

**Theorem:** We have \( C(\phi) = M(\phi) = 0 \) so \( D(\phi) = 0 \). The participation cost \( C(\phi) \) is twice continuously differentiable strictly increasing and strictly convex. The monitoring cost \( M(\phi) \) is continuously differentiable. If \( \overline{y} = 1 \) (that is \( c(1) \leq \overline{P} \) so that full participation is possible) the monitoring cost \( M(\phi) \) cannot be concave, must be decreasing over part of its range and \( M(1) = 0 \) so \( D(1) = C(1) \).

at \( \underline{y} \) no punishment cost since punishment is not needed to turn out the committed voters

at \( \overline{y} = 1 \) everybody votes so nobody is actually punished.
two parties $k = S, L$ of size $N_S < N_L$

assume that the side that produces the greatest **expected** number of votes wins a prize worth $v_L > 0$ and $v_S > 0$ per capita

both parties face identical per capita costs of turning out voters characterized by $y, \bar{y}$ and $D(\phi)$

assume that $D'(\phi) > 0$ since the non-increasing case is harder to characterize and seems less interesting.
Advantaged and Disadvantaged Parties

\[ \Phi_k = N_k \phi_k \text{ expected absolute number of voters party turns out.} \]

\[ \Phi_k = N_k y \text{ number of committed voters in each party} \]

\[ \Phi_k = N_k \bar{y} \text{ most possible voters in each party} \]

\[ \hat{\Phi}_k \text{ satisfies } D(\hat{\Phi}_k / N_k) = v_k \text{ or } \hat{\Phi}_k = \Phi_k \text{ if no solution: most number of voters party is willing and able to turnout} \]

generic assumption \( \hat{\Phi}_L \neq \hat{\Phi}_S \)

d (the “disadvantaged”) party is the one with \( \hat{\Phi}_d < \hat{\Phi}_{-d} \)
Conceding and Taking Elections

a party *concedes* the election if it makes a bid that has zero probability of winning in equilibrium.

a party *takes* the election if it makes a bid that has probability one of winning in equilibrium.

the election is *contested* if neither party either concedes or takes the election.
Main Theorem

There is a unique mixed equilibrium. The disadvantaged party earns zero and the advantaged party earns \( v_d - D(\hat{\Phi}_d/N_d) > 0 \).

If \( \hat{\Phi}_S \leq \hat{\Phi}_L \) then the small party is disadvantaged, always concedes the election by bidding \( \hat{\Phi}_S \) and the large party always takes the election by bidding \( \hat{\Phi}_L \).
Main Theorem (continued)

If $\hat{\Phi}_S > \Phi_L$ then for $\Phi \in (\Phi_L, \hat{\Phi}_d)$ the mixed strategies of the players have no atoms, and are given by continuous densities

$$p_k(\Phi_k) = \left(1/N_{-k}\right)D'(\Phi_k/N_{-k})/v_{-k}.$$  

Only the disadvantaged party concedes the election by bidding $\Phi_d$ with probability $1 - D(\hat{\Phi}_d/N_{-d})/v_{-d} + D(\Phi_L/N_{-d})/v_{-d}$ and it has no other atom.

If the small party is advantaged it has no atom and cannot take the election with positive probability. If the large party is advantaged, it turns out only committed voters (that is, bids $\Phi_L$) with probability $D(\Phi_L/N_S)/v_S$ and, if $\hat{\Phi}_S = \Phi_S$, the party takes the election by bidding $\Phi_S$ with probability $1 - D(\Phi_S/N_S)/v_S$. 
Comparative Statics

1. Only the relative sizes of the parties matters.

2. If the value of the prize to the small party is small enough or the value to the large party is large enough then the small party is disadvantaged and concedes the election with very high probability. Specifically, if $v_S \leq D(yN_L/N_S)$, it follows that $\Phi_S \leq \Phi_L$ and the small party always concedes the election. Furthermore, as $v_L \to \infty$, the probability $P_S(\Phi_S)$ that the small party concedes goes to one at a rate that is bounded independent of the value of the prize to the small party. In other words, in a very high value election, the small party turns out only its committed voters and the large party acts preemptively turning as many voters as the small party is capable of turning out.

3. If the advantaged party has a higher probability of winning a contested election than the disadvantaged party, then it has an overall higher probability of winning the election, but the converse need not be true. (by example) The disadvantaged party can have a better than 50% chance of winning the election.
Common Prize

\[ N_S v_S = N_L v_L = V \]

\( D(\phi) \) strictly increasing and twice differentiable in \([y, \bar{y}]\) and \( D''(\phi) \) univalent meaning \( D(\phi) \) either convex or concave on \([y, \bar{y}]\), but not both.

**Theorem:** If \( D(\phi) \) is convex then the large party is advantaged. If \( D(\phi) \) is concave, and and for some \( y < \hat{y} < \bar{y} \) we have \( D(\hat{y}) < V/N_L < D(\bar{y}) \) and

\[
\frac{y D'(y)}{\hat{y} - y} < - \max_{y \leq y \leq \hat{y}} D''(y)
\]

then for for \( N_L \bar{y} > \hat{\Phi}_L \) and in particular for \( N_S \) close enough to \( N_L \) the small party is advantaged.
Small Party Advantaged

for small party to be advantaged (Olsonian case):

• intermediate value of prize
• small $y$
• not too constrained by $\bar{y}$
• high costs of monitoring (generates high concavity)
• homogeneous costs of participation (generates low convexity)
**Efficiency**

measured by surplus $v_{-d} - D(\hat{\Phi}_d / \hat{N}_{-d}) > 0$

(not by whether the party with the largest $v_j$ won)

worst case: when parties are very similar and $\bar{y}$ constraint does not bind
Interpretation of $\hat{\Phi}_k$

$\hat{\Phi}_k$ in general (not just for voting) measures willingness to pay when there is a 0-1 decision

- demonstrate, do not demonstrate
- strike, do not strike
- lobbying effort

Remark: the disadvantaged party gets a surplus of zero, the advantaged party gets the surplus of winning minus of submitting a bid equal to the willingness to pay of the disadvantaged part

exactly the same surpluses as a second price auction in weakly undominated strategies; same true for first price auction if equilibrium exists

- in the case of lobbying $\Phi$ is not “lost” but may be in part income to politicians
Interpretation of \( y \)

\( y \) are “committed voters”

may in fact be due to a different social norm: “civic duty to vote” also enforced by monitoring but independent of party

- seems less likely to be a factor in non-voting situations such as lobbying, demonstrations, or striking
- not that there wouldn't be people committed to demonstrating, etc. but just that there are probably few of them compared to committed voters)

in the case of lobbying we expect \( y < 0 \), that is the lowest individual cost is positive

\[ D(0) = 0 \text{ but } \lim_{y \to 0^+} M(0) > 0 \]

fixed cost of getting anybody to contribute – studied by Levine/Modica

much more favorable to small group
Sampling Error

we assumed that the outcome is determined by the expected turnout not the actual turnout

conjectures:

• as $N_L \to \infty$ holding fixed $N_S/N_L$ all equilibria converges to the one we have computed (good approximation for large population)

• as $V_L \to \infty$ turnout of the large group approaches $N_L \bar{y}$ and not $N_S \bar{y}$

why? if the large group turns out some $\Phi_L < N_L \bar{y}$ then it has a chance of losing bounded away from zero independent of $V_L$, while if it turns out $N_L \bar{y}$ it wins for sure, so for a large enough prize it cannot be optimal to choose a turnout of $\Phi_L$ or smaller

• in the case of a large prize the implications is that turnout must decline asymptotically with population

• working on examples of how turnout depends on population