Information Aggregation in Auctions

(based on Phil Reny's slides)

(Wilson, *Restud* (1977), Milgrom, *Econometrica* (1979, 1981))

- ♦ n bidders, single indivisible good, 2nd-price auction
- ♦ state of the commodity, $\omega \sim g(\omega)$, drawn from [0,1]
- ♦ signals, $x \sim f(x/\omega)$, drawn indep. from [0,1], given ω
- ♦ unit value, $v(x,\omega)$, nondecreasing (strict in x or ω)
- ♦ $f(x/\omega)$ satisfies strict MLRP:

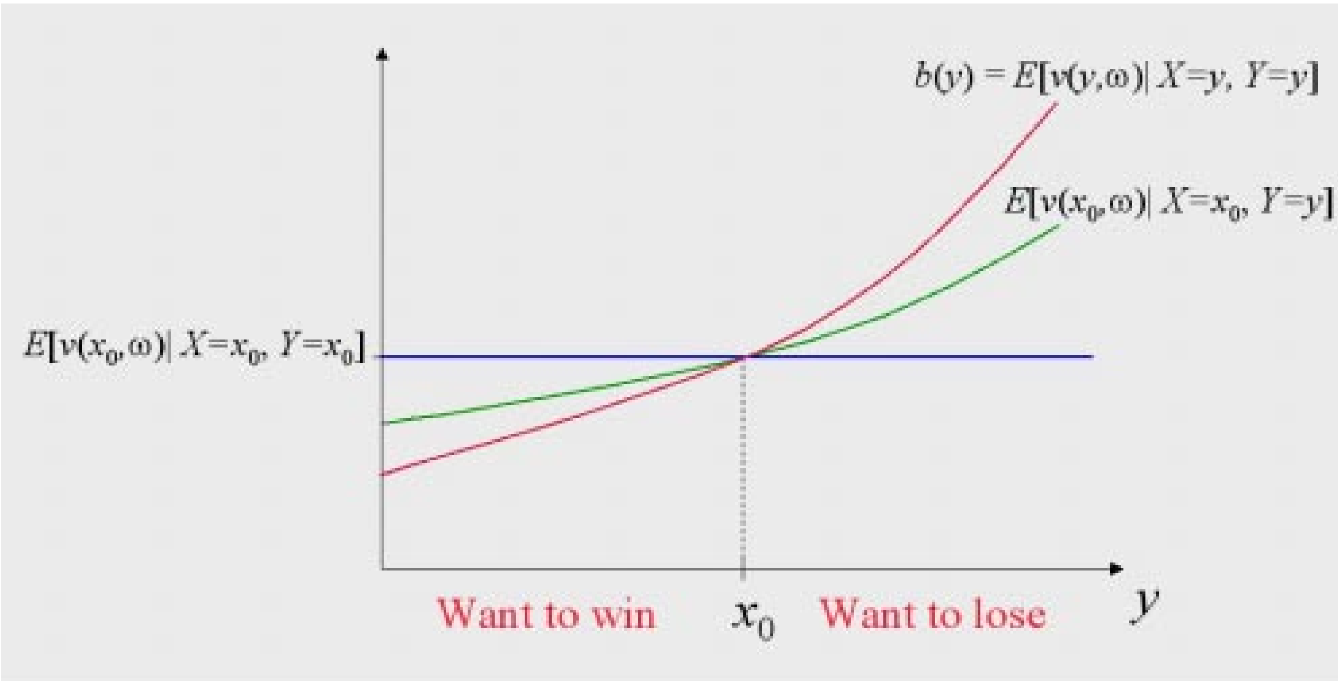
$$x > y \Rightarrow \frac{f(x \mid \omega)}{f(y \mid \omega)} \text{ \textit{strictly} } \uparrow \text{ \textit{in} } \omega$$

♦ Equilibrium: $b(x) = E[v(x,\omega)|\ X=x,\ Y=x]$

(X is owner's signal, Y is highest signal of others)

♦ Claim: $b(x) = E[v(x,\omega)|\ X=x,\ Y=x]$ is an equilibrium.

♦ Suppose signal is x_0. Is optimal bid $E[v(x_0,\omega)|\ X=x_0,\ Y=x_0]$?



- ♦ Equilibrium: $b(x) = E[v(x,\omega)|\ X=x,\ Y=x]$

  (X is owner's signal, Y is highest signal of others)
- ♦ outcome efficient for all n
- ♦ Equilibrium Price: $\mathrm{P} = E[v(\mathrm{z},\omega)|\ \mathrm{X}=z,\ Y=z]$,

  where z is the 2^{nd}-highest signal.

- if ω is U[0,1] and x is U[0,ω], then P->$v(\omega,\omega)$

the competitive limit, and information is aggregated.

(fails if conditional density is continuous and positive.)

Principal-Agent Problem

A risk neutral principal

A risk averse agent with utility $u(c)$, where $u(0) = 0$, $u(v) = 1$

Agent may take one of two actions $e = 0,1$ (effort level)

Total utility of agent is $u(w) - e$ where w is payment from principal

Two possible output levels $0, y$ accrue to the principal

If agent takes effort $e = 0$ then probability of y output is $\pi_0 > 0$; if agent takes effort $e = 1$ then probability is $1 > \pi_1 > \pi_0$

Assume that $\pi_1 y - 1 > \pi_0 y$ so that it is efficient for the agent to make an effort

Agent's reservation utility is 0

With complete observability

Maximize principal's utility

Pay the agent a fixed fee of v if he provides effort, nothing if he does not. So agent is indifferent gets $u(v) - 1 = 0$ if effort, $u(0) = 0$ if no effort. So he is willing to provide effort, but not if he is paid less

With incomplete observability

Principal only observes output, pays w_y, w_0

Incentive constraint for agent:

$$\pi_1 u(w_y) + (1-\pi_1)u(w_0) - 1 \geq \pi_0 u(w_y) + (1-\pi_0)u(w_0)$$

individual rationality constraint for agent:

$$\pi_1 u(w_y) + (1-\pi_1)u(w_0) - 1 \geq 0$$

Principal may pay 0, get 0, or minimize $\pi_1 w_y + (1-\pi_1)w_0$ subject to these constraints

Rewrite IC

$$(\pi_1 - \pi_0)\left[u(w_y) - u(w_0)\right] \geq 1$$

implies IR constraint must hold with equality, since otherwise could lower w_0 while maintaining IC

IR $\pi_1 u(w_y) + (1-\pi_1)u(w_0) - 1 = 0$

objective function $\pi_1 w_y + (1-\pi_1)w_0 = c$

IC $(\pi_1 - \pi_0)\left[u(w_y) - u(w_0)\right] \geq 1$

$w_y = \dfrac{c - (1-\pi_1)w_0}{\pi_1}$ [from objective function]

substitute objective into IR

$$\pi_1 u\left(\frac{c-(1-\pi_1)w_0}{\pi_1}\right) + (1-\pi_1)u(w_0) - 1 = 0$$

differentiate

$$\frac{dc}{dw_0} = -\frac{-\pi_1(1-\pi_1)u'(w_y) + (1-\pi_1)u'(w_0)}{\pi_1 u'(w_y)}$$

$$= -\frac{(1-\pi_1)\left[u'(w_0) - \pi_1 u'(w_y)\right]}{\pi_1 u'(w_y)} \leq 0$$

non-negative since $w_y \geq w_0$ implies $u'(w_0) \geq u'(w_y)$

because $dc/dw_0 \leq 0$ should increase w_0 until the IC binds

combining the IC binding with the IR

$$\left(\pi_1 - \pi_0\right)\left(1 - u(w_0)\right) = \pi_1$$

which is possible only if $u(w_0) < 0$, that is $w_0 < 0$

notice that IC implies $w_y > w_0$ so no full insurance what if constrained to $w_0 \geq 0$? ("limited liability *ex post*")

The constraint binds, so optimum has $w_0 = 0$

(IC) $(\pi_1 - \pi_0)u(w_y) \geq 1$

(IR) $\pi_1 u(w_y) \geq 1$ does not bind if (IC) holds

so objective is to minimize $\pi_1 w_y$ subject to IC

namely IC should bind $(\pi_1 - \pi_0)u(w_y) = 1$

agent earns an "informational" rent because IR does not bind

since IC binds, still have $w_y > w_0$ and no full insurance

Macro Mechanism Design: The Insurance Problem

Kehoe, Levine and Prescott [2000]

continuum of traders ex ante identical

two goods $j = 1,2$

c_j consumption of good *j*

utility is given by $\tilde{u}_1(c_1) + \tilde{u}_2(c_2)$

each household has an independent 50% chance of being in one of two states, $s = 1,2$

endowment of good 1 is state dependent

$\omega_1(2) > \omega_1(1)$

endowment of good 2 fixed at ω_2.

In the aggregate: after state is realized half of the population has high endowment half low endowment

Gains to Trade

after state is realized

low endowment types purchase good 1 and sell good 2

before state is realized

traders wish to purchase insurance against bad state

unique first best allocation

all traders consume $(\omega_1(1) + \omega_1(2))/2$ of good 1, and ω_2 of good 2.

Private Information

idiosyncratic realization private information known only to the household

first best solution is not incentive compatible

low endowment types receive payment

$(\omega_1(2) - \omega_1(1))/2$

high endowment types make payment of same amount

high endowment types misrepresent type to receive rather than make payment

Incomplete Markets

prohibit trading insurance contracts

consider only trading ex post after state realized

resulting competitive equilibrium

- equalization of marginal rates of substitution between the two goods for the two types

- low endowment type less utility than the high endowment type

Mechanism Design

purchase $x_1(1) > 0$ in exchange for $x_1(2) < 0$

no trader allowed to buy a contract that would later lead him to misrepresent his state

assume endowment may be revealed voluntarily, so low endowment may not imitate high endowment

incentive constraint for high endowment

$$\begin{aligned}&\tilde{u}_1(\omega_1(2) + x_1(2)) + \tilde{u}_2(\omega_2 + x_2(2))\\ &\geq \tilde{u}_1(\omega_1(2) + x_1(1)) + \tilde{u}_2(\omega_2 + x_2(1))\end{aligned}$$

- Pareto improvement over incomplete market equilibrium possible since high endowment strictly satisfies this constraint at IM equilibrium

Need to monitor transactions

Lotteries and Incentive Constraints

one approach: X space of triples of net trades satisfying incentive constraint

use this as consumption set

enrich the commodity space by allowing sunspot contracts (or lotteries)

1) X may fail to be convex

2) incentive constraints can be weakened - they need only hold on average

$$E\,|_2\ \tilde{u}_1(\omega_1(2)+x_1(2))+\tilde{u}_2(\omega_2+x_2(2))$$
$$\geq E\,|_1\ \tilde{u}_1(\omega_1(2)+x_1(1))+\tilde{u}_2(\omega_2+x_2(1))$$

General Equilibrium and Mechanism Design

consider a pure exchange economy with a continuum of consumers of N different types

consumer of type i has utility $u(x^i)$ and endowment of w^i

utility functions are known but endowments are private information

consider the mechanism in which each consumer announces his endowment and is assigned the competitive equilibrium net trade $\hat{x}^i - w^i$ that corresponds to a competitive equilibrium at prices p with respect to the announced endowments

[provided consumers announce no more than they have, this is feasible]

because consumers are “small” the competitive equilibrium and prices don’t change when a consumer makes a different announcement so given that everyone else is telling the truth, you should do the same since it maximizes your utility in your budget set, which is all you can hope to get

Makowski and Ostroy: with finitely many consumers this is dominant strategy if the economy is “perfectly competitive” meaning that no individual consumer can effect price through his presence in the economy

“the marginal social benefit of an extra person is zero”

Bargaining and The Core

In the continuum pure exchange economy there we have that the core=the set of competitive equilibria

Definition of the core: no coalition can improve the equilibrium allocation

CE is in the core follows from the usual first welfare theorem argument: given the equilibrium prices, if the coalition could improve themselves (using the Pareto criterion) it would cost strictly more than they can afford at equilibrium prices, hence is not feasible for their given endowments

Large numbers of “identical” or similar consumers narrows the core down to the CE

So is there a “non-cooperative” theory of the core?

Perry and Reny “A Non-Cooperative View of Coalition Formation and The Core” *Econometrica* 1994

Transferable utility economies only

Time is continuous, no discounting

A proposal is a coalition and an allocation for that coalition

Choices at each moment of time

- ♦ Be quiet
- ♦ Make a proposal (may be done anonomously)
- ♦ Accept the current proposal
- ♦ Leave and consume

A proposal is binding when accepted by all members of the coalition to which it applies

If anyone leaves then they all leave and get what is accepted

Proposals must either exclude a coalition that has reached a binding agreement, or must include them all

A new binding proposal may nullify an existing one

Payoff from never leaving no better than existing in a coalition of yourself

Must accept proposals immediately

For a brief interval before and after a “move” everyone has to be quiet

There is a unique Markov perfect equilibrium

(they called it “stationary” but their meaning is what we now call Markov)

every such equilibrium is in the core

if the game is totally balanced (i.e. a pure exchange economy) every core allocation is an equilibrium