Contracts, Liability Restrictions and Costly Verification*

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Abstract

As counterintuitive as it may appear, this paper describes simple contract-theoretical settings where the parties optimally choose to stipulate unverifiable or unenforceable contracts. The only manner to achieve the first-best is to stipulate a "roundabout" contract that explicitly prohibits the optimal outcome, while secretly agreeing to breach this written agreement in equilibrium so as to achieve the first-best. We introduce a general representation of enforceability constraints that encompasses both technological and institutional constraints. Enforceable contracts are shown to be undominated if and only if verification is costless (so that the court's knowledge satisfies the axiom of positive introspection), and the parties are not subject to liability limitations such as a damage-compensation restriction on transfers, or individual liability rule.

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1 Introduction

Suppose that, before playing a game, the players may sign a contract to Pareto-improve upon the Nash equilibria of the game. In many instances the enforceability of contractual transfers may be limited by verifiability or liability constraints. This paper describes simple contract-theoretical settings, where the only manner for the players to achieve the first-best is to stipulate a contract that explicitly prohibits the first-best, secretly agreeing to breach the agreement in equilibrium so as to achieve the optimal outcome. While the contract prescribing the first-best outcome fails to induce the first-best, the optimal outcome can be achieved with these "roundabout," apparently contradictory contracts.[1]

These findings contradicts the natural supposition that, at least when writing contracts is not costless, it would be a waste of resources to stipulate unverifiable prescriptions. Optimal roundabout contracts are violated in equilibrium, and hence necessarily contain unverifiable and unenforceable prescriptions. This fact is of interest for the foundations of contract incompleteness, because it shows that the players should not necessarily sign an incomplete contract when complete contracts are unenforceable.[2]

Our results show that the standard definition of contracts as functions of signals verifiable in court is too restrictive, because it is equivalent to assuming that the players may only sign contracts that do not include unverifiable prescriptions.[3] Furthermore, the issue of contract enforcement cannot be settled by looking at verifiability constraints only: we introduce a general representation that captures all relevant enforceability constraints, both technological ones (such as verifiability constraints), and institutional ones (such as liability constraints).

In line with the general axiomatization of knowledge from the epistemic literature in games, we represent verifiability as an information correspondence mapping each action profile actually played

[1] In the Civil Law tradition, a contract that, by mutual agreement, does not express the true intent of the parties is defined a 'simulation', see Black's Law Dictionary, 6th. Ed. (1990), page 1384.

[2] The literature on contract incompleteness that follows the seminal contributions of Grossman and Hart (1986) and Hart and Moore (1990) is vast. The reader can consult one of the many surveys, such as Hellwig (1996), or Tirole (1999). Transaction and complexity costs have been studied by Anderlini and Felli (1999) and by Segal (1999), among others.

[3] For this basic approach, see, for instance, Laffont and Maskin (1982). Recently, Maskin and Tirole (1999) have identified sufficient conditions under which subgame-perfect implementation message mechanisms can alleviate verifiability constraints. The main difficulties that they highlight consist of the requirements of welfare neutrality and of renegotiation proofness.

into a set of action profiles that cannot be ruled out by the court.[4] In particular, correspondences whose ranges include non-singleton sets represent cases of imperfect verifiability, and product correspondences represent cases where the action taken by each player can be verified independently of the opponents' choices. Besides enabling us to analyze the court's knowledge with general epistemic tools, this modelling choice allows us to describe contracts as transfers that directly depend on the parties' conducts (and possibly on state contingencies), and to provide a simple representation of the manner in which any written transfer scheme is transformed into the actually court-enforced transfers, in the presence of the relevant enforceability constraints.

A preliminary result (implicitly proved in Bernheim and Winston, 1998) is that enforceable contracts are undominated if liability is unrestricted and the verifiability correspondence is product and partitional. While prima facie non-partitional verifiability may seem unnatural, we will show that these structures have not been uncommon in contract theory, even though they have not been formally defined as non-partitional correspondences.[5] Looking in depth at the epistemic axioms that induce non-partitional structures, Geanakoplos (1989) shows that either the axiom of positive introspection, or the axiom of negative introspection (or both) must be violated. In this context, the key axiom is the first one, which is expressed by requiring that the information correspondence is transitive, and informally states that if one "knows" that an event has occurred, then she must also "know that she knows" that the event has occurred. Specifically, we show that when verifiability is product and liability is unrestricted, enforceable contracts are undominated if and only if the verifiability correspondence is transitive.

While some research suggests that the axiom of negative introspection is unreasonable in the context of court's knowledge, there is no consensus on the validity of positive introspection.[6] This paper presents a simple court model that satisfies positive introspection. The main restricting

[4]This approach has been adopted by Bernheim and Winston (1998), who restrict attention to product and partitional correspondences. For a general survey of the game-theoretical literature on epistemic knowledge, the reader may consult Dekel and Gul (1997).

[5]Also note that, while a player's information structure may be non-partitional only if she does not know it (otherwise, she will resolve it into a partition by means of counterfactual reasoning), the court is constrained by rules of legal procedure that may limit the use of counterfactual arguments in trial decisions, thus preventing the resolution of a non-partitional structure.

[6]The court is an institution that needs to present conclusive evidence for all the statements presented in sentences. As Shin (1993) remarks, such a provability requirement may break the negative-introspection (*Know That You Don't Know)* axiom: when one is unable to prove a statement, she may also be unable to prove that the statement cannot be proved. Dimitri (2000) presents an epistemic model of observability and verifiability, where verifiability does not necessarily satisfy positive introspection.

assumption is that all evidence is costlessly produced in trial. When evidence production and fact-finding are costly activities, intransitive verifiability naturally arises. Intuitively this occurs because verifying an event is often less costly than verifying that the event can be verified, as this introduces one additional layer of costly verification.

Non-product verifiability structures are fairly common in contract theory, and arise when the court cannot verify the action taken by a player independently of the other players' choices. When the verifiability structure is non-product, it is often the case that if a player violates a contract, the court verifies the breach of contract but cannot identify the violator. In order to analyze these instances, we introduce a distinction based on the liability rule. As a natural liability principle, we propose that an individual may be held liable for a contract violation only if she is identified as the party breaching a contract. Specifically, we denote by *individual liability* rule the case where each player can be held accountable only for her own actions, and not for a contract violation that may be due to another player. If instead any player may be penalized by the court regardless of the identity of the contract violator, we say that the rule is one of *joint (and several) liability.*[7]

For the case of individual liability, this paper establishes that the court does not settle the trial on the basis of the verifiability structure, but on the basis of the projections of the verifiability correspondence onto the players' action spaces. Roundabout contracts dominate enforceable contracts when these projections are intransitive, even if the verifiability structure is itself partitional. For the case of joint liability, instead, our results are the same as in the case of product verifiability: enforceable contracts are undominated if and only if the verifiability correspondence is transitive.

We finally consider the possibility that liability is restricted to damage compensation, because the court may not enforce contractual transfers (liquidated damages clauses) that are punitive in nature, meaning that they are in excess of the actual harm or in excess of the foregone profits caused by the breach of contract.[8] We obtain a strong negative result: the first-best can only be

[7]While contractual liability usually follows from individual contractual commitments, the default rule may be joint and several liability in some special cases. For instance, the Universal Commercial Code 3-116, regulating 'negotiable instruments' (i.e. unconditional promises or orders to pay a fixed amount of money) states that "Except as otherwise provided in the instrument, two or more persons who have the same liability on an instrument [...] are jointly and severally liable [...]." While not predominant in contract law, joint and several liability may apply in tort law, e.g. the Comprehensive Environmental Response Compensation and Liability Act. For a survey on research in law and economics assessing the efficiency of difference liability rules, see Shavell (1987).

[8]For references to this, see the Uniform Commercial Code (sections 2-718 and 2A-504), the Restatement (Second) of Contracts section 356, and 347 cmt. a: "Contract damages ... are intended to give [the plaintiff] the benefit of his bargain by awarding him a sum of money that will, to the extent possible, put him in as good a position as he would

achieved by means of roundabout contracts even in basic contract-theoretical settings, as long as the verifiability correspondence is slightly more complex than a 'trivial' correspondence,[9] even if it is product and partitional.

While it is not easy to uncover empirical data on contractual parties secretly violating their written agreements, it is not too arduous to find anecdotal evidence in support of our thesis. Consider, for a simple instance borrowed from university life, the students' code of honor. One of the aims of the code is to prevent students from cheating and earning an undeserved high grade. However, it is usually difficult to directly verify whether a student cheats, and often the only instance in which cheating can be verified is when two or more students turn in identical exam answers. But in this event, it is impossible to establish which student copied from which. The first-best can still be achieved by overspecifying the code of honor (i.e. the "contract") by requiring students not only not to cheat, but also to take steps to prevent anyone else from cheating from their work. Students are penalized for failing to take such steps only if there is evidence that cheating has occurred and, in practice, no such steps are ever actually taken.[10]

Roundabout contracts play a role when enforceability or verifiability costs or limitations make it possible to deter only large deviations from the letter of the contract: it is optimal in some cases to write an overdemanding contract, and achieve the first-best with small violations. For instance, casual observation suggests that in some areas of the US it is considered unlikely to be sanctioned when violating the speed limit by less than 10 M.p.h. on a highway. In fact, in the faster lanes the traffic often flows at a speed that is roughly 10 M.p.h. above the limit. Furthermore, many drivers believe that this is justifiable because speed limits are too restrictive. This phenomenon allows for a simple equilibrium explanation. Suppose that the socially optimal top speed on a highway is x M.p.h., but that enforceability and verifiability costs make it unlikely that a speed violation is sanctioned unless it is at least 10 M.p.h. above the speed limit. The social contract crafted by

have been in had the contract been performed." According to Feess and Hege (1997), "In virtually all legal systems outside the United States, punitive damages are either excluded or play a very minor role. Even in the US, punitive damages are normally restricted to cases of reckless conduct, e.g. drunken driving." We are indebted to Bill Zame for pointing us out the relevance of damage compensation for this study.

[9] A verifiability correspondence is trivial if either nothing can be verified, or everything can be verified.

[10] On a similar account, we would like to thank Faruk Gul for telling us that, if a joint task is to be accomplished by a team in the Turkish army, each member of the team is often explicitly requested to accomplish the whole task on his own. These apparently absurd instructions are given so as to avoid the need for identifying which members of the teams shirked the assignment in case that the task is not accomplished.

the legislator will incorporate this limitation and set the speed limit at $x - 10$ M.p.h., so that the faster traffic flows at x M.p.h.

Beyond these simple casual examples, it is well documented from the legal literature that verifying breach of contract is not a simple matter in practice, in particular in areas where substantial performance is sufficient for compliance (e.g. construction building, insurance). Also, the determination of damages may be problematic and costly, and is often a conservative remedy for the breach of agreement.[11] These limitations make it unlikely that even a carefully written agreement effectively deters all possible contractual deviations. Overspecifying the contract improves the final outcome by deterring large deviations, without precluding Pareto-improving deviations. In fact, legal scholars have long recognized the possibility that written contracts include overspecified clauses that are seldom exercised in practice. Following the seminal paper by Macaulay (1963), a striving literature underlines the major difference between "the contours of transactors' contracting relationship [and] the scope of the rights and duties memorialized in their written, legally enforced contracts".[12]

The paper is presented as follows. After reviewing related literature in the second section, the third section presents the model, the fourth section shows the suboptimality of enforceable contracts vis a vis roundabout contracts in simple contract-theoretical settings, the fifth section derives our general game-theoretical characterization, and the sixth section concludes. The most complex aspects of our analysis are relegated to the Appendix.

2 Literature Review

Our work is closely related to some of the contributions in the game-theoretical literature on knowledge. It is immediate to compare our results with the analysis of Geanakoplos (1989) on the

[11] These findings, based on reviews of legal cases concerning breach of contracts, damage compensation, and other remedies, are reported, for instance, in the textbook edited by Macaulay, Kidwell, Galanter and Whitford (1995), particularly Vol. 1, chapter 2, and Vol. 2, chapter 5.

[12] Possibly this phenomenon is best synthetized in the text-book edited by Macaulay, Kidwell, Galanter and Whitford (1995), page 512: "even when lawyers prepare elaborate contractual documents, often business people who carry out the transaction follow conventional practices rather than reading the written contract". In the same vein, the study by Keating (1997) quotes: "My investigation revealed that there are two, only slightly overlapping, worlds out there: the world of business practice and the world of law." and "Buyer prefer boilerplate language that is favorable to them, and sellers prefer boilerplate language that is favorable to them. However, in the majority of cases business people know that these difference simply do not matter."; whereas the case-study by Bernstein (1995) quotes: "In many contexts, transactors accept late payment, vary quantity terms, assume new obligations, waive covenants, and adjust prices in ways that their written contracts do not require".

value of information and on the "no-trade" theorems in the presence of (truthful) non-partitional information correspondences. The axiom of transitivity is necessary for the value of information to be positive, and to rule out speculative trade; together with the axiom of nestedness, it is also sufficient to establish these results. In our framework, instead, nestedness does not play any role, as transitivity is necessary and sufficient for enforceable contracts to be optimal.

Our characterization shares some similarities with the analysis of Green and Laffont (1986). They study an environment where an informed agent communicates to a principal that precommits to a decision rule specifying a single choice for any received message. The agent's message, however, is constrained by a correspondence, mapping the states of the world into subsets of the state space. In any state, the message sent must belong to the image of the state through the correspondence. If the correspondence is trivial, and thus the agent's message is unconstrained, by the Revelation Principle,[13] one can restrict attention without loss of generality to truthful (direct) mechanisms. Green and Laffont (1986) show that the key axiom is indeed transitivity: the Revelation Principle holds in this environment if and only if the correspondence is transitive.[14]

While this paper determines conditions under which optimal contracts include unverifiable prescriptions, Bernheim and Winston (1998) follow the 'opposite route' of identifying games where the optimal contracts exclude verifiable prescriptions. According to a commonly held view, imperfect verifiability could make complete contracts ineffective, because it constrains enforceability. Bernheim and Winston (1998) identify a further (purely strategic) source of contract incompleteness whereby the players may optimally choose contracts that do not restrict their choice to the maximal extent allowed by verifiability constraints. Their analysis of static games may be considered precursory to this paper's characterization, as it states (translated in this paper's terminology) that enforceable contracts are optimal in static settings with product and partitional verifiability.

Among recent models of evidence production and verification, Bull (2001) shows that costly verification may induce non-partitional verifiability, whereas Bull and Watson (2002a) show that when evidence production is costless, the parties can restrict attention to contracts where all privately controlled evidence is disclosed in court. The results of Sanchirico and Triantis (2002) are much in

[13]See Green and Laffont (1977), Dasgupta, Hammond and Maskin (1979), and Myerson (1982).

[14]Deneckere and Severinov (2002), however, show how to resurrect the revelation principle when communication is costly but unconstrained, and Bull and Watson (2002a) prove a revelation principle with private control of hard evidence.

the spirit of this paper's analysis, as they show that, despite undermining truth finding, evidence fabrication may improve the outcome of contractual relationships (see also Sanchirico, 2000). In our quest for a general axiomatic construction to represent enforceability and verifiability, we are conceptually indebted to Daughety and Reinganum (2000a), who introduce a Bayesian framework where axioms represent constraints imposed at the trial level by rules of evidence, procedure, and higher court review.[15]

Research in law and economics compares the efficiency of different liability rules in cases with multiple defendants. Shavell (1987) provides a summary of this literature, which shows the existence of liability rules that implement the first-best when actions are observable.[16] Besides being detrimental to defendants, Kornhauser and Revesz (1993, 1994) show that the joint and several liability rule may also be detrimental to plaintiffs because it may stifle or complicate out-of-court settlement.

In the economics literature on damage compensation, Aghion and Bolton (1987) demonstrate the role for stipulated damage provisions in excluding competitors, and Spier and Whinston (1995) incorporate renegotiation and market power in their analysis. Stole (1992) provides a signalling explanation for the court enforcement of liquidated damage terms (as long as they do not significantly exceed actual losses). Spier (1994) compares the welfare properties of finely tuned and flat damages. Lewis and Sappington (1999) show that lender's deep pockets mitigate judgment-proof problems arising with budget-constrained producers.

Finally, to the extent that this paper advocates 'hard' game-theoretical foundations to appropriately incorporate institutional and legal features in contract-theoretical analysis, our work is conceptually indebted to the models of trials that appear in the law and economics literature (such as Milgrom and Roberts, 1986; Rubinfeld and Sappington, 1987; Daughety and Reinganum, 2000b), and to the explicit modeling of inalienable decisions and technological constraints in contract-

[15] In the legal literature, Posner (1999) proposes both a search model and a cost-minimization model of evidence production, and compares their features both in the inquisitorial and in the adversarial system; similar in purpose is the analysis of Lewis and Poitevin (1997) who examine how different rules for presentation of evidence affect verdicts in regulatory hearings modeled as games of imperfect information.

[16] When the court is fully informed, Kornhauser and Revesz (1989) show that a variety of negligence rules implement the first-best. However, Feess and Hege (1998) show that an efficient liability rule may not exist when multiple tortfeasors interact in a non-separable way, there is imperfect information about their actions, and punitive damages are not feasible. See Emons (1990) and Green (1976) on the case of additive separability, and Emons and Sobel (1991) on the case where agents are not identical.

theoretical relations (see Reinganum, 1988 and 2000, on plea bargaining, and Watson, 2001, on the timing of renegotiation).

3 The Model

Consider a normal-form game $G = (I, A, u)$, where I is the set of players, A is the (finite) action space, and u are the utility functions.[17] In the literature on contract theory, verifiability constraints are usually modeled by introducing a set of verifiable signals, whose realizations depend on the profile played a. In line with the general axiomatization of knowledge from the epistemic literature in games, we represent the court's verifiability constraints with the information correspondence $P : A \to 2^A$; if the players play profile a, then the court can only conclude that no profile $b \notin P(a)$ has been played, but cannot distinguish among the action profiles contained in $P(a)$.[18]

Throughout the paper we consider *truthful* verifiability correspondences: we assume that $a \in P(a)$, for any $a \in A$. On the other hand, we allow for the possibility that verifiability is non-partitional, and we also introduce a further distinction: we say that verifiability is product whenever each player's choice is verified independently of the other players' choices.

Definition 1 *The correspondence P is* partitional *if for any a and b, $P(a) \cap P(b) \neq \emptyset$ implies $P(a) = P(b)$. The correspondence P is* product *if there is a collection $(P_1, \cdots, P_I)$ with $P_i : A_i \to 2^{A_i}$ such that $P(a) = \times_{i=1}^{I} P_i(a_i)$ for any $a \in A$.*

In anticipation of playing game G, the players may choose to stipulate a contract in order to Pareto improve upon the Nash Equilibria of G. In general, a contract is a transfer scheme that modifies the incentives in the game. To simplify our exposition, we focus on simple *contracts,* [19] i.e. agreements by which the players commit not to take some actions, and appoint an agent who does not directly participates in the game, with the right of collecting fines when the contract

[17]To the extent that the underlying game G may be expanded so as to include the possibility of sending messages, our model subsumes the message game approach of, for instance, Green and Laffont (1977). The restriction to games of complete information is made for expositional ease only, it is immediate to expand our model to allow for general Bayesian games.

[18]Following the epistemic game-theoretical approach, in the first part of the paper we take the correspondence P as a primitive construct. We reconsider the issue in Section 5, where we show how the correspondence P may also be derived from a simple model of court decision with verifiable signals: stating that $a \notin P(b)$ is equivalent to state that when b is played, the court is likely to observe verifiable signals x that induce a small enough posterior probability assessment $\Pr(a|x)$.

[19]Bernheim and Winston (1998) introduce the concept of simple contracts for product and partitional verifiability.

is violated.[20] The function of the collector of fines is to make sure that any verifiable breach of contract is deterred: it is in fact in her best interest to go to court whenever a contract violation can be verified.[21] Formally a simple contract is a set of allowable action profiles $C \subseteq A$. We say that action profile b is a violation of C if $b \notin C$. However, if the profile b is played, and $P(b) \cap C \neq \emptyset$, the court cannot rule out that an action profile allowed by the contract was played.

Definition 2 *Given a contract C, the violation $b \notin C$ is* verifiable *if $P(b) \cap C = \emptyset$.*

When a contract violation b is verified, under the joint liability rule, each player i can be held liable regardless of whether the violation can be imputed to her; we thus set $u_i(b|C, J) = -\infty$ if b is a verifiable violation of C, and $u_i(b|C, J) = u_i(b)$ otherwise.[22] Under the individual liability rule, instead, the court will sanction player i only if it identifies her as a contract violator. When b is played, the court cannot rule out the possibility that player i intended to comply with the contract C, whenever there is a profile c allowed by the contract C such that $(c_i, a_{-i}) \in P(b)$ for some $a_{-i} \in A_{-i}$. Equivalently put, player i's personal liability cannot be established if $\pi_i(P(b)) \cap \pi_i(C) \neq \emptyset$, where for any set of profiles $B \subseteq A$, the set $\pi_i(B) = \{a_i \in A_i | (a_i, a_{-i}) \in B \text{ for some } a_{-i} \in A_{-i}\}$ denotes the *projection* of B on A_i.

Definition 3 *Given any contract C, and profile b, player i is* individually liable *if $\pi_i(P(b)) \cap \pi_i(C) = \emptyset$.*

For each player i, we set $u_i(b|C, I) = -\infty$ if player i is individually liable for b under contract C, and $u_i(b|C, I) = u_i(b)$ otherwise.[23]

[20] For ease of the non-technical reader, we relegate our general construction and results to the Appendix. As well as dealing with general transfer schemes, we will introduce the *budget-balancedness* constraint, which restricts attention to zero-sum transfers among the players in the game. We will show that budget balancedness is *immaterial* for our results.

[21] In Holmstrom (1982) for instance, the members of the team achieve the cooperative outcome by appointing an agent, external to a team, with the right of collecting fines from the members of the team if any of them has failed to cooperate.

[22] For ease of the exposition, it is implicitly assumed that the players are unlimitedly liable for breaking the contract, so that if a violation is verified, the players can be sanctioned in an arbitrarily harsh manner. It is immediate to see how to modify our construction to accomodate for bounded liability.

[23] It is immediate to see that in order for a player i to be individually liable when b is played and C is signed, it must also be the case that b is a verifiable violation of C. Furthermore, under individual liability, we can restrict attention without loss of generality to product contracts. A player i in fact is liable for a profile b under a contract C if and only if she is liable for the profile b under the contract $\times_{i \in I} \pi_i(C)$. As a result $u_i(b|C, I) = u_i(b| \times_{i \in I} \pi_i(C), I)$, for any $b \in A$. This is equivalent to say that each contract should be understood as a collection of individual commitments C_i for each each player i.

When transfers are limited to damage compensation, players should not be rewarded in excess of actual harm, or foregone profits, vis a vis the stipulated outcomes in the contract C.[24] For any profile b, and any player i, we set $u_i(b|C,\mathbf{t}) = u_i(b) + t_i(b)$ where $t_i(b)$ denote the net transfers to player i. First, we assume that net transfers are budget balanced, $\sum_{i\in I} t^i(b) = 0$, because the players cannot commit to pay fines to an external collector whose payoff is independent of their choices. Second, we assume that players cannot be rewarded in excess of the difference between the (minimum) payoff guaranteed by contract C and the (maximum) payoff that they may have achieved.

Definition 4 *Given contract C, and profile b, the* maximum damage compensation *of player i is*

$$\bar{t}_i(b) = \max\{0, \min_{c\in C} u_i(c) - \max_{a\in P(b)} u_i(a)\}.$$

We define the action profile a to be *achievable by contract C* under *joint liability* if a is a Nash equilibrium of the game $G|C, J = (I, A, u(\cdot|C, J))$ and $u(a|C, J) = u(a)$.[25] We analogously say that the profile a is *achievable by contract C* under *individual liability* if a is a Nash equilibrium of $G|C, I = (I, A, u(\cdot|C, I))$ and $u(a|C, I) = u(a)$; and that a is *achievable by C* under *damage compensation* restrictions if a is a Nash equilibrium of a game $G|C, \mathbf{t} = (I, A, u(\cdot|C, \mathbf{t}))$ and $u(a|C, \mathbf{t}) = u(a)$ for some $\mathbf{t}$ such that $\sum_{i\in I} t_i(b) = 0$ and $t_i(b) < \bar{t}_i(b)$ for every i, b. For brevity, we say that $a \in A^J(G, P)$, respectively $a \in A^I(G, P)$, and $a \in A^D(G, P)$, if a is achievable with some contract C under joint liability, respectively individual liability, and damage-compensation restrictions.

A contract is usually defined *enforceable* if it depends only on verifiable events. Intuitively, the court would not be able to enforce its prescriptions, if it cannot verify which instance has occurred. This definition however would imply that a simple contract C is not to be deemed enforceable if it admits undeterred joint violations. Such a requirement may be perceived as over-restrictive because Nash equilibrium does not allow joint deviations. Since this paper aims at characterizing the optimality of enforceable contracts, an over-restrictive definition may undermine our results.

[24]While in many settings this restriction is likely to be associated with individual liability restrictions, in the interest of clarity we introduce a logically independent constraint.

[25]If this last condition does not hold, a may be an equilibrium of G when C is signed only because some player i is always penalized when her opponents play a_{-i}. Obviously, player i would never sign the contract C. While in principle contract C could achieve profile a, this outcome would not be individually rational.

To avoid any ambiguity, we introduce a definition that requires only individual deviations to be deterred.

Definition 5 *The contract C is* enforceable *under* joint liability *(respectively,* individual liability, *and* damage compensation restrictions) *if for any $c \in C$, any $i \in I$, and any $(b_i, c_{-i}) \notin C$, it is the case that $P(b_i, c_{-i}) \cap C = \emptyset$, and respectively that $\pi_i(P(b)) \cap \pi_i(C) = \emptyset$, and that $u_i(b_i, c_{-i}) - u_i(c) \leq \sum_{j \neq i} \bar{t}_j(b_i, c_{-i})$.*[26]

As well as checking whether a contract's prescriptions are enforceable, we are also interested in determining whether in a specific game it will be violated in equilibrium. In such a case, we say that the contract is not *equilibrium compatible.* Obviously, any enforceable contract is equilibrium compatible in all games where the contract allows for a pure strategy equilibrium.[27] But our results will show that in some games, the players choose in equilibrium not to violate contracts that include unenforceable prescriptions.

Definition 6 *A contract C is* equilibrium compatible in game G *if there is a profile $a \in C$ that is achieved by C.*

4 The Suboptimality of Enforceable Contracts

In order to introduce our results, we first note that if verifiability is product and partitional, and transfers are not restricted to damage compensation, then any achievable profile a can be achieved with the most restrictive enforceable contract, which coincides with $P(a)$.[28] The following analysis

[26] When an enforceable contract is also immune to joint verifiable deviations, inspired by the concept of strong equilibrium, we will define it as strongly enforceable. Formally, C is *strongly enforceable under joint liability* (resp. *individual liability,* and *damage compensation restrictions)* if $P(b) \cap C = \emptyset$ for any $b \notin C$, and respectively, if $\pi_i(P(b_J, c_{-J})) \cap \pi_i(C) = \emptyset$ for any $c \in C$, any $J \subseteq I$, any $(b_J, c_{-J}) \notin C$, and any $i \in J$, and if for any $b \notin C$, there is j such that $t_j(b) > 0$. When proving our result, we will show that, in fact, the distinction between enforceability and strong enforceability is irrelevant in our analysis.

[27] It is easy to extend our construction to allow for mixed equilibria. Once this is done, since mixed strategy equilibria exist in all games, it is easy to show that every enforceable contract is equilibrium compatible in every game.

[28] Whenever the players sign a contract $C \subseteq P(a)$, in fact, they behave as if they were playing the game G restricted to the product action set $P(a)$, because the court cannot distinguish the choices in $P(a)$ allowed by C from those ruled out by C. Thus if a is achievable with C it must also be achievable by signing the contract $P(a)$. At the same time, signing a contract C that allows profiles not included in $P(a)$ may only give the players a larger set of undeterred deviation from a. Thus it cannot be the case that a is achievable with C and not achievable with $P(a)$. See for instance Bernheim and Winston (1998), Definition 1 and Proposition 1.

shows that this result is not necessarily true when restricting transfers to damage compensation, or if one allows for non-product or for non-partitional correspondences.

4.1 Non-Product Verifiability and Individual Liability

Non-product verifiability structures arise when the court cannot verify the action taken by a player independently of the other players' choices. It may then be the case that a player violates a contract, and the court verifies that the contract is breached but cannot identify the violator. In multi-agency problems such as Hölmstrom (1982), for instance, the court can only verify whether the agents cooperated or not, but it cannot identify which agents shirked. The cooperative outcome may then be achieved only if the agents are subject to joint and several liability. If any of the agents violated the agreement, in fact, an external counterpart could take all the agents to trial, and the court would sanction them jointly. Under the individual liability rule, however, each agent is only accountable for her own actions, and cannot be fully penalized if the court cannot verify that she violated her contract. As a result it may not be possible to effectively deter underinvestment. Building on this intuition, we will show that in the following contract-theoretical setting, the first-best can be achieved in equilibrium only with a contract that explicitly prohibits the first-best.

Multi-Agency Problem with Monitoring. We begin the exposition by laying down a simple multi-agency problem. Each of two agents makes a private unverifiable investment that determines the quality of jointly produced output sold to a buyer, who may choose to buy (action B) or not (action N). For instance, each agent may be a producer of intermediate products necessary in the production process of the buyer; or each agent may be a professional, and the agents' services are complementary for the buyer. Joint output is verifiable in court, and it is high if and only if both agents' investment is high (action H). In the socially-optimal outcome, the two agents play H and the buyer buys the output. However each of the two agents has a private incentive to underinvest (action L, for low investment), and the court cannot distinguish private investments. The verifiability structure associated with the agents' choices is described in Figure 1.

If agents are only individually liable, it will not be possible to effectively deter underinvestment. Consider, in fact, the contract between agent 1 and the buyer, and suppose that agent 1 commits to play H, but then breaches the agreement and plays L. If taken to court, agent 1 can defend

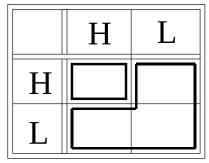

Figure 1: Simple Team Problem

herself by blaming agent 2 for the low output. Since investments cannot be identified, there is no way to show that agent 1 violated her contract: Formally, $H \in \pi_1(P(L, a_2, a_3))$, regardless of agent 2's action a_2, and of the buyer's choice a_3. Note that in this multi-agency problem, individual liability requires that the principal signs bilateral contracts with each agent separately, as in Prat and Rustichini (2003).

We now elaborate this simple model, and suppose that each agent can costly monitor the other agent's investment (action M), or choose to overlook her conduct (O). Besides productive effort, also the choice of monitoring is a private action, so that it cannot be directly verified. Despite this, if agent i monitors agent j, then she is able to gather enough hard evidence to verify that agent j has shirked, whenever this is the case. Specifically, we introduce a binary verifiable signal x_i for each agent i, and say that $x_i = l$ if and only if agent i plays L and agent j plays M, otherwise $x_i = 0$. This is appropriate, for instance, to represent the choice by agent j to gather all available evidence that i shirked and collect it in a report, without including any evidence that i worked hard. If j's report fails to show that i shirked, it can be either because i worked hard or because j did not spend enough effort in gathering evidence.[29]

The verifiability structure associated with the agents' choices is described in Figure 2, where, for instance, the upper-left box gathers all action profiles inducing high output, and the step-like set includes all action profiles where output is low, but neither of the agents can be blamed for this. The payoffs are as follows. Each agent pays a cost c when playing H, and suffers a loss l if playing M. If the output is of high quality, it is worth y to the buyer, otherwise it is worth 0. In either case, it is worth 0 to the agents. If the transaction is concluded, the buyer receives the output and pays

[29] We discuss later the alternative possibility that agents' monitoring efforts are directly verifiable.

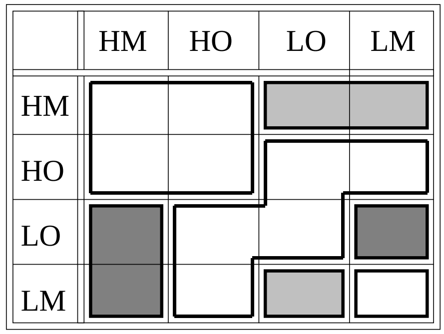

Figure 2: Team Problem with Monitoring

off p to each agent.[30] To make the problem meaningful, we assume that $p > c$, and that $y > 2p$.[31]

The first-best outcome is (HO, HO, B): the two agents invest at the cooperative levels, without wasting resources monitoring each other, and the counterpart buys the output. But as in the simpler multi-agency problem, the contract $\{(HO, HO, B)\}$, prescribing cooperative investments without monitoring, will fail to deter underinvestment. Again, if an agent breaches the agreement and underinvests, there is no way to blame her for low output. Formally, since $HO \in \pi_1 (P(LO, HO, B))$, if agent 1 violates her contract and plays LO, then the court cannot conclude that agent 1 has breached her commitment. Since it cannot be achieved by the contract $\{(HO, HO, B)\}$, the outcome (HO, HO, B) cannot be achieved with any equilibrium-compatible contract, or with any enforceable contract.

The players can achieve the first-best outcome (HO, HO, B) only by signing the contract $\{(HM, HM, B)\}$, that prescribes cooperative investment levels, and that requires the agents to monitor each other. In equilibrium, each agent will secretly violate the contract, and play HO, so as to cooperate without wasting her effort on monitoring. Knowing this, the buyer will play B, and the first-best will be achieved. The agents' contract violation cannot be verified in court: since output is high, no agent can be blamed for low output, and hence the absence of monitoring efforts cannot be verified. Formally, if both agents play HO, since $(HM, HM, B) \in P(HO, HO, B)$, it will be impossible to rule out in court that any agent played HM.

[30] As is often the case in these settings, we implicitly assume that the quality of final output is not contractible in the sale contract; possibly because of unforeseen contingencies.

[31] Besides making trade beneficial to all parties, this implies that the damage incurred by the buyer if buying low-quality output is larger than the joint gains of the agents for playing L. Hence this example is not affected when contractual transfers are restricted to damage compensation.

Moreover in equilibrium this "roundabout", apparently contradictory mechanism is effective in deterring underinvestment. Suppose that the buyer plays B, agent 2 plays HO, and agent 1 deviates from the candidate equilibrium to play LO. When taken to court by the buyer, agent 1 will not be able to discharge herself of the accusation of breaching the contract, because the court will conclude that the play belongs to the set $P(LO, HO, B)$. Since $HM \notin \pi_1 (P(LO, HO, B))$, it is impossible that agent 1 played HM. Intuitively, agent 1 will be unable to show in court that agent 2 has underinvested. Hence the court will conclude that either she did not gather enough evidence on agent 2's conduct, or that she is really the one who underinvested. In either case agent 1 is liable for breaching her contractual commitment which requires her both to make the high investment, and to monitor agent 2.[32]

We have shown a simple contract-theoretical setting where the first-best can be achieved only by signing a contract that explicitly prohibits the first-best. This prescription is not verifiable, and thus the optimal contract is not an enforceable contract. The optimal contract is not equilibrium compatible either: it will be breached in equilibrium so that the first-best is achieved.[33] $\diamond$

In the above multi-agency problem, the agents will not able to achieve the first-best by contractually committing to play H and to produce "complete" reports, that include all evidence that the other agent worked hard, as well as all the evidence that she shirked. In case that an agent does not monitor, in fact, the buyer will directly verify in court that she did not monitor, and will collect the fine. As a result, only a second-best outcome can be achieved: to avoid being penalized, in equilibrium, each agent fully complies with her contractual commitment of working hard and monitoring.[34] The first-best is restored only if transfers are restricted to damage compensation, because when the output is high, the buyer does not suffer any damage and cannot collect agents'

[32] One interesting feature of this equilibrium is that, although each agent is only individually liable, if one agent underinvest, then both agents are penalized. This occurs because the buyer can independently verify that *each of the agents* has breached contract, either because she has underinvested, or because she has not monitored the other agent. Under joint liability, it would be possible to punish both agents if the buyer could show that *at least one of them* has breached contract.

[33] Peer monitoring has been suggested as a possible means to alleviate the moral-hazard (see for instance Laffont and N'Guessan, 2000, and Che, 2002), but if agents monitor each other in equilibrium, the first-best cannot be achieved. Our analysis identifies a setting where there is no need for actual monitoring in equilibrium, so that the first-best is achieved.

[34] The same reasoning shows that the first-best cannot be achieved with any contract by which agents commit to a verifiable action incorrelated with output. The inclusion of monitoring in the contract does not only serve the role of allowing the buyer to collect fines even though effort is not verifiable, it is crucial that the contract allows collection if and only if output is low.

fines. Hence there is no need for any agent to monitor, as long as the other agent plays H, and the first-best is again achieved in equilibrium.

As well as multi-agency problems, also the hold-up models following Hart and Moore (1990) have a non-product verifiability structure, because they consider trading problems in which the court can verify whether trade takes place or not, but cannot verify which party refused to trade (see also Lewis and Sappington, 1991). We build on this feature to show a simple contract-theoretical setting where, under individual liability, the only contract that achieves the first-best (while not breached in equilibrium) necessarily contains prescriptions that cannot be verified, and hence is not an enforceable contracts.

Hold Up Problem and Willingness to Trade. In the game represented in Figure 3, player 1 is the producer of an intermediate good that can either be employed in player 1's or in player 2's final production. Player 1 may make an unobservable and unverifiable investment of cost c, that improves the good in a characteristic specific to player 2's final production. Specifically, the value of the good if utilized by player 1 is v, regardless of whether the cost of production is high (action H) or low (action L). Instead, if player 1 plays H, the value of the good for player 2 is $b > v$.

At the same time, player 1 decides whether to employ the good in her own production (action E), or to try and sell the good to player 2 for a price p (action S). Trade occurs if and only if both parties agree to trade: if player 2 agrees to buy (action B) and player 1 plays S. In order to make the problem meaningful, we say that $b - p > 0 > v - p$. If player 1 plays H, player 2 would like to buy the good, but when player 1 plays L, the buyer would rather not to trade (action N). If the deal is not reached, player 1 can still employ the good in her own production, but she will not be able to recapture the cost t of the delay and of the failed transaction with player 2. In order to make the problem meaningful, we say that $p - t - c > v$; and for simplicity, we say that player 2 does not bear any transaction cost.

While the court can verify whether the transaction has taken place or not, it cannot verify which party refused to trade. Moreover, it cannot verify player 1's investment, or ascertain the good's characteristics before it is sold. However, if the good has not been improved, it will verifiably reduce

	B	N
HS	p-t-c, b-p	v-t-c, 0
LS	p-t, v-p	v-t, 0
HE	v-c, 0	v-c, 0
LE	v, 0	v, 0

Figure 3: Trade of Intermediate Good

the value of player 2's final production. In that case, the court will conclude that the intermediate good had not been improved.

It is straightforward to show that the efficient outcome (HS, B) may be achieved by signing the contract $\{(HS, B)\}$, by which player 1 takes the commitment to make the investment and to sell to player 2, and by which player 2 makes the commitment to buy the intermediate good from player 1. In fact, when player 1 is playing H, so as to improve the characteristics of the good, it is in the best interest of player 2 to play B, and buy the intermediate good from her. Given this, it is in the best interest of player 1 to play S, and try to sell the good to player 2. Moreover, whenever trade takes place, it is the best interest of player 1 to play H, as she would be found liable for breach of the contract $\{(HS, B)\}$ if she played L.

The contract $\{(HS, B)\}$ however cannot be considered enforceable under individual liability. This contract in fact, prescribes that the two parties should trade, but if any of them unilaterally decides not to trade, the court cannot identify who has refused to trade. As a result, player 1's contractual commitment to play T is not verifiable, nor is player 2's commitment to play B. In fact, it is easy to verify that, since $\pi_1(P(LE, B)) = A_1$ and $\pi_1(P(HS, N)) = A_2$, any enforceable contract that allows for the profile (HS, B) does not restrict the players' choice at all, and thus can only achieve the Nash equilibrium (LE, N). The good will not be improved, and the two players will not trade. $\diamond$

4.2 Damage Compensation

We now show that damage-compensation restrictions disrupt the optimality of enforceable contracts even in very simple contract-theoretical settings: notably, the next one does not hinge on the

	P	O
IH	y-k-c, w+v	-k-c, r
IL	y-k, w	-k, r
H	y-c, v	-c, r
L	y, 0	0, r

Figure 4: Partially Verifiable Hold-Up Problem

information structure being non-product or non-partitional.

Partially Verifiable Hold Up Problem. Consider the game represented in Figure 4. Player 1's productive effort or investment is non-contractible and may be either high (H), at cost c, or low (L). High effort has value v for player 2. Before the choice between H and L, or contextually with this choice, player 1 may also make a *verifiable* investment (I), of cost k to herself, and value w to player 2. For instance, the investment I may consist in a technology upgrade, in a capacity increment, or in the adoption of a simpler (hence contractible) technology than the one identified by H.

Player 2 may either participate in the relation (P), or opt out (O) and collect the reservation value $r < w$. The value of player 2's participation to player 1 is $y > k$; and to make the problem meaningful, we set $w > k + c$.[35] We assume that $v \geq w$ and that $c \geq k$: the non-contractible effort/investment is at least as valuable to player 2, and at least as costly to player 1, as is the verifiable up-front investment. Whether player 2's action is contractible or not is immaterial for our results. In either cases, the verifiability structure is partitional and product, with $P_1 = \{\{IH, IL\}, \{H, L\}\}$ and $P_2 = \{\{P\}, \{O\}\}$, or $P_2 = \{P, O\}$, respectively.

It is immediate to see that the only equilibrium without any contracts is the profile (L, O): player 1 does not invest and player 2 opts out. The parties would like to Pareto improve upon this outcome. They cannot achieve the outcome (H, P) by means of the contract $\{(H, P)\}$ because player 1 would deviate and play L, and this deviation would be undetected. Most importantly,

[35]This description may be appropriate to represent several hold up problems: player 2 may be a buyer undecided on whether to buy from player 1 or not; or a producer who must choose whether to make a relation specific investment or not. The key feature of these one-sided prisoner's dilemmas is that while player 1 always prefers to defect, player 2 prefers to defect if and only if player 1 defects.

if transfers are restricted to damage compensation, the players cannot even achieve the outcome (IL, P), in which player 1 makes the verifiable investment and player 2 commits to participate, with any contract C allowing for (IL, P). If player 1 violates C and plays L, player 2 cannot show that she suffered a damage, because it cannot be excluded that player 1 played H, and hence that player 2's payoff is $v > w$. Formally, it is the case that $\bar{t}_2(L, P) = 0$ under any contract C such that $(IL, P) \in C$. Hence the outcome (IL, P) cannot be achieved with any enforceable or equilibrium compatible contract.

On the other hand, the outcome (IL, P) may be achieved by means of the contract $\{(IH, P)\}$, by which player 1 commits both to make the contractible investment and to exert non-verifiable effort. This contract stipulates that player 2's payoff should be $w + v$. If player 2 breaches the contract and plays H or L, the court verifies that player 2 has been harmed, because it concludes that 2's payoff is at most v, and allows a transfer $t_2(H, P) = (w + v) - v = w$; this transfer is sufficient to deter player 1 from shirking the verifiable investment. At the same time, since the court cannot distinguish between IH and IL, if player 1 violates the contract $\{(IH, P)\}$ and play IL, her violation is undected and hence $\bar{t}_2(IL, P) = 0$. Formally, this concludes that (IL, P) is an equilibrium when the contract $\{(IH, P)\}$ is stipulated with transfers $\mathbf{t} = \bar{\mathbf{t}}$, because,

$$u_1(IL, P) + t_1(IL, P) = y - k > y - k - c = u_1(IH, P)$$
$$u_1(IL, P) + t_1(IL, P) = y - k > y - w = u_1(L, P) + t_1(L, P) > y - c - w = u_1(H, P) + t_1(H, P),$$

and clearly player 2 plays O if player 1 plays IL. ◇

4.3 Non-Partitional Verifiability: Costly Verification

While they have not been formally represented as verifiability correspondences, non-partitional structures have not been uncommon in the contract theory literature. In Example 2 of Okuno-Fujiwara, Postlewaite and Suzumura (1990), two oligopolists may or may not make investments that reduce marginal costs. A firm i may verify in court that its marginal cost is low (we denote this action by L) by running its production line very fast: in our notation, $P_i(L) = \{L\}$. Running the line slowly however does not demonstrate that it cannot run faster, and thus that the costs of production are high (action H). As a result, if costs are high, the court will not be able to draw any conclusion, so that $P_i(H) = \{L, H\}$. This structure is clearly non-partitional.[36] In Tirole (1986, 1992), a verifiable signal is observed with positive probability if the cost of production is low, but nothing is observed if the cost is high. Whenever players are unlimitedly liable, this yields the same incentives as an instance where it can certainly be verified that the cost is low, $P_i(L) = \{L\}$, but it can never be verified that the cost is high, $P_i(H) = \{L, H\}$. The structure is non-partitional because of the stochastic signal structure from which it is derived.

Among the institutional motivations for the study of non-partitional correspondences, we point out that not all evidence may be admissible in court. For instance, whereas a defendant's testimony discharging herself from an accusation is admissible evidence in court to support the defendant's innocence, the absence of a discharging testimony is not admissible as evidence that the defendant is guilty. It is immediate to see that this yields the structure $P(I) = \{I\}$, $P(G) = \{I, G\}$, where I stands for innocent and G stands for guilty. Most importantly, Bull (2001) underlines a key technological motivation for the study of non-partitional verifiability structures. He shows that this feature may arise because verification is a costly activity, both when undertaken by contractual parties, and when accomplished by the judicial system. We now show a simple contract-theoretical setting where equilibrium-compatible and enforceable contracts are suboptimal because the verifiability correspondence is non-partitional, in the presence of costly verifiability.

Verifiability with Costly Precision. Consider a basic "tragedy of the commons" problem of private contribution to the common good. Since these games are well understood, we will not describe the problem in details: It should suffice to set up the notation x_i to denote each player i's chosen contribution, let the unique Nash equilibrium be $\hat{x}$, and the first-best outcome be $x^* > \hat{x}$. Suppose that outcome verification and contract enforcement are costly. When player i's choice is x_i, the judicial system may only legally determine that her action belongs to the interval $[x_i - M(c), x_i + M(c)]$ at cost c, but will not be able to determine which action in the set has been played. The function M is a margin of error that can only be narrowed if the judicial system increases expenditure in verification and enforcement. It is reasonable to suppose

[36] This paper takes the correspondence P as primitive instead of deriving it as the outcome of an explicitly modeled trial game. In principle, we could expand the game G endowing the players with the additional action of producing or conceiling evidence. The information correspondence defined on this expanded action set will be partitional whenever the *only* source of non-partitional information is the private control of determinist evidence costlessly produced: see Bull and Watson (2002b).

that M is decreasing, with $\lim_{c\to\infty} M(c) = 0^{+}$. As a result, the court will choose an optimal level of enforcement c^{*} such that $M(c^{*}) > 0$, and it will not be able to precisely pin down agent i's action. The induced verifiability correspondence is product but non-partitional because, if two distinct values x_i and x_i' are less than $M(c^{*})$ apart, the sets $[x_i - M(c^{*}), x_i + M(c^{*})]$ and $[x_i' - M(c^{*}), x_i' + M(c^{*})]$ overlap but do not coincide.[37]

Any contract C such that $x^{*} \in C$ will not achieve the social optimum, because it fails to deter each player i from choosing any x_i such that $x_i^{*} - M(c^{*}) < x_i < x_i^{*}$: since $x_i^{*} \in P_i(x_i)$, it cannot be concluded in court that i violated the agreement. However, it is immediate to see that the social optimum x^{*} will be achieved, if each player i contractually commits to contribute at least $x_i^{*} + M(c^{*})$, for instance by stipulating the contract $C_i = \{(x_i^{*} + M(c^{*}))\}$. The smallest contribution that player i can make without verifiably breaching such an agreement is x_i^{*}. The first-best outcome is achieved only with contracts that explicitly prohibit the first-best. $\diamond$

5 The Complete Characterization

We begin our characterization by pointing out that, in this environment, any outcome a achievable by means of an enforceable contract C, can also be achieved by means of any enforceable contract C' that allows for a and that is more restrictive than C. The contract C' in fact, cannot induce a set of undeterred deviations from a larger than C. In this sense, there is no scope for contract incompleteness, because any outcome a achievable by means of an enforceable contract, can also be achieved by an enforceable contract that does not leave any verifiable aspect of the interaction unspecified.[38] This class of contracts is derived by assigning to each action profile a the most restrictive enforceable contracts that allows the players to play a. The next Lemma shows that for any profile a, there exists only one such contract, both in the case of joint and of individual liability. The proof is constructive.

Lemma 1 *For any verifiability correspondence P and any action profile $a \in A$, there exists a unique* minimal *(in terms of set inclusion) contract $C(a, J)$, respectively $C(a, I)$,* enforceable *under*

[37] This framework is apt to represent, for instance, tax evasion, traffic violations, and in general, problems of private contribution to the common good, where precise verification and enforcement of the social contract is costly for the judicial system. The literature on (static) voluntary contribution games includes, for instance, Bernheim (1986).

[38] These contracts are studied for instance by Bernheim and Winston (1998), who define them as "complete."

joint (respectively individual) liability, such that $a \in C(a, J)$, *and* $a \in C(a, I)$.

We now show that both in the environments of joint liability, and of product verifiability, the axiom of positive introspection *(Know-that-You-Know)* is sufficient to insure that the players cannot improve upon the most restrictive enforceable contracts, and hence upon enforceable and equilibrium-compatible contracts. As is well known (see Geanakoplos 1989), the axiom of positive introspection is expressed by requiring that the information structure P is *transitive:* i.e. that $a \in P(b)$ and $b \in P(c)$ implies $a \in P(c)$, for any a, b, and c.

Theorem 1 *Take any game* G, *and any* transitive *verifiability correspondence* P. *If* $a \in A^J(G, P)$, *then* a *can be achieved with contract* $C(a, J)$. *If* P *is also* product, *then any* $a \in A^I(G, P)$ *can be achieved with* $C(a, I)$.

Beyond the product case, under individual liability, it can be established that equilibrium-compatible contracts are optimal if the court's knowledge of each agent i's action a_i (given any possible opponents' profile a_{-i}) is a transitive operator. Formally, for each a_{-i}, we introduce the correspondence $Q_{a_{-i}} : A_i \to 2^{A_i}$ such that for any b_i, $Q_{a_{-i}}(b_i) = \pi_i (P(b_i, a_{-i}))$, and say that the collection $Q = \left(Q_{a_{-i}}\right)_{A_{-i}, I}$ is transitive if $Q_{a_{-i}}$ is transitive for any i and a_{-i}. Informally, the court's knowledge of each player's choice needs to satisfy positive introspection, when conditioning on every possible choice of the other players. The multi-agency problem with monitoring studied in the previous section shows that this assumption is much stronger than requiring transitivity of P and that it is easily violated in settings of interest: while P is partitional (and hence transitive) in that problem, the correspondence Q_{HO} is intransitive.

Proposition 2 *For any game* G *and verifiability correspondence* P, *if* $a \in A^I(G, P)$ *and* Q *is* transitive, *then* a *can be achieved with the* (minimal equilibrium-compatible) *contract* $C = \{a\}$.

Conversely to the above results, Proposition 3 determines conditions on the verifiability correspondence P, such that one can construct generic games in which all equilibrium-compatible and all enforceable contracts are suboptimal. Essentially, we find that the intransitivity of P is sufficient for the failure of equilibrium-compatible contracts in general, and that the intransitivity of Q is sufficient under individual liability. For any game G, we say that an action profile a is *desirable* if it is Pareto-undominated and if it is individually-rational, i.e. it weakly Pareto dominates all Nash equilibria of game G.

Proposition 3 *If $c \in P(a)$, $a \in P(b_i, a_{-i})$ and $c \notin P(b_i, a_{-i})$ for some a, c, i, b_i, and respectively if Q is intransitive, then there is an open set of utility vectors $u \in \mathbb{R}^{I \times A}$, such that each $G = (I, A, u)$ has a desirable $a \in A^J(G, P)$, respectively $a \in A^I(G, P)$, that cannot be achieved with any equilibrium-compatible contract. When P is product, both conditions are equivalent to the intransitivity of P.*

Finally, Corollary 4 shows that for any game where the action space is rich enough, one can construct intransitive verifiability structures for which all incentive-compatible and all enforceable contracts are suboptimal. This result shows that incentive-compatible contracts may be suboptimal in a very unrestricted class of games, and underlines that the source of this suboptimality does not reside in the specifics of the game played, but rather in the intransitivity of the verifiability structure.[39]

Proposition 4 *Fix any game $G = (I, A, u)$ with a desirable profile a such that at least one player i has actions b_i and d_i satisfying $u_i(b_i, a_{-i}) < u_i(a) < u_i(d_i, a_{-i})$. For some intransitive verifiability structure P, the outcome a is achievable only with contracts that are not equilibrium compatible. Under individual liability, this is also the case for some intransitive collection Q.*

The above characterization determines the key role of positive introspection. In order to assess how likely one is to encounter intransitive verifiability structures, we introduce a simple model of the court's decision process. While we allow for stochastic evidence, the key restricting assumption is that *all evidence realized is costlessly brought to trial.* We will show that this induces transitive verifiability structures. But when evidence production is a costly activity, as in the tragedy of the commons problem of the previous section, verifying an event may be less costly than verifying that one event can be verified, because this introduces one additional layer of costly verification.

Without loss of generality, the evidence can be represented as a random signal in the finite set X. When an action profile b is played, signal x is brought to court with probability $\Pr(x|b) \in [0, 1]$.

[39]Hence the failure of enforceable contracts in this environment is more general than the failure of complete contracts in Bernheim and Winston (1998), which depends on whether the game is of strategic substitutes or complements.

If the signal x is brought to court and $\Pr(x|a) = 0$, the court concludes that it is impossible that a was played.[40] Suppose that the players contractually committed to play a. When playing action b they anticipate that, with positive probability, the signal x will be brought to court and that they will be penalized.[41] When the punishment is very harsh, the players behave as if they were sure that when taking action b the court would be able to rule out action a. In other terms, they behave as if facing a court with verifiability structure P such that $a \notin P(b)$.

Definition 7 *The* evidence model $(A,\ X,\ \{\Pr(x|a)|a \in A, x \in X\})$ *induces the verifiability structure P if for any $a, b \in A$,*

$$a \notin P(b) \text{ if and only if } \Pr(x|b) > 0 \text{ and } \Pr(x|a) = 0 \text{ for any } x \in X.$$

We now show that any verifiability structure derived from an evidence model is transitive, but not necessarily partitional.

Lemma 2 *If the correspondence P is derived from an evidence model, then P is transitive, but not necessarily partitional.*

We conclude the analysis by characterizing under which conditions enforceable and equilibrium-compatible contracts restricted by damage compensation are undominated. Consider first the case of at least 3 players and of product verifiability: Proposition 5 below shows that all equilibrium-compatible contracts are suboptimal in generic games, as long as P is non-trivial (i.e. it is neither the case that nothing can be verified, nor that everything can be verified). When P is non-product (but partitional), the same implication obtains if there are two profiles a and (d_i, a_{-i}) that induce different non-singleton information sets; an even weaker condition applies for the non-partitional case. For the case of 2 players, the failure of equilibrium-compatible and enforceable contracts is only slightly less general.[42]

[40] For the sake of simplicity, this model assumes that the court holds a full-support prior and that it concludes that an event has occurred only if it assesses that probability equal to 1. We could also require the court to state that an event has occurred when it assesses that probability larger than p, for some given p.

[41] In order to separate the issue of intrasitive verifiability from liability restrictions, we assume that the players are jointly liable. It is immediate to modify the construction to account for individual liability.

[42] Transitivity of the correspondence P is imposed in Proposition 5 to insure that the failure of equilibrium-compatible contracts is due to the damage-compensation restriction only.

Proposition 5 *Say that $I \geq 3$ and the correspondence P is transitive. If $b \in P(a) \setminus P(d)$, and $c \in P(d)$ for some distinct profiles a, b, c and d such that $a_{-i} = d_{-i}$ for some i, then there is an open set of utility vectors $u \in \mathbb{R}^{I \times A}$, such that each game $G = (I, A, u)$ has a desirable profile $a \in A^D(G, P)$ that cannot be achieved with any equilibrium-compatible contract. When $I = 2$, the only additional requirement is that $b_i \neq a_i$.*

6 Conclusion

In this paper, we have identified simple contract-theoretical settings where the only manner for the players of a game to achieve the first-best is to stipulate a contract that explicitly prohibits the first-best, and secretly agree to breach the agreement in equilibrium. Enforceable contracts are fully efficient if and only if verification is costless (so that the court's knowledge satisfies the axiom of positive introspection) and the parties are not subject to liability limitations. Our results are of interest for the foundations of contract incompleteness, because they contradict the natural supposition that it would be a waste of resources to stipulate unverifiable prescriptions in a contract.

The general message we learn from our analysis is that 'hard' game-theoretical foundations are required in order to appropriately incorporate technological and institutional features in contract-theoretical analysis. A definition of contracts as functions of signals that are verifiable in court may be too restrictive, as it is equivalent to assuming that the players may only sign contracts that do not include unverifiable prescriptions. Moreover, the issue of contract enforcement cannot be settled by looking at verifiability constraints only: we introduce a general construction where liability constraints, as well as verifiability constraints, transform stipulated contractual transfers into actually enforced transfers.

Appendix A: Omitted Proofs

Proof of Lemma 1. Let the contract $C(a, J)$ be the limit of the sequence $\{C_n\}_{n \geq 0}$ constructed as follows. Let $C^0 = \{a\}$ and $C^n = f^J(C^{n-1})$ for any $n \geq 1$, where the relation $f^J : 2^A \to 2^A$ is defined by the rule:

$$f^J(C) = \cup_{i=1}^{I} \{(b_i, a_{-i}) | P(b_i, a_{-i}) \cap C \neq \emptyset, \text{ for some } a \in C\}, \text{ for every } C \subseteq A. \tag{1}$$

The limit $C(a, J)$ exists because the relation f^J is non-decreasing and 2^A is finite.

Suppose that C is an arbitrary enforceable contract and that $a \in C$. For any player i, any profile (b_i, a_{-i}) such that $a \in P(b_i, a_{-i})$ must be included in C, by the definition of enforceable contracts. Once (b_i, a_{-i}) is included in C, also any c such that $(b_i, a_{-i}) \in P(c)$ must be included, and so on. Therefore, it must be the case that $C(a, J) \subseteq C$.

The unique minimal contract $C(a, I)$, enforceable under individual liability, and for which $a \in C(a, I)$, is the limit of the sequence $\{C^n\}_{n \geq 0}$ such that $C^0 = \{a\}$ and $C^n = f^I(C^{n-1})$ for any $n \geq 1$, where the relation $f^I : 2^A \to 2^A$ is defined by the rule:

$$f^I(C) = \times_{i=1}^I \{b_i | \pi_i(P(b_i, a_{-i})) \cap C_i \neq \emptyset, \text{ for some } a_{-i} \in C_{-i}\}, \text{ for every } C \subseteq A. \tag{2}$$

■

Proof of Theorem 1. Fix any contract C, a is a Nash equilibrium of game $G|C$ such that $u(a|C) = u(a)$, i.e. a is achieved by C, if and only if $P(a) \cap C \neq \emptyset$ and $P(d_i, a_{-i}) \cap C = \emptyset$ for any i and $d_i : u_i(d_i, a_{-i}) > u_i(a)$. Hence, picking any $c \in P(a) \cap C$, we obtain that if a is achieved by C, then a is achieved also by $\{c\}$.

Say that P is transitive and contracts are subject to joint liability. We want to show that any achievable profile a can also be achieved with both the minimal enforceable contract $C(a, J)$, and with the minimal strongly-enforceable contract, which we denote by $C^S(a, J)$, and that is uniquely derived as the limit of $\{C^n\}_{n \geq 0}$ recursively defined with $C^n = f^S(C^{n-1})$ for any $n \geq 1$, with $C^0 = \{a\}$, and with $f^S : 2^A \to 2^A$ such that $f^S(C) = \{b | P(b) \cap C \neq \emptyset\}$. Note, immediately, that for any C, $f(C) \subseteq f^S(C)$, and thus $C(a, J) \subseteq C^S(a, J)$.

Take any pair of profiles b and q different from a. By transitivity, if $a \in P(b)$ and $a \notin P(q)$, then $b \notin P(q)$. By definition, $f^S(\{a\}) = \{b | a \in P(b)\}$. Thus if $q \notin f^S(\{a\})$ then $q \notin f^S(f^S(\{a\}))$, and so $C^S(a, J) = f^S(\{a\})$.

Observe that $a \in P(a) \cap C^S(a, J)$, so $u_i(a|C^S(a, J), J) = u_i(a)$. Thus, if $u_i(a) \geq u_i(d_i, a_{-i})$, then player i will not play the deviation d_i from a, when $C^S(a, J)$ is signed. So we only need to consider pairs i and d_i, such that $u_i(d_i, a_{-i}) > u_i(a)$ and $c \notin P(d_i, a_{-i})$. But since $c \in P(a)$, transitivity implies $a \notin P(d_i, a_{-i})$. One further application of transitivity implies that for any b such that $a \in P(b)$, $b \notin P(d_i, a_{-i})$. Since $C^S(a, J) = f^S(\{a\}) = \{b | a \in P(b)\}$, we have obtained that $P(d_i, a_{-i}) \cap C^S(a, J) = \emptyset$. Hence a is a Nash Equilibrium of the game $G|C^S(a, J), J$ and $u(a|C^S(a, J), J) = u(a)$.

To show that a is also a Nash Equilibrium of the game $G|C(a, J), J$ and $u(a|C(a, J), J) = u(a)$, since $a \in C(a, J)$ and $a \in C^S(a, J)$, we just need to show that, for any i and $d_i : u_i(d_i, a_{-i}) > u_i(a)$, if $P(d_i, a_{-i}) \cap C^S(a, J) = \emptyset$ then $P(d_i, a_{-i}) \cap C(a, J) = \emptyset$. This immediately follows from the relation $C(a, J) \subseteq C^S(a, J)$.

If P is product and transitive, then P_i is transitive for any i and $\pi_i(P(b_i, a_{-i})) = P_i(b_i)$ regardless of a_{-i}. If a can be achieved by a contract $\{c\}$, applying transitivity of P_i in the same way as the transitivity of P is applied above, one shows that $C(a,I)_i = f_i^I(\{a\})$ for any i and that $P_i(d_i) \cap C(a,I)_i = \emptyset$ for any i and d_i, such that $u_i(d_i, a_{-i}) > u_i(a)$. Since $a_i \in C(a,I)_i$ for every i, it follows that $C(a,I)$ achieves a. ■

Proof of Proposition 2. If the profile a is achieved by some contract C, then there is c such that for any i, $c_i \in \pi_i(P(a_i, a_{-i}))$ but $c_i \notin \pi_i(P(d_i, a_{-i}))$ for any d_i with $u_i(d_i, a_{-i}) > u_i(a)$. By transitivity of $\pi_i(P(\cdot, a_{-i}))$ it must be that $a_i \notin \pi_i(P(d_i, a_{-i}))$, and hence that $\pi_i(P(d_i, a_{-i})) \cap \{a_i\} = \emptyset$. Since $\pi_i(P(a)) \cap \{a_i\} = \{a_i\} \neq \emptyset$, it follows that a is a Nash equilibrium of $G|\{a\}$, hence it is the minimal equilibrium-compatible contract that achieves a. ■

Proof of Proposition 3. Suppose, first, that there is a profile a such that $c \in P(a)$ and for some i and b_i, $a \in P(b_i, a_{-i})$, but $c \notin P(b_i, a_{-i})$ for some c.

We construct u as follows. We say that $u_i(q_i, a_{-i}) < u_i(a) < u_i(b_i, a_{-i})$ for any action $q_i \notin \{a_i, b_i\}$ and that, for any $j \neq i$ and any $q_j \neq a_j$, $u_j(a) > u_j(q_j, a_{-j})$. So as to have a Pareto-dominate all Nash Equilibria of G, we select a profile e such that for at least one $j \neq i$, $e_j \neq a_j$, and require that, for any i, $u_i(a) > u_i(e) > u_i(q_i, e_{-i})$ for all $q_i \neq e_i$, and that for any $q \neq e$, there is a player j and an action $q_j' \neq q_j$ such that $u_j(q_j', q_{-j}) > u_j(q)$. In particular we are saying that there is a player $j \neq i$ and an action $q_j' \neq a_j$ such that $u_j(q_j', b_i, a_{-ij}) > u_j(a_j, b_i, a_{-ij})$. To make a Pareto undominated, we further extend the requirements on u, by assuming that, for each $q \neq a$ there is a player j such that $u_j(q) < u_j(a)$. In particular, there must be a player $j \neq i$ such that $u_j(b_i, a_{-i}) < u_j(a)$. Given our payoff construction, the profile a is desirable.

Since $a \in P(b_i, a_{-i})$ and $u_i(a) < u_i(b_i, a_{-i})$, the profile a cannot be achieved under joint liability by $\{a\}$. Consider contract $\{c\}$: by assumption, $u_i(a|\{c\}, J) = u_i(a) > u_i(q_i, a_{-i}) \geq u_i(q_i, a_{-i}|\{c\}, J)$, for any $q_i \notin \{a_i, b_i\}$. At the same time, for any player $j \neq i$, $u_j(q|\{c\}, J) = u_j(q)$, and by assumption, for any $q_j \neq a_j$, $u_j(a) > u_j(q_j, a_{-j})$. Therefore, a is a Nash Equilibrium of $G|\{c\}$ and $u_i(a|\{c\}, J) = u_i(a)$ for all i.

Second, suppose that $a_i \in \pi_i(P(b_i, a_{-i}))$ and $c_i \in \pi_i(P(a_i, a_{-i}))$, but $c_i \notin \pi_i(P(b_i, a_{-i}))$. With the same payoff construction, we obtain that a is desirable, it can be achieved under individual liability by $\{c\}$ but not by $\{a\}$. ■

Proof of Proposition 4. For the first claim, construct P such that for some $c \in P(a)$ and for some i and b_i, $a \in P(b_i, a_{-i})$, but $c \notin P(b_i, a_{-i})$. For the second claim, construct $\pi_i(P(\cdot, a_{-i}))$ such that $a_i \in \pi_i(P(b_i, a_{-i}))$ and $c_i \in \pi_i(P(a_i, a_{-i}))$, but $c_i \notin \pi_i(P(b_i, a_{-i}))$. The remainder of the proof is identical to the last part of the proof of Proposition 3, and is thus omitted. ■

Proof of Lemma 2. We will show that if $c \in P(a)$ and $c \notin P(b)$ then $a \notin P(b)$. Since $c \notin P(b)$, there must exist an $x \in X$ such that $\Pr(x|b) > 0$ and $\Pr(x|c) = 0$. As $c \in P(a)$, it follows that $\forall y \in X$, $\Pr(y|a) = 0$ or $\Pr(y|c) > 0$ or both: in particular, $\Pr(x|c) = 0$ implies that $\Pr(x|a) = 0$. So we have concluded that $\Pr(x|b) > 0$ and $\Pr(x|a) = 0$, i.e. $a \notin P(b)$.

For an example where P is not partitional, set $A = \{c, d\}$, and $X = \{x, y\}$, with $\Pr(y|c) = 0$, $\Pr(x|d) > 0$: this induces the verifiability structure $P(c) = \{c, d\}$, $P(d) = \{d\}$. ■

Proof of Proposition 5. Suppose that there are distinct profiles a, b, c and d such that $a_{-i} = d_{-i}$ for some i, and $b \in P(a) \backslash P(d)$, $c \in P(d)$. Construct u so that $\{u_i(b), u_i(c)\} < u_i(a) < u_i(d)$, and $u_j(c) \geq u_j(a) > u_j(d)$ for any $j \neq i$. Set $u_k(b) > [u_i(d) - u_i(a)]/(I-2) + u_k(c)$ for any $k \neq i$ such that $b_k = a_k$, and let $u_j(b) < u_j(a)$ for any other j (including i). For any other z, and any k, let $\min\{u_k(a), u_k(b), u_k(c), u_k(d)\} > u_k(z)$.

Given this construction, a is desirable. Furthermore, any contract such that $a \in C$ fails to achieve a. For any $j \neq i$, since $u_j(c) \geq u_j(a)$ and $c \in P(d)$, it must be the case that $\bar{t}_j(d) = 0$; and hence i's deviation from a to d cannot be deterred.

We will nevertheless show that contract $C = \{b\}$ achieves a. If any player $j \neq i$ plays a_j, in fact, i's deviation d_i is deterred because $\bar{t}_k(d) > [u_i(d) - u_i(a)]/(I-2)$ for any $k \neq i$ such that $b_k = a_k$ and hence $\sum_{j \neq i} \bar{t}_j(d) > u_i(d) - u_i(a)$. Since $u_i(a) > u_i(z)$ for any other z, this shows that i will play a_i. Since the profiles c and d cannot obtain unless i plays d_i, and $u_j(a) > u_j(z)$ for any other z, and $u_j(b) < u_j(a)$ for the one j such that $b_j \neq a_j$, we conclude that also any player $j \neq i$ chooses to play a_j.

When $I = 2$, the only additional requirement is that $b_i \neq a_i$, if not there cannot be any $k \neq i$ such that $b_k = a_k$.

Finally, note that if P is product, and there are no profiles a, b, c and d such that $a_{-i} = d_{-i}$ for some i, and $b \in P(a) \backslash P(d)$, $c \in P(d)$, then it must be the case that P is trivial.

Suppose that P is non-trivial. Then there is $d : P(d) \neq \{d\}$ and $P \neq A$. I.e. there is $w \neq a$ such that $w \in P(a)$ and $z \neq a$ such that $z \notin P(a)$. Since P is product, $w_k \in P_k(d_k)$ for any k, whereas there is i such that $z_i \notin P_i(d_i)$: let a be such that $a_{-i} = d_{-i}$ and $a_i = z_i$, let c be such that $c_{-j} = d_{-j}$ and $c_j = w_j$ for one $j \neq i$, and finally, let b be such that $b_{-j} = a_{-j}$ and $b_j = c_j$. It follows that $c_k \in P_k(d_k)$ and $b_k \in P_k(a_k)$ for all k, and that $b_i = a_i \notin P_i(d_i)$. ■

Appendix B: General Contracts

This Appendix extends our construction to general transfer schemes, and introduces the budget-balancedness constraint. Maintaining the restriction to games of complete information, for simplicity, a contract is a transfer scheme $t : A \to \mathbb{R}^I$, such that $\sum_{i \in I} t_i(a) \leq 0$ for any a, where strict inequalities are possible due to the presence of the external collector of fines. If $\sum_{i \in I} t_i(a) = 0$ for all a, then the contract t is called *budget-balanced.* Let $t^{-1} : \mathbb{R}^I \to 2^A$ be the correspondence such that $a \in t^{-1}(\mathbf{t})$ if and only if $t(a) = \mathbf{t}$. Were the contract t enforced, the players utility when taking their actions would be $u + t$, and we introduce the notation $G|t = (I, A, u + t)$.

When P is partitional (and liability is unrestricted), the standard definition of enforceable transfers is that t is measurable with respect to P. This is extended to the present environment where P is possibly non-partitional by stipulating that the court will not enforce any transfer $\mathbf{t}$ when a is played unless $P(a) \subseteq t^{-1}(\mathbf{t})$, i.e. unless the court concludes that the transfer $\mathbf{t}$ should be enforced. Formally, we introduce the *verification* operator $v : \mathbb{R}^{I\times A} \to \mathbb{R}^{I\times A}$, that maps stipulated transfer schemes into actually enforced transfer schemes, such that for any $a \in A$, it is the case that $v(t)(a) = t(a)$ if $P(a) \subseteq t^{-1}(\mathbf{t})$ and $v(t)(a) = 0$ otherwise. When liability is unrestricted, an action profile a is *achievable* with a contract t, if it is a Nash equilibrium of the game $G|v \circ t = (I, A, u + v(t))$. A contract t is *enforceable* if all its non-trivial transfers can be enforced by a court, i.e. $v \circ t = t$. A contract t is *equilibrium compatible* if its transfers are enforced in equilibrium, i.e. there is a Nash equilibrium a of the game $G|v \circ t$, such that $t(a) = v(t)(a)$.

In order to account for individual liability, it is convenient to describe a contract as a family of transfers $\left(t^i\right)_{i\in I}$ such that $t^i : A_i \to \mathbb{R}^I$ for any i, with $t^i_j(a_i) \geq 0$ for any $j \neq i$ and $\sum_{j\in I} t^i_j(a_i) \leq 0$ for any a. Intuitively, each player i takes the personal commitment to make the transfers $t^i(a_i)$ if she plays action a_i. If $\sum_{j\in I} t^i_j(a) = 0$ for all a, and i, then $\left(t^i\right)_{i\in I}$ is budget-balanced. For any i, let $t_i^{-1} : \mathbb{R}^I \to 2^{A_i}$ be the correspondence such that $a_i \in t_i^{-1}(\mathbf{t})$ if and only if $t^i(a_i) = \mathbf{t}$. The court will not enforce any transfer $\mathbf{t}$ when the profile a is played unless $\pi_i(P(a)) \subseteq t_i^{-1}(\mathbf{t})$, i.e. the court concludes that the transfer $\mathbf{t}$ should be enforced. We introduce the *individual liability* map $l : \mathbb{R}^{I\times I\times A} \to \mathbb{R}^{I\times I\times A}$ such that for any a, and transfer family $\left(t^i\right)_{i\in I}$, it is the case that $l^i\left(t^i\right)(a) = t^i(a)$ if $\pi_i(P(a)) \subseteq t_i^{-1}(\mathbf{t})$, and $l^i\left(t^i\right)(a) = 0$ otherwise. An action profile a is *achievable under individual liability* with a contract $\left(t^i\right)_{i\in I}$, if it is a Nash equilibrium of the game $G|\left(l^i \circ t^i\right)_{i\in I} = \left(I, A, u + \sum_i l^i\left(t^i\right)\right)$; a contract $\left(t^i\right)_{i\in I}$ is *enforceable* if $l^i \circ t^i = t^i$ for all i, whereas it is *equilibrium compatible* if it achieves an outcome a such that $l^i\left(t^i\right)(a) = t^i(a)$.[43]

When transfers are limited to damage compensation, the court will rule out net contractual transfers that exceed verified damage. Formally, we introduce the *damage compensation* map $d : \mathbb{R}^{I\times I\times A} \to \mathbb{R}^{I\times I\times A}$ such that given the budget-balanced transfers t, $d(t)(a) = t(a)$ only if $t^i(a) - t^i(c) \leq \max\{0, u_i(c) - \max_{b\in P(a)} u_i(b)\}$ for any i, c; and $d(t)(a) = 0$ otherwise. An action profile a is *achievable* with a contract t, if it is a Nash equilibrium of the game $G|v \circ d \circ t = (I, A, u + v(d(t)))$; a contract t is *enforceable* in game G if $v \circ d \circ t = t$, and *equilibrium compatible* if it achieves a profile a such that $v(d(t))(a) = t(a)$.

It is easy to check that all our negative results are still valid when allowing for general transfer schemes and budget balancedness. We extend our main positive result, Theorem 1, to the present environment. We should point out that the result below holds both in the case that an external collector of fines may be appointed, and in the case that only budget-balanced contracts are allowed: *the distinction is immaterial.*

[43] It is easy to see that in this formulation, verifiability does not impose any additional enforcement requirements over individual liability. In fact, when deriving the transfer schemes $t : A \to \mathbb{R}^I$ and $l(t) : A \to \mathbb{R}^I$ such that $t(a) = \sum_i t^i(a)$ and $l(t)(a) = \sum_i l^i(t^i)(a)$, it is immediate to obtain $v(l(t)) = l(t)$.

Proposition 6 *For any game G, and* transitive *verifiability structure P, any action profile a achievable under* joint liability *can be achieved with an* enforceable *(hence equilibrium-compatible) contract. If P is also* product, *then also any profile a achievable under* individual liability *can be achieved with an enforceable contract. If the game G has no ties in the payoffs, then a can also be achieved with a* finest enforceable *contract.*[44]

Proof. For any contract $\tilde{t}$, we introduce the sets $Z(\tilde{t}) = \{c : u(c|\tilde{t}, J) = u(c)\}$, and $Y(\tilde{t}) = \{c \in Z(\tilde{t}) : \tilde{t}(c) \neq \mathbf{0}\}$. By definition, the contract $\tilde{t}$ is enforceable if and only if $Y(\tilde{t}) = \emptyset$. Take the contract $\hat{t}$ such that for any c, $\hat{t}(c) = u(c|t, J) - u(c)$. It is immediate that $t(c) = \hat{t}(c)$ for any $c \notin Y(t)$ and that for any $c \in Z(t)$, it is the case that $u(c|t, J) = u(c|\hat{t}, J)$ and that $c \notin Y(\hat{t})$.

Pick any $d \notin Z(t)$, we want to show that $d \notin Y(\hat{t})$. Suppose by contradiction that $d \in Y(\hat{t})$, then there must exist $c \in P(d)$, such that $c \notin \hat{t}^{-1}\left(\hat{t}(d)\right)$. But since $d \notin Y(t)$, it must be the case that $P(d) \subseteq t^{-1}(t(d))$, and hence that $c \in \hat{t}^{-1}\left(\hat{t}(d)\right)$. It follows that $\hat{t}(c) \neq t(c) = t(d)$. Since $t(b) = \hat{t}(b)$ for any $b \notin Y(t)$, it follows that $c \in Y(t)$, and hence that there is a profile b such that $b \in P(c)$, but $b \notin t^{-1}(t(c)) = t^{-1}(t(d))$. Since $b \notin t^{-1}(t(d))$ and $d \notin Y(t)$ it follows that $b \notin P(d)$. We have obtained a violation of transitivity: $b \in P(c)$ and $c \in P(d)$ but $b \notin P(d)$.

Since $d \notin Z(t)$, by construction $\hat{t}(d) = t(d)$, thus $u(d|t, J) = u(d|\hat{t}, J)$. This concludes that $u(c|t, J) = u(c|\hat{t}, J)$ for all $c \in A$. Since a is a Nash equilibria of $G|t, J$, it must also be a Nash Equilibrium of $G|\hat{t}, J$.

In order to construct a finest enforceable contract $\check{t}$ simply note that if the game has no relevant ties then there is a uniform bound ε such that $|u_i(b_i, a_{-i}) + \hat{t}_i(b_i, a_{-i}) - \left(u_i(a) + \hat{t}_i(a)\right)| > \varepsilon$ for any i, b_i and a_{-i}. Hence there is a δ small enough such that if $\left|\check{t}_i(a) - \hat{t}_i(a)\right| < \delta$ for all a and i, then any profile achievable with $\hat{t}$ is also achievable with $\check{t}$, which may be constructed so as to be finest enforceable.

Suppose that P is product. For any contract $\tilde{\mathbf{t}} = \left(\tilde{t}^i\right)_{i \in I}$, and any i, we introduce the sets $Z_i(\tilde{t}^i) = \{c_i : t_i(c_i) = \mathbf{0}$ or $P_i(c_i) \nsubseteq t_i^{-1}\left(\tilde{t}_i(c_i)\right)\}$, and $Y_i(\tilde{t}^i) = \{c_i : \tilde{t}^i(c_i) \neq \mathbf{0}$ and $P_i(c_i) \nsubseteq t_i^{-1}\left(\tilde{t}_i(c_i)\right)\}$. By definition, the contract $\tilde{\mathbf{t}}$ is enforceable if and only if $Y_i(\tilde{t}^i) = \emptyset$ for every i.

Take any contract $\hat{\mathbf{t}}$ such that for any i and any c_i, $\hat{t}^i(c_i) = t_i(c_i)$ if $c_i \notin Z_i(t^i)$, and $\hat{t}^i(c_i) = 0$ if $c_i \in Z_i(t^i)$. Since P is transitive, P_i is transitive for each i. The same argument presented in the first part of the proof shows that for any i, if $d_i \notin Z_i(t_i)$,then $d_i \notin Y_i(\hat{t}_i)$. This concludes that $\hat{\mathbf{t}}$ is enforceable, and that $\hat{\mathbf{t}}$ and $\mathbf{t}$ achieve the same set of equilibria.

Both in the case of joint liability and in the case of product verifiability and individual liability, the transformation from t to $\hat{t}$ and $\check{t}$ does not violate the budget-balancedness restriction. It immediately follows that this restriction is immaterial for our results. ■

[44] When allowing for general transfer schemes, the concept of minimal enforceable contracts generalizes as follows. For any contract t, let $EC(t) = \{t^{-1}(t(a)) : a \in A\}$ be the set of equivalence classes generated by t; a contract t is a *finest enforceable contract* if it is enforceable and there is no enforceable contract t' such that $EC(t')$ is a refinement of $EC(t)$. Unlike minimal enforceable simple contracts, finest enforceable contracts need not be unique.

We conclude by presenting our positive results concerning damage compensation.

Proposition 7 *If P is trivial, or if $I = 2$, P is product and P_i is trivial for all i, then any action profile achievable* with damage compensation transfers *can be achieved with an enforceable (hence equilibrium-compatible) contract.*

Proof. Say that a is the Nash equilibrium of a game $G|t$ for some contract t under damage compensation.

If $P = \{A\}$ than $d(t)$ is constant across a, and hence a may either be achieved with a constant contract t, which is clearly enforceable, or cannot be achieved at all.

If $P = \{\{a\} : a \in A\}$, then let $t' = d(t)$. Any such transfer satisfies the requirement that $P(b) \subseteq t^{-1}(t(b))$. Any b such that $d(t)(b) = 0$ clearly satisfies the requirement that $t'_i(b) - t'_i(c) \leq \max\{0, u_i(c) - u_i(b)\}$. Any b such that $d(t)(b) = t(b)$ satisfies the requirement that $t_i(b) - t_i(c) \leq \max\{0, u_i(c) - u_i(b)\}$ hence a fortiori $t'_i(b) - 0 \leq \max\{0, u_i(c) - u_i(b)\}$ for any c such that $d(t)(c) = 0$.

To show that if $I = 2$ and P product and partitional, then enforceable contracts are also optimal as long as for any i, either $P_i = \{A_i\}$, or $P_i = \{\{a\} : a \in A\}$, on top of the above result, I need only consider the case where $P_2 = \{A_2\}$ and $P_1 = \{\{a_1\} : a_1 \in A_1\}$.

Pick any profile a, if $u_2(a_1, d_2) > u_2(a)$ for any d_2, then since $a_2 \in P_2(a_1, d_2)$, a cannot be a Nash equilibrium with any contract t.

If $u_1(d_1, a_2) \leq u_1(a)$ for any d_1, then a can be achieved with a constant contract t, which is clearly enforceable.

Suppose that for any $d_1 : u_1(d_1, a_2) > u_1(a)$ it is also the case that $u_2(a) > u_2(b)$ for any $b : b_1 = d_1$. Then set $\hat{t}(c) = d(t)(a)$ for any $c \in P(a)$ and $\hat{t}(b) = d(t)(d_1, a_2)$ for any $b : b_1 = d_1$. Since $u_2(a_1, d_2) \leq u_2(a)$ for any d_2, $\min_{c \in P(a)} u_2(c) = u_2(a)$ and $\max_{b \in P(d_1, a_2)} u_2(b) = u_2(d_1, a_2)$; hence all transfers introduced in $\hat{t}$ are enforceable. ■

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