Two-Class Voting: A Mechanism for Conflict Resolution?[1]

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Abstract

This paper discusses the merits of two-class voting procedures where voters are separated into classes that vote separately. A prominent example are Chapter 11 bankruptcy proceedings, where claim-holders who decide on a workout proposal are divided into classes, and the approval of the proposal needs a majority of the votes in each class. Some political mechanisms employ a similar process. We analyze two-class voting in a context where voters have conflicts of interest as well as differences of opinion regarding a proposal. We investigate how voting mechanisms aggregate information dispersed among voters. We find that two-class voting provides a significant improvement over one-class voting in all those situations where voters have significant conflicts of interests, and where the electorate is relatively evenly divided between different interest groups. Then voting in homogeneous groups provides voters with protection against expropriation and allows them to reveal their information through voting. However, two-class voting is inefficient for relatively homogenous electorates and if interest groups are very uneven in size.

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1. Introduction

In many situations voting contests are resolved by grouping voters into separate classes, so that the proposal put to a vote is only passed if it receives a majority from both classes of voters:

- Under Chapter 11 of the US Federal Bankruptcy Reform Act of 1978, creditors and shareholders are divided into classes following a reorganization proposal. The proposal is only accepted if it receives a majority from both classes of claim-holders.

- In some countries constitutional amendments need separate approval from different bodies: Belgium requires that the Walloon and the Flemish region of the country approve certain laws separately.[4]

- Covenants of preferred stock sometimes require that mergers are agreed separately by common stock and preferred stock holders.

- Bicameral legislatures, where two chambers of a parliament have to agree to the same law.[5]

The purpose of this paper is to study two-class voting as a mechanism to resolve differences of opinion and conflicts of interest. Consider the creditors of the firm in Chapter 11 bankruptcy who have to determine whether a proposal for a workout

[4] This "strengthened special majority"–procedure applies only to some laws that affect the power of the regions. These require that (i) 50% of each language group should be present, (ii) 50% of each language group votes in favor and (iii) the total of yes votes in both groups exceeds 2/3 of all votes cast. We are grateful to Marc Zenner for providing us with this information.

[5] See Tsebelis and Money (1997) for a survey of the institutions of bicamerialism and analytic approaches to two-chamber parliaments. Their table 2.1 (pp. 48-52) summarizes the institutions of 52 countries with a bicameral parliament as of December 1995.

should be accepted, or whether the firm should be liquidated. Each creditor has some information and therefore an opinion on the question which alternative would maximize the value of the firm and make *all* claim-holders collectively better off. However, suppose senior creditors have to give up more of their claims than junior creditors, creating a conflict of interest between creditors of different classes. Then senior creditors may vote against a workout proposal even though they have received private information indicating that accepting the workout would increase aggregate value.[6] Similarly, junior creditors may vote in favor of an inefficient workout because they receive sufficient transfers. In short, individuals are called to vote on a subject where two things matter: the information they have on the proposal, and specific interests that set them apart from other voters. If the conflict of interest between different groups of voters is sufficiently strong, then they will vote according to their preferences and ignore the information they have.[7] The resulting allocation is inefficient.

We analyze the efficiency of voting mechanisms in this context. We show how two-class voting may significantly improve the outcome, where improvements are measured with respect to the outcome chosen by a social planner who is only interested in increasing aggregate welfare and ignores distributional consequences of the decision.[8] The reason is that two-class voting induces voters to vote predominantly based on their information, so that a larger amount of valuable information is re-

[6] Similarly, a referendum on a change in the tax regime may have different implications for different geographical regions, and a merger proposal in a company may have different implications for holders of different securities.

[7] The tension between information-based and partisan voting under one-class voting was analyzed by Feddersen and Pesendorfer (1997).

[8] Note that two-class voting is very different from weighted voting, where holders of different securities receive different numbers of votes. Then the information or the interests of certain voters simply receive a higher weight, but the procedure is fundamentally a one-class voting procedure.

flected in the final decision. Reconsider the example of junior and senior creditors in chapter 11, and suppose that 40% of the votes are held by senior creditors, and the remaining 60% by junior creditors. One-class voting with a simple majority leads to a situation where the votes of the junior creditors carry more weight for the final outcome. They can therefore expropriate senior creditors by voting in favor of a workout proposal that favors junior creditors. This vulnerability to expropriation may lead senior creditors to oppose the workout even if they expect acceptance would increase the outcome for all creditors as a whole. In this case two-class voting protects the interests of senior creditors, since then the workout proposal needs a majority of all senior creditors separately. By removing the vulnerability of senior creditors to expropriation, they are now more likely to vote in favor of the workout proposal if they receive favorable information. Hence, two-class voting removes voter's focus on their interests by protecting them, and therefore allows them to vote more according to the information they receive. The result is a lower incidence of incorrect decisions and an improvement of overall welfare.

We conclude that in a large number of circumstances it is beneficial to seek the approval of each class of voters (creditors of the same priority) separately.[9] This is true especially if voters are relatively evenly distributed among interest groups. Otherwise it may be socially efficient - though not necessarily equitable - to ignore the interests of the smaller group altogether.[10] Then one-class voting is more effective, since a majority can only be obtained by convincing a significant proportion of the larger interest group. Also, a mechanism that eliminates conflicts

[9] Mechanisms may refine this simple requirement by imposing additional constraints, for example that each group has to approve with a specified majority, and that the number of votes as a whole has to pass a separate majority.

[10] We use a social welfare function that maximizes the expected payoffs of all voters and ignores distributional consequences.

of interest by mandating compensating side payments would be more effective than two-class voting.

We also analyze two-class voting when preferences are homogeneous and no conflicts of interest exist. It is well known that one-class voting can induce informative behavior for the entire electorate if preferences are homogenous.[11] In this case one-class voting dominates two-class voting. We investigate the best response functions of voters in a two-class voting as a function of majority requirements and the other class's behavior. We find that voters vote more in favor of the proposal if (i) the majority requirements are stronger and (ii) the voters of the other class vote less in favor of the proposal. Furthermore, we show that one needs weaker majority requirements in two class voting in comparison to one-class voting in order to induce identical voting behavior in both mechanisms.

We discuss the literature in the next section 2. Section 3 introduces the model and discusses the social planning problem. In section 4 we introduce two-class voting and discuss some of its properties. In section 5 we analyze the efficiency properties of two-class voting and state our main result. The following section 6 illustrates some additional properties of the model. Section 7 concludes. All proofs and technical results are collected in the appendix. A table with the notation can be found on page 29.

2. Related Literature

Information aggregation through voting goes back to the Condorcet Jury Theorem which states that a majority based rule is less likely to make a "mistake" than any single decision-maker. Several proofs have been offered for variations and extensions

[11] See Austin-Smith and Banks (1996) and Feddersen and Pesendorfer (1998).

of this claim.[12] One shortcoming of these papers is that they implicitly assume that each voter behaves sincerely, conditioning her vote only on her own information. Recently, a second generation of papers showed that "naive" voting is often irrational.[13]Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) investigate the features of strategic voting with homogenous preferences. From their analysis we know that strategic voting can aggregate information perfectly if the correct majority rule is used. Coughlan (2000) extends this model and analyzes the implications of modifying the voting mechanism to allow for mistrials and pre-vote communications among voters. Feddersen and Pesendorfer (1996a, 1998) study the asymptotic features of information aggregation in models with heterogenous preferences. They show that the probability of electing the wrong candidate converges to zero in an arbitrarily large electorate. Our modelling framework is closer to Feddersen and Pesendorfer (1998) and Austen-Smith and Banks (1996). However, our approach differs from them in two ways. First, we allow for conflicts of interest among the electorate and focus on non-asymptotic features of information aggregation. Consequently, we show that one-class voting wastes large amounts of private information. More importantly, we discuss the possibility to enhance information aggregation through a bicameral system.

In a more recent paper, Chwe (1999) also analyzes the problem of conflicts of interest between a majority and a minority with different preferences or beliefs. He derives optimal mechanisms and introduces the possibility that minorities obtain special privileges that increase their inclination to reveal information. However, Chwe (1999) does not discuss two-class voting, we discuss his results in greater detail below. Wolinsky (2000) discusses a set-up where a decision-maker extracts

[12]See Klevorick, Rothschild and Winship (1984), Ladha (1992), Miller (1986), Young (1988).

[13]See Austin-Smith and Banks (1996); Feddersen and Pesendorfer (1996a, 1996b, 1997, 1998); Myerson (1994,1997a, 1997b) and McLennan (1996).

information from a number of experts whose preference differ from those of the decision-maker. He shows that separating experts into groups can enhance information aggregation. However, he does not discuss two-class voting and the experts in his model do not have conflicting interests.

The literature on bicameralism has addressed similar institutions but from a different angle than our paper does. Diermeier and Myerson (1999) show in a model of self-interested legislative chambers how supermajority requirements and committees can enhance the bargaining position of a chamber in a bicameral system. Tsebelis and Money (1997) survey institutional and theoretical findings. Their models focus on intercameral bargaining between legislative chambers and on the process of obtaining agreement in case of a stallmate. Their findings include that bicameral legislatures tend to preserve the status quo compared to unicameral legislatures and that bicameral systems tend to concentrate multi-dimensional conflicts of interest on one privileged dimension. This literature is complementary to our approach as none of these approaches is related to information aggregation and the Condorcet Jury Theorem, whereas our model simplifies by focusing on one-dimensional conflicts of interest and assuming mechanical rules of conflict resolution.

3. The Model

Assume there are $M_J + M_S$ voters who jointly decide on a proposal that affects all of their preferences, where voters can be of one of two types $\tau \in \{J, S\}$, where 'S' refers to "senior" and 'J' refers to junior. The impact of the proposal on each voter's welfare depends on a state of nature $s \in \{l, h\}$, where both states are equally probable. Let D be a dummy variable that indicates acceptance of the proposal ($D = 1$ if the proposal is accepted, $D = 0$ if it is rejected). Voters' utility depends on the state of nature s, their type τ, and acceptance of the proposal D. Utility is denoted by $U_\tau(s, D)$. Then the M_τ voters of type τ have payoffs that satisfy:

$$\begin{aligned} U_\tau(h,1) - U_\tau(h,0) &= H + A_\tau \\ U_\tau(l,1) - U_\tau(l,0) &= -L + A_\tau \end{aligned} \tag{3.1}$$

This specification sacrifices some generality in order to simplify the argument.[14] We assume without loss of generality that:

$$M_J * A_J + M_S * A_S = 0 \tag{3.2}$$

Hence, the voters have to reach a decision on a proposal that has two effects: accepting the proposal yields a gain $H > 0$ for all voters if the state is h, and a loss $-L < 0$ for all voters if the state is l. In addition to this, accepting the proposal implies a transfer (positive or negative) from the junior to the senior voters. The size of A_J and A_S is a measure for the conflict of interest between senior and junior voters. If $A_J = A_S = 0$, then there is no conflict of interest at all. If $A_J > 0$, then $A_S = -\frac{M_S}{M_J} A_J < 0$ and the junior voters have a relative advantage from the proposal at the expense of the senior voters, and vice versa if $A_J < 0$ and $A_S > 0$. We also assume that $H + A_\tau > 0 > -L + A_\tau$ so that voters do not choose entirely on the basis of their preferences.

We consider two types of voting rules. If voting proceeds in one-class, then all voters have one vote, and acceptance of the proposal requires that at least a proportion α of all voters vote in favor of the proposal. Define $\mathbf{a} \equiv \alpha (M_J + M_S)$ and assume that $\mathbf{a}$ is an integer. We refer to this mechanism as one-class or unicameral voting. Alternatively, voters are grouped in classes according to their type, with the junior voters in one and the senior voters in the other class. Then acceptance of the

[14] The restriction is that we assume that H and L do not depend on the type. This has some impact for the social welfare functions we can consider. We will discuss this aspect in greater detail below.

proposal requires that at least a proportion α_J of the junior and α_S of the senior voters vote in favor of the proposal. (again, $\mathbf{a}_\tau \equiv \alpha_\tau M_\tau$ are assumed to be integers) We refer to this mechanism as two-class voting or bicameral voting. Clearly, this assumes that voters' type is associated with some observable characteristic.

Each voter observes a signal $\sigma_i \in \{g, b\}$ with the properties that:

$$\Pr(\sigma_i = g \,|s = h) = \Pr(\sigma_i = b \,|s = l) = 1 - \varepsilon \qquad 0 < \varepsilon < 1/2 \tag{3.3}$$

A pure strategy for each voter is a mapping from the signal space into the binary action space: $\{g, b\} \rightarrow \{'yes', 'no'\}$. A mixed strategy is a mapping from the signal space $\{g, b\}$ into [0; 1], a probability of voting 'yes,' ω_τ^σ. We know from the voting literature that there exists a large number of asymmetric equilibria in this type of game, where two voters of the same type τ observe the same signal and still vote differently.[15] We will analyze the symmetric equilibria of this game. Every symmetric equilibrium has that voters of the same type who observe the same signal follow the same strategy. Symmetric equilibria are completely characterized by two pairs of randomizing probabilities $\left\{\omega_\tau^g, \omega_\tau^b\right\}$, where ω_τ^σ is the probability of a type τ-voter to vote 'yes' subsequent to observing signal σ. We also know from the related literature on unicameral voting that no voter will randomize after both signals, since expected utility depends on the signal and no voter can therefore be indifferent at both values of the signal. (see Lemma 9.8 in the appendix for a proof.) More specifically, each voter randomizes after observing the bad signal only if she always votes 'yes' after observing the good signal. ($\omega_\tau^b > 0 \rightarrow \omega_\tau^g = 1$) Also, she votes 'no' after observing the good signal only if she always votes 'no' after observing

[15] Maug (1999) analyzes the asymmetric pure strategy equilibria for a similar game and shows that their properties are very similar to the symmetric mixed strategy equilibria characterized by Feddersen and Pesendorfer (1998). Myerson (1998) objects to asymmetric equilibria as being non-robust.

the bad signal. ($\omega_\tau^g < 1 \rightarrow \omega_\tau^b = 0$) It is now convenient to define

$$\omega_\tau \equiv \omega_\tau^g + \omega_\tau^b \in [0;2] \quad . \tag{3.4}$$

ω_τ characterizes the strategy of the type $\tau-$voters completely, and we can always infer the randomizing probabilities as $\omega_\tau^g = Min\{\omega_\tau, 1\}$, $\omega_\tau^b = Max\{\omega_\tau - 1, 0\}$. In order to simplify, we will abuse notation and refer to ω_τ as the strategy of type-τ voters. Denote by $\pi_\tau(s)$ the probability of a type $\tau-$voter to vote 'yes' if the state of nature is s. We have:

$$\pi_\tau^h(\omega_\tau) = \begin{cases} (1-\varepsilon)\omega_\tau & if\ 0 \leq \omega_\tau \leq 1 \\ (1-\varepsilon) + \varepsilon(\omega_\tau - 1) & otherwise \end{cases} \quad . \tag{3.5}$$

Similarly, we have

$$\pi_\tau^l(\omega_\tau) = \begin{cases} \varepsilon\omega_\tau & if\ 0 \leq \omega_\tau \leq 1 \\ \varepsilon + (1-\varepsilon)(\omega_\tau - 1) & otherwise \end{cases} \quad . \tag{3.6}$$

In all voting models there are trivial equilibria where each shareholder votes for the same candidate or proposal irrespective of his private information. Then no voter can influence the outcome, so this outcome is always an equilibrium. These equilibria are not very interesting and trivial to analyze, so we will restrict attention to equilibria in which each voter's decision is a function of her private information. We use the following definition:

Definition 3.1. *We say type τ votes responsively if $\pi_\tau(h) \neq \pi_\tau(l)$.*

We should note that this definition implies that $\pi_\tau(h), \pi_\tau(l)$ are strictly inside the unit interval, and that $\omega_\tau^g \neq \omega_\tau^b$. We follow the convention in the literature and refer to a responsive voting strategy where $\omega_\tau^g = 1$ and $\omega_\tau^b = 0$ as "sincere" voting. Let β denote voters' beliefs of being in the "good" state $s = h$. Denote the beliefs of voters of type τ conditional on (1) the signal σ they have observed and (2) on being pivotal by β_{τ}^{σ}. We will analyze later how these beliefs are formed. Voters of type τ vote in favor of the proposal if and only if:

$$\beta_{\tau}^{\sigma} H - (1 - \beta_{\tau}^{\sigma}) L + A_{\tau} \geq 0 \Rightarrow \beta_{\tau}^{\sigma} \geq \frac{L - A_{\tau}}{H + L} \equiv \beta_{0\tau} \tag{3.7}$$

otherwise they reject the proposal. Voters' beliefs β^{σ} have to pass a hurdle $\beta_{0\tau}$ for them to accept the proposal. As an equilibrium concept we use responsive symmetric Nash equilibrium, defined as follows:

Definition 3.2. *An equilibrium is a Symmetric Responsive Nash Equilibrium if (i) at least one voter votes responsively, (ii) all voters of the same type choose the same strategy ω_{τ}, and (iii) this strategy maximizes:*

$$\omega_{\tau}^{\sigma} \left[\beta_{\tau}^{\sigma} H - (1 - \beta_{\tau}^{\sigma}) L + A_{\tau} \right] \quad \tau \in \{J, S\} \quad \sigma \in \{g, b\} \tag{3.8}$$

Hence, if the expression in square brackets in (3.8) is positive, $\omega_{\tau}^{\sigma} = 1$, if it is negative, then $\omega_{\tau}^{\sigma} = 0$, and the voter randomizes with some strategy strictly between zero and one if the expression in square brackets is zero. The symmetry requirements rules out those equilibria where voters of the same type and with the same information vote differently. The responsiveness requirement rules out those equilibria where voters do not respond to their signal.

Our efficiency criterion which we use as a benchmark for comparing voting mechanisms is:

$$V(s, D(s)) \equiv M_J * U_J(s, D(s)) + M_S * U_S(s, D(s)) \tag{3.9}$$

The objective function (3.9) is clearly plausible in the context of chapter 11 bankruptcy regulation, where the social objective is to maximize the value of the firm. Extending our analysis to a more general social welfare function that would give different weights to different agents would require a substantially higher complexity of analysis than we have undertaken.[16]

The notation $D(s)$ emphasizes that the decision to implement the proposal is a function of the state s. Hence, the optimal decision rule maximizes $E[V(s, D(s))]$. Define by $P(s)$ the common component of the proposal: $P(h) = H$, $P(l) = -L$. Then we have:

$$\begin{aligned} V(s, D(s)) &= M_J * U_J(s, 0) + M_S * U_S(s, 0) \\ &\quad + (M_J + M_S) * P(s) * D(s) \end{aligned}$$

Hence, from the point of view of all voters as a whole, maximizing $E[V(s, D(s))]$ is equivalent to maximizing:

$$E[P(s) * D(s)] \tag{3.10}$$

Now, suppose a social planner could observe all $M_J + M_S$ signals. Then the planner's beliefs are a function of the number of signals and the number of good signals, where $a \in \{0, 1, ..., M_J + M_S\}$. We denote the planner's beliefs by $\beta(a, M_J + M_S)$. It is easy to show that β decreases in M_τ and increases in a. If the planner maximizes (3.9), she chooses to accept the project whenever:

$$\begin{aligned} & \beta(a, .) H - (1 - \beta(a, .)) L \geq 0 \\ \Rightarrow \quad & \beta(a, .) \geq \frac{L}{H + L} \equiv \beta_0 \end{aligned} \tag{3.11}$$

[16] Consider a social welfare function with weights w_τ given to type $\tau-$voters. Then we would have to change the tuple $\{L, H, A_J, A_S\}$ into a new tuple $\left\{\tilde{L}, \tilde{H}, \tilde{A}_J, \tilde{A}_S\right\}$ such that the conditions (3.1) are still satisfied and condition (3.2) is replaced by $w_J M_J A_J + w_S M_S A_S = 0$. The resulting conditions can be satisfied in general only if L and H are assumed to be different across types. If we were to generalize our results in this direction, then a modified version of proposition 5.1 and corollary 5.2 would still hold. However, for our main results in proposition 5.3 and theorem 5.7 we require symmetry ($H = L$), hence we could not extend the proofs of our main results to a generalized version of the model. Hence, we have not pursued this generalization.

We could characterize the solution to this particular problem directly. However, for subsequent propositions we need a different approach. Therefore, in order to introduce our methodology of proof for subsequent sections, we now introduce some notation and terminology.

Define the conditional probabilities of observing a good signals out of a total of $M_J + M_S$ signals, $\Pr(a|h)$ and $\Pr(a|l)$, as:

$$\Pr(a|h) = \binom{M_J + M_S}{a}(1-\varepsilon)^a \varepsilon^{M_J+M_S-a} \tag{3.12}$$

$$\Pr(a|l) = \binom{M_J + M_S}{a}(1-\varepsilon)^{M_J+M_S-a} \varepsilon^a \tag{3.13}$$

Then the probability of observing at least a goods signals conditional on state s is:

$$E(a|s) = \sum_{j=a}^{M_J+M_S} \Pr(j|s) \tag{3.14}$$

Note that the probability of a type I-error of accepting a proposal that is expected to reduce the objective (3.9) and (3.10) is $\frac{E(a|l)}{2}$. Conversely, the probability of a type II-error of rejecting a proposal that is expected to increase (3.10) is $\frac{1-E(a|h)}{2}$. Hence, (3.10) can be restated as:

$$Max_a \quad \frac{1}{2}(H \times E(a|h) + L \times (1 - E(a|l))) \tag{3.15}$$

This is equivalent to minimizing the weighted sum of type-I and type-II errors. From maximizing (3.15) we can see that there exists an optimal cutoff value a^* such that the social planner will accept the proposal if and only if the number of good signals exceeds a^*, and reject it otherwise. Please note that we can derive this result in two different ways: Firstly, it follows from the maximization of (3.15) directly. However, we can also solve condition (3.11) and observe that $\beta(a, .)$ is monotonically increasing in a.

4. Two-Class voting

There are several types of voting rules that can be applied (in order to accept a proposal) if voters vote in classes:

1. Both classes need to agree (supermajority or submajority in both classes).

2. One class agrees (supermajority or submajority in one-class).

3. Contingent majorities: the majority requirement in one-class depends on the outcome in the other class (e. g., accept if both classes have voted 1/2 in favor, or if one-class has voted 3/4 in favor).[17]

4. Both classes need to agree, and all voters together must agree.[18]

In this paper we will restrict attention to the first case given its frequent use. For example, Chapter 11 restructuring proposals are voted this way.

A voter is pivotal whenever two conditions are met: (1) the voters of the other class have agreed to the proposal, and (2) exactly $\mathbf{a}_\tau - 1$ voters of her own class have voted 'yes', and all others have voted no. It is convenient (but not necessary) to analyze the updating process in two steps. Now we make use of our definitions of (3.5) and (3.6) above and define F_τ to be the cumulative distribution function of the $\mathbf{a}_\tau - th$ order statistic from a population with cdf $\pi^h_\tau(\omega_\tau)$[19]:

$$F_\tau = \sum_{i=\mathbf{a}_\tau}^{M_\tau} \binom{M_\tau}{i} [\pi^h_\tau(\omega_\tau)]^i [1 - \pi^h_\tau(\omega_\tau)]^{M_\tau - i}. \qquad (4.1)$$

[17] Effectively, the "minority caucus" proposed by Chwe (1999) is such a contingent majority requirement: in his mechanism the proposal is accepted if either (1) it receives an overall majority or (2) it falls short of the overall majority by at most one vote, but obtains a supermajority in the smaller class ("minority").

[18] See the remarks on the Belgian constitution in the introduction.

[19] Note that F_τ is a function of several variables. Therefore, it is more appropriate to write $F_\tau(\omega_\tau, \mathbf{a}_\tau)$. For simplicity, we will abuse the notation and use F_τ whenever it is not confusing.

Similarly, we define:

$$G_\tau = \sum_{i=\mathbf{a}_\tau}^{M_\tau} \binom{M_\tau}{i} [\pi_\tau^l(\omega_\tau)]^i [1-\pi_\tau^l(\omega_\tau)]^{M_\tau - i}. \tag{4.2}$$

Note that, F_τ and G_τ are the probabilities of having at least $\mathbf{a}_\tau$ 'yes' votes given the state is h and l respectively. Now we can define β_τ, the probability of having state h conditional on class τ approving the proposal:

$$\beta_\tau = \frac{F_\tau}{F_\tau + G_\tau} \quad . \tag{4.3}$$

We would like to note that $\beta_\tau > \frac{1}{2}$, since $F_\tau > G_\tau$. Therefore, a voter conditioning on the other class approving the proposal is more optimistic about having state h. We denote the probability distribution function of the order statistics (4.1) and (4.2) by f_τ and g_τ respectively:

$$f_\tau = \frac{\partial F_\tau}{\partial \omega_\tau}, \quad g_\tau = \frac{\partial G_\tau}{\partial \omega_\tau} \quad . \tag{4.4}$$

We characterize the properties of these cumulative and probability distribution functions in lemmas 9.1-9.6 in the appendix. The only important property here is that f and g are not defined for pure strategy equilibria ($\omega_\tau = 1$). With the help of these definitions we can now state the following result:

Lemma 4.1. ***(Beliefs):*** *The beliefs of voters of type τ who condition on (1) the fact that class τ' has approved the proposal, (2) signal σ and (3) the fact that they are pivotal in their own class, β_τ^σ, satisfy:*

$$\frac{\beta_\tau^g}{1-\beta_\tau^g} = \frac{F_{\tau'}}{G_{\tau'}} \frac{f_\tau}{g_\tau}\bigg|_{\omega_\tau<1} \quad , \tag{4.5}$$

$$\frac{\beta_\tau^b}{1-\beta_\tau^b} = \frac{F_{\tau'}}{G_{\tau'}} \frac{f_\tau}{g_\tau}\bigg|_{\omega_\tau>1} \quad . \tag{4.6}$$

We express beliefs as odds rather than as probabilities for simplicity of exposition. It is also convenient, because we can now state condition (3.7) more concisely. Voters of type τ who have observed σ will vote to accept the proposal only if:

$$\frac{\beta_\tau^\sigma}{1-\beta_\tau^\sigma} \geq \frac{L-A_\tau}{H+A_\tau} \tag{4.7}$$

If $\tau-$voters randomize after observing the signal ($0 < \omega_\tau^\sigma < 1$), then (4.7) has to be satisfied as an equality. If the inequality is strict, then $\omega_\tau^\sigma = 1$, and if (4.7) is violated, then $\omega_\tau^\sigma = 0$, Hence, (4.7) defines the optimal voting strategy.

It is instructive to see and also important for the strategy of our proofs that condition (4.7) can also be derived as a first order condition of the following optimization problem:

$$\omega_\tau^{**} = \arg\max_{\omega_\tau} \left[\beta_{\tau'} F_\tau(\omega_\tau)(H+A_\tau) + (1-\beta_{\tau'}) G_\tau(\omega_\tau)(-L+A_\tau)\right] \tag{4.8}$$

Voters of type τ condition on the fact that class τ' has already accepted, hence, class τ believes to be in the high state with probability $\beta_{\tau'}$, as given in (**??**). In any Symmetric Responsive Nash Equilibrium where all $\tau-$voters use strategy ω_τ, the probability of accepting the proposal in the high state is $F_\tau(\omega_\tau)$, then the payoff is $H + A_\tau$. The probability of accepting the proposal in the bad state is $G_\tau(\omega_\tau)$, in which case the payoff is $-L + A_\tau$. The solution to (4.8) is characterized by the first-order condition:[20]

$$\frac{F_{\tau'}}{G_{\tau'}}\frac{f_\tau}{g_\tau} = \frac{L-A_\tau}{H+A_\tau} \tag{4.9}$$

We therefore have the following result:

Proposition 4.2. ***(Equilibrium)****: (i) Fix any* $(\alpha_J, \alpha_S) \in (0,1)\times(0,1)$. *Then, there exist* M_J' *and* M_S' *such that for all* $M_J > M_J'$ *and* $M_S > M_S'$ *there is a*

[20]Note that F and G are not differentiable at $\omega_\tau = 1$, and f and g are not defined at that point. This is also reflected in conditions (4.5) and (4.6). The correct statement of the first order condition for this case leads to (4.10).

symmetric responsive equilibrium. (ii) The first order conditions (4.9) follow from the optimization problem (4.8), or, equivalently, from the optimality condition (4.7). (iii) Any Symmetric Responsive Nash Equilibrium where $\omega_\tau \neq 1$ is characterized by the first order condition (4.9) for each class of voters. If voters of class τ vote sincerely ($\omega_\tau = 1$), then the first order condition becomes:

$$\frac{\beta_\tau^g}{1-\beta_\tau^g} \geq \frac{L-A_\tau}{H+A_\tau} \geq \frac{\beta_\tau^b}{1-\beta_\tau^b} \tag{4.10}$$

where at least one inequality must be strict.

We need the minimum size of the electorate for existence (part (i)) since for a given prior and given payoffs to each class of voters, a certain number of signals is required to convince voters to vote against the alternative implied to be optimal given their priors. Part (ii) shows that the first order conditions can be approached from two different perspectives, namely, the optimization of (4.8) and the optimality condition (4.7). Note that beliefs β_τ^σ condition on $\tau-$voters being pivotal (as a group) and each $\tau-$voter conditioning on being the marginal voter in her class, whereas the optimization problem (4.8) makes no reference to pivotality within a class. Take any strategy ω_τ^{**} that solves (4.8) and assume all $M_\tau - 1$ voters in class τ follow this strategy. Then it is optimal for the $M_\tau - th$ voter in class τ to follow this strategy as well. This fact parallels results on unicameral voting where voters who condition on being pivotal behave as if they were maximizing the payoff of all voters. Here, each voter behaves as if she maximized the payoff of all voters in her class.

It is instructive to look at the best response functions ω_τ^{**} in a little more detail.

Proposition 4.3. ***(Best response functions):*** *(i) The best response function ω_τ^{**} is a downward sloping function of $\omega_{\tau'}$. (ii) For any Symmetric Responsive Nash*

Equilibrium the best response functions ω_J^{**} and ω_S^{**} are increasing functions of a_J and a_S.

The intuition for the first result is simple. Consider the case of a junior voter who observes that each senior voter votes for the proposal with a higher probability. Then the junior voter becomes less optimistic about being in the good state if she knows that the senior class has approved: Approval by the senior voters is less indicative of being in the good state, hence β_τ is decreasing in ω_τ. This implies immediately that the junior voter now requires more positive information about the proposal from her own class in order to stay indifferent between her choices, so that (4.7) is still satisfied. This is only possible if all other junior voters now vote more conservatively, so that being pivotal in her own class increases the odds of being in the good state. Then she becomes pivotal only if more junior voters have observed the good signal. Part (ii) of proposition 4.3 follows from the same intuition: if the majority requirement for class τ increases, then each voter becomes more optimistic conditional on being pivotal as a larger number of positive votes is required for approval. In order to be indifferent, these votes must now be less indicative of being in the good state, so ω_τ has to increase.

Proposition 4.4. ***Bicameral vs. Unicameral Voting:*** *Consider the case where voters have no conflicts of interest ($A_J = A_S = 0$). Then consider the Symmetric Responsive Nash Equilibrium of the one-class voting game with majority requirement α and strategy ω^* chosen by all voters. Compare this with any Symmetric Responsive Nash Equilibrium of the two-class voting game with majority requirements α_J, α_S chosen so that $\omega_J^{**} = \omega_S^{**} = \omega^*$. Then we have that the one-class majority requirement provides an upper bound so that $\alpha_J + \alpha_S < \alpha$ if each electorate is sufficiently large.*

The intuition here is that in two-class voting, being marginal conveys more optimistic information than in one-class voting. This is because each voter conditions on acceptance by the other class, which can be marginal acceptance with $a_{\tau'}$ votes, but it could also be acceptance with a much higher majority. In the model, $\beta_{\tau'} > \frac{1}{2}$. From proposition 4.3 we know that more optimistic beliefs lead to higher randomizing probabilities, so that indifference at the same randomizing probabilities implies that the majority requirements have to be lower.

Two-class voting rules that require a majority of all voters *in addition to* a majority of each class reflect a concern that two-class voting with a simple majority requirement for each class passes proposals too easily.[21] Proposition 4.4 contradicts this notion by showing that two-class voting leads to a *lower* rather than a higher majority requirement compared to one-class voting: the two-class mechanism itself establishes an additional hurdle the proposal needs to take.

5. Efficiency of Voting Rules

In this section we discuss the ability of different voting rules to achieve social optimality. We already know from the literature that one-class voting without conflicts of interest can implement the social optimum. We have:

Proposition 5.1. *There is no set of majority rules* $(\mathbf{a}_S,\ \mathbf{a}_J)$ *such that voting in two classes aggregates information perfectly, and the likelihood of a mistake is always strictly larger than that attained by the social planner.*

The proof of proposition 5.1 is instructive and conveys also the intuition of the result, hence we put it in the text:

[21] See point 4 in the list on page 13 above. The Belgium constitution noted in the introduction is an example of this rule.

Proof of proposition 5.1:

The proof is illustrated in Figure 5.1.The horizontal axis of Figure 5.1 has the

Figure 5.1:

number g_J of good signals received by junior voters, the vertical axis the number g_S of those received by senior voters. The diagonal line indicates the location of all (g_J, g_S) −tuples so that $g_J + g_S = a^*$. The proposal should only be accepted if (g_J, g_S) lies to the right and above the line. Two-class voting can only implement the optimum if all voters vote sincerely, since some information is lost through randomization. Moreover, the tuple of majority-requirements (a_J^*, a_S^*) that supports sincere voting so that $\omega_J = \omega_S = 1$ must satisfy $\mathbf{a}_J^* + \mathbf{a}_S^* = a^*$, otherwise the social optimum cannot be implemented. Suppose now that we have a tuple $(\mathbf{a}_J^*, \mathbf{a}_S^*)$ that supports sincere voting by all voters. Then with two-class voting and sincere voting by all voters the proposal is accepted whenever the tuple (g_J, g_S) lies to the right of $\mathbf{a}_J$ and above $\mathbf{a}_S$. Then areas I and III in the graph indicate those areas where the proposal is incorrectly rejected ($g_\tau < \mathbf{a}_\tau^*$ for one class), whereas area III indicates distributions of signals where the proposal is incorrectly accepted (since $\mathbf{a}_J^* + \mathbf{a}_S^* < a^*$, which is true whenever proposition 4.4 applies. Clearly, there is no combination of arranging horizontal and vertical lines that could replicate the diagonal line which represents the social planning solution.[22] Hence, two-class voting cannot implement the social optimum. ■

We immediately have the following corollary:

Corollary 5.2. *If there are no conflicts of interest so that $A_J = A_S = 0$, then one-class voting is strictly better than two-class voting with respect to maximizing the social objective function (3.9).*

This implication is immediate and shows that two-class voting can be an optimal mechanism only if there are sufficient conflicts of interest so that unicameral voting is no longer optimal. Hence, our next step is to investigate the impact of conflicts of interest on the efficiency of one-class voting. A voter of type τ approves only if his posterior belief, β_τ^σ, is greater than or equal to his hurdle value $\beta_{0\tau}$, and rejects otherwise. A voter votes responsively only if his hurdle rate is in the interval $[\beta_\tau^b, \beta_\tau^g]$. With a high level of conflict of interest, these hurdle values are very different for both types. (see also (4.7) and (4.10)) On the other hand, being pivotal carries similar information for both types in one-class voting. For large enough electorates, the similarity is very high. Therefore, for larger electorates two voters with different preferences but identical signals have virtually identical posterior beliefs and the interval $[\beta_\tau^b, \beta_\tau^g]$ is very similar for both types. Hence, if the conflict of interest is

[22] This result is also general and not dependent on the social welfare function chosen, see footnote 16 on page 11 above. A different social welfare function would only change a^*, hence the location of the diagonal line in Figure 5.1.

large enough, then it is not possible for both types to have their hurdle rate in this interval, and only one interest group can vote responsively.

We can now state the major implication for conflicts of interest:

Proposition 5.3. ***(Conflicts of Interest)*** *(i) For a given $\alpha \in (0,1)$, a Symmetric Responsive Nash Equilibrium of the one-class voting game exists for any conflict of interest, $A_J > 0$, whenever the electorate is large enough. (ii) There exists a constant $\overline{A_J} > 0$ so that for all $A_J > \overline{A_J}$, only one type of voters votes responsively in the unique Symmetric Responsive Nash Equilibrium of the one-class voting game. In particular, if $(M_J + M_S)\alpha > M_J$, then type $J-$voters vote for the proposal independently of their signal. Otherwise, type $S-$voters vote against the proposal independently of their signal.*

Hence, under the conditions of proposition 5.3, conflicts of interest can be sufficiently significant so that only one class of voters votes responsively, whereas voters of the other class vote only according to their preferences, independently of the signal they receive. In this case two-class voting is potentially interesting, since now the standard results on information aggregation in one-class voting games with homogeneous electorates no longer apply. In fact, we know from proposition 5.3 that one type of voters completely ignore their information if the conflict of interest is large enough. In this case two-class voting can duplicate the outcome and efficiency level of one-class voting simply by setting $\alpha_\tau = 0$ and $\alpha_{\tau'} = \frac{\alpha(M_J+M_S)-M_\tau}{M_J+M_S}$ where τ stand for the type of voters who vote non-responsively with one-class voting and majority rule α. We obtain:

Proposition 5.4. Corollary 5.5. *Whenever conflicts of interest are sufficiently strong so that there is no Symmetric Responsive Nash Equilibrium where both types of voters vote responsively, then two-class voting is weakly better than one-class voting.*

In fact, we will show next that two-class voting can do strictly better than one-class voting if the conflict of interest is significant in the sense of proposition 5.3. In order to state our main result, we need to focus on a special case of our model where payoffs and electorates are symmetric, so that $M_J = M_S = M$ and $H = L$. Without loss of generality we now assume that $A_J = A > 0$ and $A_S = -A$. However, we first state more formally what we mean by "significant" conflicts of interest:

Definition 5.6. *We say that conflicts of interest are significant if* $A > L(1 - 2\varepsilon)$.

We are now ready to state the main result of the paper:

Theorem 5.7. ***Efficiency of Two-class voting:*** *If conflicts of interest are significant and the electorate is sufficiently large, then two-class voting is strictly better than one-class voting with respect to maximizing the social objective (3.9).*

While the proof of this result is somewhat lengthy, the intuition is rather simple. Whenever there are conflicts of interest, voters will use their information in a way that deviates from the optimum. If conflicts of interest are significant, then unicameral voting becomes inefficient as voters pay a lot of attention to their interests and ignore their information, so that one group of voters disregards their information completely. Two-class voting offers an alternative if voters are grouped according to their preferences. Now voters' specific interests are protected by the fact that approval of their whole group is required, independently of how other voters of the other group decide. Since all voters within a group are homogeneous, collectively they wish to accept the proposal if it benefits their group. This protection reduces the importance of their preferences and increases the importance of their private information for their voting behavior.[23] Therefore, more information is incorporated

[23] Chwe (1999) discusses other mechanisms to protect minorities to enhance their inclination to reveal information.

in the final outcome and this information is used in a way consistent with the social objective.

6. Numerical Illustrations

The proof of our main result relies critically on the assumption that the electorate is evenly split between two classes so that $M_J = M_S$. Clearly, the fact that we obtain strict dominance shows that we can expect the result to hold also if we perturb the game in a neighborhood of the parameters chosen. We can also expect that symmetry is not strictly necessary for our main result to hold.

In this section we complement our discussion with some numerical examples in order to illustrate how far we can perturb our parameters and to what extent we can deviate from symmetry. We also illustrate some additional features that cannot be demonstrated analytically.

For this we start by specifying $\varepsilon = 0.4$ and $H = L = 5$. We use a grid of different values for the electorate where M_τ ranges from 10 to 500. Recall from our discussion before proposition 5.3 that the important consideration is the difference between the hurdle rates β_{0J} and β_{0S}. We therefore define conflicts of interest as constant whenever $\beta_{0S} - \beta_{0J}$ is constant across different electorates, and we choose $\beta_{0S} - \beta_{0J} = 0.25$. We cannot choose β_{0J} and β_{0S} (respectively, A_J and A_S) independently since we lose one degree of freedom through condition (3.2). We can compute the two-class voting equilibrium for each vector of parameters $\{H, L, \varepsilon, A_J, A_S, M_J, M_S, \alpha_J, \alpha_S\}$ by solving a system of two equations in two unknowns. ((4.9), respectively, (4.10)) We could not prove uniqueness of equilibrium, but we have never encountered multiple equilibria. Since $H = L$ we can define the minimum of the error the social planner would make by restating (3.9) and (3.15) as:

$$E^*_{Social} \equiv Min_a \ \{1 - E\,(a|h) + E\,(a|l)\} \tag{6.1}$$

Similarly, we can define the optimum that can be achieved with two-class voting as:

$$E^*_{Twoclass} \equiv Min_{\{\mathbf{a}_J,\mathbf{a}_S\}} \ \{1 - F_J F_S + G_J G_S\} \tag{6.2}$$

Here, $1 - F_J F_S$ (respectively, $1 - E(a^*|h)$) is the probability of a type-II-error (rejecting a good proposal), $G_J G_S$ (respectively, $E(a^*|l)$) is the probability of a type-I-error (accepting a bad proposal). For each electorate we determine the optimal majority rules α_J, α_S with respect to minimizing (6.1). We use the minimized $E^*_{Twoclass}$ as our statistic to evaluate two-class voting.

We do the same for one-class voting for the same parameters, where we also use the optimal majority rule $\mathbf{a}^*$ for one-class voting. For one-class voting we define:

$$E^*_{Oneclass} \equiv Min_{\{\mathbf{a},\tau\}} \ \{1 - F_\tau + G_\tau\} \tag{6.3}$$

where τ is the type of voters who votes responsively. That is, we compute the errors for one-class voting by establishing first that only one class can vote responsively, and then checking which class should vote responsively so that the error is minimized. Hence our notion of one-class voting assumes that the majority rule for one-class voting is chosen so as to minimize the loss function of the social planner.

In order to compare two-class and one-class voting we compute a number of statistics. The first set of statistics compares absolute errors, i. e., the difference in $E^*_{Twoclass}$ and $E^*_{Oneclass}$. Figure 6.1 displays the *absolute* difference in errors between one-class versus two-class voting (left panel), and one-class voting versus the error with perfect information aggregation (right panel). The contour chart (left panel of Figure 6.1) clearly demonstrates that two-class voting makes the largest difference when (1) the electorate is more or less evenly split ($M_J \approx M_S$), and (2) the electorate is small. In this case the absolute difference in errors are between 4% - 5%. It is also clear from the right hand panel that these are precisely the cases where one-class voting deviates most strongly from the social optimum. The chart suggests

Figure 6.1:

that both differences are of about the same order of magnitude. We investigate this further by looking at the *relative* differences. For this we compute two statistics. The first is simply the relative improvement from two-class voting:

$$Criterion\ I = \frac{E^*_{Oneclass}}{E^*_{Twoclass}} \tag{6.4}$$

For our parameter values this statistic ranges from 1 (for very unevenly split electorates) to 79.2 for $M_J = M_S = 500$. The second statistic normalizes the absolute improvement by relating it to the gap between one-class voting and the social optimum:

$$Criterion\ II = \frac{E^*_{Oneclass} - E^*_{Twoclass}}{E^*_{Oneclass} - E^*_{Social}} \tag{6.5}$$

The motivation behind this statistic is that it is normalized to lie between zero and one. If two-class voting achieves the social optimum, then Criterion II equals 1.

Figure 6.2:

If two-class voting is ineffective and leads to the same error as one-class voting, then Criterion II equals zero. We can interpret Criterion II as measuring the extent to which two-class voting helps to close the gap between one-class voting and the social optimum. Figure 6.2 displays both criteria. Both panels of Figure 6.2 show that the relative improvement is larger if the electorate is large and evenly split between the two classes. Criterion II approaches 1 as the electorate becomes large: for our parameters it becomes .984 for $M_S = M_J = 500$. This criterion converges fast for evenly split electorates, and more slowly for unevenly split electorates.

The values of $E^*_{Twoclass}$ and $E^*_{Oneclass}$ and the social optimum, all approach zero as the electorate becomes sufficiently large. Criterion II allows us to look at the relative speeds of convergence, and shows that for small electorates the relative advantage of two-class over one-class voting is small, since both mechanisms lead to a significantly higher incidence of errors relative to the social optimum. For large electorates, the two-class voting mechanism behaves like the social planner: the difference in the incidence of errors between the social planner and two-class voting becomes negligible relative to the incidence of errors with one-class voting.

7. Discussion and Conclusion

This paper has analyzed the comparative advantages of one-class voting and multi-class voting of Chapter 11 bankruptcy procedures in a situation where individuals do not only have differential information, but also different interests with respect to the proposal decided by a vote. The general finding is that in a large number of cases it is useful to segregate voters into different classes which are then more homogeneous with respect to voters' preferences. This also lends support to some institutions found in practice in addition to Chapter 11 bankruptcy proceedings, like bicameral parliamentary systems, and some constitutional assemblies.

At this point we need to emphasize that we have not established that two-class voting is in any sense the optimal mechanism. Clearly, a superior mechanism can always be found where voters agree on the proposal in conjunction with a scheme of offsetting transfers. Effectively, transfers would compensate for the distributional consequences of a proposal and amount to proposals on which voters do not have conflicting interests. We do not see examples of transfers of this kind, probably because eliminating conflicting interests is not generally possible.

Another limitation is the sequential nature of many voting processes, since, for example, the different chambers of parliaments vote often sequentially rather than simultaneously. Dekel and Piccione (1999) show that simultaneous and sequential voting games produce identical symmetric equilibria within a one-class framework. In one-class voting, observing prior votes does not give useful information in addition to being pivotal. However, this result does not extend to two-class voting games since observing the first class' vote gives useful information for the second class in addition to being pivotal. Hence, it is an open issue how the election outcomes and efficiency differ in a sequential two-class voting game.

8. Notation

Symbol	Interpretation
$\tau \in \{J, S\}$	Class of voters (**J**unior or **S**enior)
M_τ	Number of voters in one class (size of electorate)
$s \in \{h, l\}$	State of nature (**h**igh or **l**ow)
$P(s) \in \{H, -L\}$	state-contingent pay off
H	payoff in **h**igh-state
$-L$	payoff in **l**ow state
A_τ	class-dependent payoff to voters of class τ
$\sigma \in \{g, b\}$	signal (**g**ood or **b**ad)
ε	Error of signal σ
π_τ^s	probability that voter of type τ votes 'yes' in state s
ω_τ^σ	probability that voter of type τ votes 'yes' after observing σ
$\omega_\tau = \omega_\tau^g + \omega_\tau^b$	strategy of voters of type τ
$F_\tau,\ f_\tau$	cdf and pdf of order statistic in high state
$G_\tau,\ g_\tau$	cdf and pdf of order statistic in low state
β_τ	beliefs conditional on class τ approving proposal
β_τ^σ	beliefs conditional on voter of class τ being pivotal,
	and observing signal σ
$\beta_{0\tau}$	hurdle rate for voters of class τ
$D(s) \in \{0, 1\}$	acceptance / rejection dummy as function of s
$V(s, D(s))$	objective of social planner
$U_\tau(s, D(s))$	utility of voter of class τ
$\omega_\tau^{**},\ \omega_\tau^*$	Best response function in two-class / one-class voting

9. Appendix: Proofs and Technical Results

9.1. Technical Results on Order Statistics

This section summarizes some technical results of order statistics. Recall the definitions from equations (4.1) and (4.2) above. Furthermore, we define

$$F_\tau^i = \binom{M_\tau}{i}[\pi_\tau^h(\omega_\tau)]^i[1-\pi_\tau^h(\omega_\tau)]^{M_\tau - i}. \tag{9.1}$$

Similarly, we define:

$$G_\tau^i = \binom{M_\tau}{i}[\pi_\tau^l(\omega_\tau)]^i[1-\pi_\tau^l(\omega_\tau)]^{M_\tau - i}. \tag{9.2}$$

Therefore, we have $F_\tau = \sum_{i=\mathbf{a}_\tau}^{M_\tau} F_\tau^i$ and $G_\tau = \sum_{i=\mathbf{a}_\tau}^{M_\tau} G_\tau^i$.

Lemma 9.1. *For any $\mathbf{a}_\tau$ and ω_τ, $\frac{F_\tau^i}{G_\tau^i}$ is strictly less than $\frac{F_\tau^{i+1}}{G_\tau^{i+1}}$.*

Proof of Lemma 9.1: $\frac{\frac{F_\tau^{i+1}}{G_\tau^{i+1}}}{\frac{F_\tau^i}{G_\tau^i}} = \frac{\pi_\tau^h(\omega_\tau)\left(1-\pi_\tau^l(\omega_\tau)\right)}{\pi_\tau^l(\omega_\tau)\left(1-\pi_\tau^h(\omega_\tau)\right)} > 1$ since $\frac{\pi_\tau^h(\omega_\tau)}{\pi_\tau^l(\omega_\tau)} > 1$ and $\frac{\left(1-\pi_\tau^l(\omega_\tau)\right)}{\left(1-\pi_\tau^h(\omega_\tau)\right)} > 1$ given ε is less than $\frac{1}{2}$. ■

Lemma 9.2. *For any $\mathbf{a}_\tau$ and $\omega_\tau \neq 1$, $\frac{f_\tau}{g_\tau}$ is (weakly) less than $\frac{F_\tau^{\mathbf{a}_\tau}}{G_\tau^{\mathbf{a}_\tau}}$.*

Proof of Lemma 9.2: Note that

$$f_\tau = \binom{M_\tau}{\mathbf{a}_\tau}\mathbf{a}_\tau[\pi_\tau^h(\omega_\tau)]^{\mathbf{a}_\tau - 1}[1-\pi_\tau^h(\omega_\tau)]^{M_\tau - \mathbf{a}_\tau}\frac{\partial \pi_\tau^h}{\partial \omega_\tau}. \tag{9.3}$$

Similarly,

$$g_\tau = \binom{M_\tau}{\mathbf{a}_\tau}\mathbf{a}_\tau[\pi_\tau^l(\omega_\tau)]^{\mathbf{a}_\tau - 1}[1-\pi_\tau^l(\omega_\tau)]^{M_\tau - \mathbf{a}_\tau}\frac{\partial \pi_\tau^l}{\partial \omega_\tau}. \tag{9.4}$$

Therefore,

$$\frac{f_\tau}{g_\tau} = \left(\frac{\pi_\tau^h}{\pi_\tau^l}\right)^{\mathbf{a}_\tau - 1}\left(\frac{1-\pi_\tau^h}{1-\pi_\tau^l}\right)^{M_\tau - \mathbf{a}_\tau}\frac{\frac{\partial \pi_\tau^h}{\partial \omega_\tau}}{\frac{\partial \pi_\tau^l}{\partial \omega_\tau}}. \tag{9.5}$$

For $\omega_\tau < 1$, we have $\frac{\frac{\partial \pi_\tau^h}{\partial \omega_\tau}}{\frac{\partial \pi_\tau^l}{\partial \omega_\tau}} = \frac{1-\varepsilon}{\varepsilon} = \frac{\pi_\tau^h}{\pi_\tau^l}$ implying $\frac{f_\tau}{g_\tau} = \frac{F_\tau^{\mathbf{a}_\tau}}{G_\tau^{\mathbf{a}_\tau}}$. For $\omega_\tau > 1$, we have $\frac{\frac{\partial \pi_\tau^h}{\partial \omega_\tau}}{\frac{\partial \pi_\tau^l}{\partial \omega_\tau}} = \frac{\varepsilon}{1-\varepsilon} < 1 < \frac{\pi_\tau^h}{\pi_\tau^l}$ implying $\frac{f_\tau}{g_\tau} < \frac{F_\tau^{\mathbf{a}_\tau}}{G_\tau^{\mathbf{a}_\tau}}$. ■

Lemma 9.3. *For any $\mathbf{a}_\tau$ and ω_τ, $\frac{F_\tau}{G_\tau}$ are strictly decreasing functions of ω_τ.*

Proof of Lemma 9.3: For any $\omega_\tau \neq 1$, the previous two lemmas imply that $\frac{f_\tau}{g_\tau}$ is strictly less than $\frac{F_\tau}{G_\tau}$. Therefore, $f_\tau G_\tau$ is strictly less than $g_\tau F_\tau$. Note that $\frac{\partial}{\partial \omega_\tau}\left(\frac{F_\tau}{G_\tau}\right) = \frac{f_\tau G_\tau - g_\tau F_\tau}{(G_\tau)^2}$ is negative, since $f_\tau G_\tau - g_\tau F_\tau < 0$. For $\omega_\tau = 1$, the fraction $\frac{F_\tau}{G_\tau}$ is countionous and non-differentiable. ■

Lemma 9.4. *For any $\mathbf{a}_\tau$ and $\omega_\tau \neq 1$, both $\frac{F_\tau^i}{G_\tau^i}$ and $\frac{f_\tau}{g_\tau}$ is a strictly decreasing function of ω_τ.*

The proof of this lemma is omitted as it follows immediately from Lemma 9.4 and differentiation of (9.5) with respect to ω_τ. ■

Lemma 9.5. *For a given ω_τ, increasing the social choice rule from $\mathbf{a}_\tau$ to $\mathbf{a}'_\tau$ (weakly) lowers both F_τ and G_τ. Furhtermore, $\frac{F_\tau(\mathbf{a}_\tau)-F_\tau(\mathbf{a}'_\tau)}{G_\tau(\mathbf{a}_\tau)-G_\tau(\mathbf{a}'_\tau)}$ is strictly less than both $\frac{F_\tau(\mathbf{a}_\tau)}{G_\tau(\mathbf{a}_\tau)}$ and $\frac{F_\tau(\mathbf{a}'_\tau)}{G_\tau(\mathbf{a}'_\tau)}$.*

Proof of Lemma 9.5: Both F_τ^i and G_τ^i are non-negative. Thus the first part of the Lemma follows. For the second part note that

$$\frac{F_\tau(\mathbf{a}_\tau) - F_\tau(\mathbf{a}'_\tau)}{G_\tau(\mathbf{a}_\tau) - G_\tau(\mathbf{a}'_\tau)} = \frac{\sum_{i=\mathbf{a}_\tau}^{\mathbf{a}'_\tau - 1} F_\tau^i}{\sum_{i=\mathbf{a}_\tau}^{\mathbf{a}'_\tau - 1} G_\tau^i}. \tag{9.6}$$

From Lemma 9.1, this expression must be strictly less than both $\frac{F_\tau(\mathbf{a}_\tau)}{G_\tau(\mathbf{a}_\tau)}$ and $\frac{F_\tau(\mathbf{a}'_\tau)}{G_\tau(\mathbf{a}'_\tau)}$. ■

Lemma 9.6. *For a given ω_τ, increasing the social choice rule from $\mathbf{a}_\tau$ to $\mathbf{a}'_\tau$ increases both $\frac{F_\tau}{G_\tau}$ and $\frac{f_\tau}{g_\tau}$.*

Proof of Lemma 9.6: The first part follow immediately from Lemma 9.5. For the second part note that $\frac{f_\tau}{g_\tau}$ is

$$\left(\frac{\pi_\tau^h}{\pi_\tau^l}\right)^{\mathbf{a}_\tau - 1} \left(\frac{1-\pi_\tau^h}{1-\pi_\tau^l}\right)^{M_\tau - \mathbf{a}_\tau} \frac{\frac{\partial \pi_\tau^h}{\partial \omega_\tau}}{\frac{\partial \pi_\tau^l}{\partial \omega_\tau}}. \tag{9.7}$$

Given that $\frac{\pi_\tau^h}{\pi_\tau^l} > 1$ and $\frac{1-\pi_\tau^h}{1-\pi_\tau^l} < 1$ $\frac{f_\tau}{g_\tau}$ must increase as we increase $\mathbf{a}_\tau$. ■

9.2. Proofs

Proof of Lemma 4.1: The probability of being in state h conditional on a type-τ voter being pivotal, $\widetilde{\beta}_\tau$, is:

$$\widetilde{\beta}_\tau = \frac{F_{\tau'}[\pi_\tau^h(\omega_\tau)]^{\mathbf{a}_\tau - 1}[1-\pi_\tau^h(\omega_\tau)]^{M_\tau - \mathbf{a}_\tau}}{F_{\tau'}[\pi_\tau^h(\omega_\tau)]^{\mathbf{a}_\tau - 1}[1-\pi_\tau^h(\omega_\tau)]^{M_\tau - \mathbf{a}_\tau} + G_{\tau'}[\pi_\tau^l(\omega_\tau)]^{\mathbf{a}_\tau - 1}[1-\pi_\tau^l(\omega_\tau)]^{M_\tau - \mathbf{a}_\tau}}. \tag{9.8}$$

The beliefs of type-τ voters conditional on (1) the signal σ they have observed and (2) on being pivotal are:

$$\beta_\tau^g = \frac{\widetilde{\beta}}{\widetilde{\beta} + (1-\widetilde{\beta})\frac{\varepsilon}{1-\varepsilon}} \tag{9.9}$$

$$\beta_\tau^b = \frac{\widetilde{\beta}}{\widetilde{\beta} + (1-\widetilde{\beta})\frac{1-\varepsilon}{\varepsilon}} \tag{9.10}$$

Then the Lemma follows immediately from (9.9), (9.10) and the derivations in Lemma 9.2. ■

Lemma 9.7. ***(Existence)*** *Fix any $\alpha \in (0,1)$. Then, there exists an electorate size M' such that for all $M > M'$ there is a unique symmetric responsive equilibrium of the one-class voting game with identical preferences.*

Proof of Lemma 9.7: Similar to (4.9), a symmetric responsive equilibrium is charactarized by the solution of $\frac{f_\tau}{g_\tau} = \frac{L}{H}$ in this case. Therefore, there does not exist a solution as long as one of the two inequalites (i) $\frac{f}{g} > \frac{L}{H}$ or (ii) $\frac{f}{g} < \frac{L}{H}$ holds for all ω. Note that

$$\frac{f}{g} = \left(\frac{\pi^h}{\pi^l}\right)^{\alpha M-1} \left(\frac{1-\pi^h}{1-\pi^l}\right)^{M(1-\alpha)} \frac{\frac{\partial \pi^h}{\partial \omega}}{\frac{\partial \pi^l}{\partial \omega}} \tag{9.11}$$

and

$$\lim_{\omega \longrightarrow 2} \left(\frac{1-\varepsilon+\varepsilon(\omega-1)}{\varepsilon+(1-\varepsilon)(\omega-1)}\right)^{\alpha M-1} \left(\frac{\varepsilon\omega}{(1-\varepsilon)\omega}\right)^{M(1-\alpha)} \frac{\frac{\partial \pi^h}{\partial \omega}}{\frac{\partial \pi^l}{\partial \omega}} \tag{9.12}$$

$$= \left(\frac{\varepsilon}{1-\varepsilon}\right)^{M(1-\alpha)} \frac{\frac{\partial \pi^h}{\partial \omega}}{\frac{\partial \pi^l}{\partial \omega}}. \tag{9.13}$$

Given that $\frac{\frac{\partial \pi^h}{\partial \omega}}{\frac{\partial \pi^l}{\partial \omega}}$ is constant (either $\frac{\varepsilon}{1-\varepsilon}$ or $\frac{1-\varepsilon}{\varepsilon}$) and $\frac{\varepsilon}{1-\varepsilon} < 1$, $\frac{f}{g}$ can get arbitrarily small for large M and large ω ($\omega \longrightarrow 2$). Similarly,

$$\lim_{\omega \longrightarrow 0} \frac{f}{g} = \left(\frac{1-\varepsilon}{\varepsilon}\right)^{\alpha M-1} \frac{\frac{\partial \pi^h}{\partial \omega}}{\frac{\partial \pi^l}{\partial \omega}}. \tag{9.14}$$

Thus, $\frac{f}{g}$ can get arbitrarily large for large M and small ω ($\omega \longrightarrow 0$). Therefore there exists M' large enough that neither (i) $\frac{f}{g} > \frac{L}{H}$ or (ii) $\frac{f}{g} < \frac{L}{H}$ holds implying the existence. Uniqueness follow immediately from Lemma 4. ■

Proof of Proposition 4.2: (i) Given the existence of symmetric responsive equilibrium in one-class voting with identical preferences from Lemma 9.7 all we need to show that the two best response functions $\omega_\tau^{**}(\omega_{\tau'})$ intersect. We first show that $\omega_\tau^{**}(\omega_{\tau'} = 2)$ is bounded away from zero. Note that $\lim_{\omega_\tau \longrightarrow 2} \frac{F_\tau}{G_\tau} = 1$. Therefore, $\lim_{\omega'_\tau \longrightarrow 2} \omega_\tau^{**}(\omega_{\tau'})$ is the solution to $\frac{f_\tau}{g_\tau} = \frac{L-A_\tau}{H+A_\tau}$. From the previous Lemma we know that there exists a M'_τ such that for all $M_\tau > M'_\tau$ the solution, $\omega_\tau^{**}(\omega_{\tau'} = 2)$, is bounded away from zero. Second, we show that $\omega_\tau^{**}(\omega_{\tau'} = 0) = 2$. From Lemma 9.1 and 9.2, we know that $\frac{f_\tau}{g_\tau} < \frac{F_\tau}{G_\tau}$ for all ω. Given that $\frac{f_\tau}{g_\tau}$ gets arbitrarily large for small ω and large M_τ, also $\frac{F_\tau}{G_\tau}$ gets arbitrarily large. Consequently, $\lim_{\omega'_\tau \longrightarrow 0} \omega_\tau^{**}(\omega_{\tau'})$ is the solution to $\frac{F_{\tau'}}{G_{\tau'}} \frac{f_\tau}{g_\tau} = \frac{L-A_\tau}{H+A_\tau}$, where $\frac{F_{\tau'}}{G_{\tau'}}$ is arbitrarily large. Therefore, $\omega_\tau^{**}(\omega_{\tau'} = 0) = 2$

since $\frac{f_\tau}{g_\tau}$ gets arbitrarily small for large ω and large M_τ. Thus, continuity of the best response functions will suffice. Note that $\frac{F_{\tau'}}{G_{\tau'}}$ is a continous function of $\omega_{\tau'}$. Therefore, the best reponse function $\omega_\tau^{**}(\omega_{\tau'})$ is a continous function. Therefore, there exists a $(\omega_J^{**}, \omega_S^{**}) \in (0,2) \times (0,2)$ such that two best response functions intersect.

Parts (ii) and (iii) follow immediately from the discussion in the text. ■

Proof of Proposition 4.3: (i) From Lemma 9.3 we know that $\frac{F_{\tau'}}{G_{\tau'}}$ decreases as we increase $\omega_{\tau'}$. Then, to satisfy the first order condition, $\frac{f_\tau}{g_\tau}$ must increase. From Lemma 9.4 this implies that ω_τ^{**} has to decrease. (ii) From Lemma 9.6 we know that $\frac{f_\tau}{g_\tau}$ increases as we increase $\mathbf{a}_\tau$. Keeping $\omega_{\tau'}$ constant impies $\frac{F_{\tau'}}{G_{\tau'}}$ remains constant so that to satisfy the first order condition $\frac{f_\tau}{g_\tau}$ must decrease. From Lemma 9.4 this implies that ω_τ^{**} has to increase. ■

Proof of Proposition 4.4: Given that $\omega_S^{**} = \omega_J^{**} = \omega^*$, we must have $\pi_S^s = \pi_J^s = \pi^s$. Therefore, multiplying the first order conditions for each of the two classes voting yields $\frac{F_J F_S f}{G_J G_S g} = \left(\frac{L}{H}\right)^2$ since $\frac{f_J}{g_J}\frac{f_S}{g_S} = \frac{f(\omega^*, \mathbf{a}_S + \mathbf{a}_J)}{g(\omega^*, \mathbf{a}_S + \mathbf{a}_J)}$ where $\frac{f}{g}$ is for one-class voting with $M_J + M_S$ voters. From Lemmas 9.1 and 9.2 we know that $\frac{f_\tau}{g_\tau} < \frac{F_\tau}{G_\tau}$ for all ω. Therefore, we must have $\frac{f(\omega^*, \mathbf{a}_S + \mathbf{a}_J)}{g(\omega^*, \mathbf{a}_S + \mathbf{a}_J)} < \frac{L}{H}$ since $\frac{F_J F_S}{G_J G_S} > \frac{f}{g}$. Therefore, to satisfy $\frac{f(\omega^*, \mathbf{a})}{g(\omega^*, \mathbf{a})} = \frac{L}{H}$ we have to increase $\frac{f}{g}$. Therefore from Lemma 9.6, for large enough M_J and M_S we must have a larger social choice rule, i.e., $\alpha^* < \alpha_J^{**} + \alpha_S^{**}$. ■

Proof of Proposition 5.3: For any $\mathbf{a} = (M_J + M_S)\alpha$, either (1) $\mathbf{a} > M_J$ or (2) $\mathbf{a} \leq M_J$ holds. We will show that in the first case type J, in the second case type $S-$voters will not vote responsively. Without loss of generality, assume that case (1) holds. If there is a conflict of interest, then the symmetry condition does not require that both types play the the same strategy. Therefore, we have to introduce new notation. Let $\rho_\tau^s = \sum_{i=\mathbf{a}-M_{\tau'}-1}^{M_\tau - 1} \rho_\tau^s(i)$ be the probability that a voter of type τ is pivotal in state s where

$$\rho_\tau^s(i) \quad = \quad \binom{M_\tau - 1}{i} [\pi_\tau^h(\omega_\tau)]^i [1 - \pi_\tau^h(\omega_\tau)]^{M_\tau - 1 - i}$$

$$\times \binom{M_{\tau'}}{i} [\pi^h_{\tau'}(\omega_{\tau'})]^{\mathbf{a}-i-1}[1-\pi^h_{\tau'}(\omega_{\tau'})]^{M_{\tau'}-\mathbf{a}+i+1}. \quad (9.15)$$

The first term of ρ^s_τ, $\rho^s_\tau(\mathbf{a}-M-1)$, is the probability of having all of type $\tau'-$voters and $\mathbf{a}-M-1$ of type τ voters voting 'yes'. Then, similar to Equation (4.9), the first order condition for a symmetric responsive equilibrium becomes $\frac{\rho^h_\tau}{\rho^l_\tau}K_\tau = \frac{L-A_\tau}{H+A_\tau}$ where K_τ is $\frac{\varepsilon}{1-\varepsilon}$ if $\omega_\tau > 1$ and $\frac{1-\varepsilon}{\varepsilon}$ if $\omega_\tau < 1$.

We start with part (ii). First, we show that if $\omega_\tau < \omega_{\tau'}$ then we have $\frac{\rho^h_\tau}{\rho^l_\tau} \leq \frac{\rho^h_{\tau'}}{\rho^l_{\tau'}} < (\frac{1-\varepsilon}{\varepsilon})\frac{\rho^h_\tau}{\rho^l_\tau}$. We start by comparing the ratio of the last terms of ρ^h_τ and ρ^l_τ with the ratio of the first term of $\rho^h_{\tau'}$ and $\rho^l_{\tau'}$, and so on, i.e., we compare $\frac{\rho^h_\tau(M_\tau-k)}{\rho^l_\tau(M_\tau-k)}$ with $\frac{\rho^h_{\tau'}(\mathbf{a}-M_{\tau'}-2+k)}{\rho^l_{\tau'}(\mathbf{a}-M_{\tau'}-2+k)}$ for all $k \in \{1, ..., M_J + M_S - \mathbf{a} + 1\}$.

$$\frac{\rho^h_\tau(M_\tau-k)}{\rho^l_\tau(M_\tau-k)} = \frac{[\pi^h_\tau(\omega_\tau)]^{M_\tau-k}[1-\pi^h_\tau(\omega_\tau)]^{k-1}[\pi^h_{\tau'}(\omega_{\tau'})]^{\mathbf{a}-M_{\tau'}+k-1}[1-\pi^h_{\tau'}(\omega_{\tau'})]^{M_\tau+M_{\tau'}-\mathbf{a}+1-k}}{[\pi^l_\tau(\omega_\tau)]^{M-k}[1-\pi^l_\tau(\omega_\tau)]^{k-1}[\pi^l_{\tau'}(\omega_{\tau'})]^{\mathbf{a}-M_{\tau'}+k-1}[1-\pi^l_{\tau'}(\omega_{\tau'})]^{M_\tau+M_{\tau'}-\mathbf{a}+1-k}}, \quad (9.16)$$

$$\frac{\rho^h_{\tau'}(\mathbf{a}-M_{\tau'}-2+k)}{\rho^l_{\tau'}(\mathbf{a}-M_{\tau'}-2+k)} = \frac{[\pi^h_\tau(\omega_\tau)]^{M_\tau-k+1}[1-\pi^h_\tau(\omega_\tau)]^{k-1}[\pi^h_{\tau'}(\omega_{\tau'})]^{\mathbf{a}-M_{\tau'}-2+k}[1-\pi^h_{\tau'}(\omega_{\tau'})]^{M_\tau+M_{\tau'}-\mathbf{a}+1-k}}{[\pi^l_\tau(\omega_\tau)]^{M_\tau-k+1}[1-\pi^l_\tau(\omega_\tau)]^{k-1}[\pi^l_{\tau'}(\omega_{\tau'})]^{\mathbf{a}-M_{\tau'}-2+k}[1-\pi^l_{\tau'}(\omega_{\tau'})]^{M_\tau+M_{\tau'}-\mathbf{a}+1-k}} \quad (9.17)$$

Therefore,

$$\frac{\frac{\rho^h_\tau(M_\tau-k)}{\rho^l_\tau(M_\tau-k)}}{\frac{\rho^h_{\tau'}(a-M_{\tau'}-2+k)}{\rho^l_{\tau'}(a-M_{\tau'}-2+k)}} = \frac{\pi^l_\tau(\omega_\tau)\pi^h_{\tau'}(\omega_{\tau'})}{\pi^h_\tau(\omega_\tau)\pi^l_{\tau'}(\omega_{\tau'})}. \quad (9.18)$$

If $\omega_\tau, \omega_{\tau'} \leq 1$ then $\frac{\pi^h_\tau(\omega_\tau)}{\pi^l_\tau(\omega_\tau)} = \frac{\pi^h_{\tau'}(\omega_{\tau'})}{\pi^l_{\tau'}(\omega_{\tau'})}$ so that

$$\frac{\frac{\rho^h_\tau(M_\tau-k)}{\rho^l_\tau(M_\tau-k)}}{\frac{\rho^h_{\tau'}(a-M_{\tau'}-2+k)}{\rho^l_{\tau'}(a-M_{\tau'}-2+k)}} = 1. \quad (9.19)$$

If $\omega_{\tau'} > \max\{1, \omega_\tau\}$ then, $\frac{\pi^h_\tau(\omega_\tau)}{\pi^l_\tau(\omega_\tau)} \in \left(\frac{\pi^h_{\tau'}(\omega_{\tau'})}{\pi^l_{\tau'}(\omega_{\tau'})}, \left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{\pi^h_{\tau'}(\omega_{\tau'})}{\pi^l_{\tau'}(\omega_{\tau'})}\right)$ so that

$$\frac{\frac{\rho^h_{\tau'}(a-M_{\tau'}-2+k)}{\rho^l_{\tau'}(a-M_{\tau'}-2+k)}}{\frac{\rho^h_\tau(M_\tau-k)}{\rho^l_\tau(M_\tau-k)}} \in \left(1, \frac{1-\varepsilon}{\varepsilon}\right). \tag{9.20}$$

Thus, $\frac{\rho^h_{\tau'}}{\rho^l_{\tau'}} \in \left[\frac{\rho^h_\tau}{\rho^l_\tau}, \left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{\rho^h_\tau}{\rho^l_\tau}\right)$ if $\omega_\tau < \omega_{\tau'}$.

Next, we argue that we cannot have $\omega_J < \omega_S$ in equilibrium. If $\omega_J < \omega_S \leq 1$ or $1 \leq \omega_J < \omega_S$, then $\frac{\rho^h_S}{\rho^l_S}K_S \leq \frac{\rho^h_J}{\rho^l_J}K_J$. Similarly, if $\omega_J < 1 < \omega_S$, then $\frac{\rho^h_S}{\rho^l_S}K_S < \frac{\rho^h_J}{\rho^l_J}K_J$. However, neither can hold in equilibrium since $\frac{L-A_S}{H+A_S} > \frac{L-A_J}{H+A_J}$. Therefore, we must have $\omega_J \geqslant \omega_S$ in equilibrium. Hence, we have $\frac{\rho^h_S}{\rho^l_S} \leq \frac{\rho^h_J}{\rho^l_J}$ in equilibrium. The ratio between the left hand side of the first order condition is bounded,

$$\frac{\frac{\rho^h_S}{\rho^l_S}K_S}{\frac{\rho^h_J}{\rho^l_J}K_J} \leq \left(\frac{1-\varepsilon}{\varepsilon}\right)^2. \tag{9.21}$$

However, if the conflict of interests is large enough than the ratio between the right hand side of the first order condition, $\frac{L-A_S}{H+A_S} / \frac{L-A_J}{H+A_J}$, can be larger than $\left(\frac{1-\varepsilon}{\varepsilon}\right)^2$. Note that $\frac{L-A_S}{H+A_S} / \frac{L-A_J}{H+A_J}$ is an increasing function of A_J (recall that $A_S = -\frac{M_J}{M_S}A_J$). Therefore, there exists $\overline{A_J} > 0$ so that for all $A_J > \overline{A_J}$ at most one of the first order conditions can hold. Thus, one type of voters vote non-responsively.

For the second half of part (ii) we will start by showing that $\frac{\rho^h_\tau}{\rho^l_\tau}$ is a decreasing function of ω_τ. Note that $\frac{\rho^h_\tau(i)}{\rho^l_\tau(i)}$ can be simplified as

$$\frac{[\pi^h_\tau(\omega_\tau)]^i[1-\pi^h_\tau(\omega_\tau)]^{M_\tau-1-i}}{[\pi^l_\tau(\omega_\tau)]^i[1-\pi^l_\tau(\omega_\tau)]^{M_\tau-1-i}}\Phi(\pi^h_{\tau'}, \pi^l_{\tau'}). \tag{9.22}$$

Recall that $\frac{[\pi^h_\tau(\omega_\tau)]^i[1-\pi^h_\tau(\omega_\tau)]^{M_\tau-1-i}}{[\pi^l_\tau(\omega_\tau)]^i[1-\pi^l_\tau(\omega_\tau)]^{M_\tau-1-i}}$ is $\frac{f_\tau(\mathbf{a}=i+1)}{g_\tau(\mathbf{a}=i+1)}$. From Lemma 9.4 we know that this term is a decreasing function of ω_τ. Therefore, $\frac{\rho^h_\tau(i)}{\rho^l_\tau(i)}$ must be a decreasing function of ω_τ. In other words, we have $\eta^h_\tau(i)\rho^l_\tau(i) - \eta^l_\tau(i)\rho^h_\tau(i) < 0$ or equivalently, $\frac{\eta^h_\tau(i)}{\eta^l_\tau(i)} < \frac{\rho^h_\tau(i)}{\rho^l_\tau(i)}$ where $\eta^s_\tau(i)$ is the partial derivative of $\rho^l_\tau(i)$ with respect to ω_τ. Since this holds for all i then we have $\frac{\sum_i \eta_\tau^h(i)}{\sum_i \eta_\tau^l(i)} < \frac{\sum_i \rho_\tau^h(i)}{\sum_i \rho_\tau^l(i)}$. Therefore, $\frac{\rho_\tau^h}{\rho_\tau^l}$ is a decreasing function of ω_τ.[24] We know that whenever the conflict of interest is large enough, then at least one type of voters votes non-responsively. Assume that there will be an equilibrium with type $J-$voters voting non-responsively. In other words, the first order condition never holds for type $J-$voters and thus their best response will be $\omega_J = 2$. Note that $\lim_{\omega_J \to 2} \frac{\rho_S^h}{\rho_S^l} K_S = \frac{f_S(\mathbf{a}_S = \mathbf{a} - M_J)}{g_S(\mathbf{a}_S = \mathbf{a} - M_J)}$. Recall that $\frac{f_S(\mathbf{a}_S = \mathbf{a} - M_J)}{g_S(\mathbf{a}_S = \mathbf{a} - M_J)} = \frac{L - A_S}{H + A_S}$ is the equilibrium condition for one-class voting with identical preferences when only type $S-$voters vote under $\mathbf{a}_S = \mathbf{a} - M_J$. Therefore, type $S-$voters' best response to $\omega_J = 2$ is ω_S^*, the equilibrium strategy in one-class voting with identical preferences when only type $S-$voters vote under $\mathbf{a}_S = \mathbf{a} - M_J$. Hence our assumption must be correct. Note that type $S-$voters voting non-responsively ($\omega_S = 0$) and type $J-$voters voting responsively cannot be an equilibrium, since $\lim_{\omega_S \to 0} \frac{\rho_J^h}{\rho_J^l} K_J > 1$ for all ω_J. Furthermore, uniqueness follows from Lemma 9.7. This concludes part (ii). For part (i) it suffices to replicate the proof of Lemma 9.7 for $\frac{\rho_\tau^h}{\rho_\tau^l}$ instead of $\frac{f_\tau}{g_\tau}$. ■

Proof of Theorem 5.7: In the symmetric case $\frac{L - A_S}{H + A_S} / \frac{L - A_J}{H + A_J} > \left(\frac{1-\varepsilon}{\varepsilon}\right)^2$ is simplified to $A > L(1 - 2\varepsilon)$. Therefore, if $A > L(1 - 2\varepsilon)$, then one type of voters vote non-responsively according to proposition 5.3. Without loss of generality let us assume that type $J-$voters vote non-responsively in a one-class voting game, i.e., type $J-$voters vote for the proposal independently of their signal. Consequently, using one-class voting with $\mathbf{a}$ as the social choice rule is equivalent to having only type $S-$voters vote on the proposal with the $\mathbf{a}_S = \mathbf{a} - M$ as the social choice rule. Therefore, the expected social loss (per capita) in a one-class voting game is equal to $\frac{L}{2}[(1 - F_S) + G_S]$, where F_S and G_S are calculated for $\mathbf{a}_S = \mathbf{a} - M$ and $\omega_S = \omega_S^*$.[25]

[24] Similarly, $\frac{\rho_\tau^h}{\rho_\tau^l}$ is a decreasing function of $\omega_{\tau'}$.

[25] Obviously, the expected loss (per capita) for type $S-$voters is $\frac{1}{2}[(1 - F_S)(L - A) + G_S(L + A)]$. However, we care about the entire social loss rather than the type $S-$voters only.

Note that the equilibrium strategy for type $S-$voters ω_S^* is:

$$\omega_S^* = \arg\max \frac{1}{2}[F_S(L-A) + (1-G_S)(L+A)] \tag{9.23}$$

so that first order condition becomes $\frac{f_S}{g_S} = \frac{L-A}{L+A}$. Next, we will show that the same expected social loss level can be achieved letting only type $J-$voters vote under $\mathbf{a}_J = M - \mathbf{a}_S + 1$. The equilibrium strategy of type $J-$voters ω_J^* is:

$$\omega_J^* = \arg\max \frac{1}{2}[F_J(L+A) + (1-G_J)(L-A)]. \tag{9.24}$$

Using $\mathbf{a}_J = M - \mathbf{a}_S + 1$, G_J becomes $\sum_{i=M-\mathbf{a}_S+1}^{M_\tau} \binom{M}{i} [\pi_J^l(\omega_J)]^i [1-\pi_J^l(\omega_J)]^{M-i}$. Note that $\sum_{i=1}^{M_\tau} \binom{M}{i} [\pi_J^l(\omega_J)]^i [1-\pi_J^l(\omega_J)]^{M-i} = 1$. Therefore $1 - G_J$ is equal to $\sum_{i=1}^{M-\mathbf{a}_S} \binom{M}{i} [\pi_J^l(\omega_J)]^i [1-\pi_J^l(\omega_J)]^{M-i}$. Next we would like to note that, if you set $\omega_J = 2 - \omega_S$, π_J^l is equivalent to $1 - \pi_S^h$, i.e., $\pi_J^l(\omega_J = 2-\omega_S) = 1 - \pi_S^h(\omega_S)$. Therefore, $1 - G_J(\omega_J = 2-\omega_S, \mathbf{a}_J = M - \mathbf{a}_S + 1) = F_S(\omega_S, \mathbf{a}_S)$. Similarly, we have $F_J(\omega_J = 2-\omega_S, \mathbf{a}_J = M-\mathbf{a}_S+1) = 1 - G_S(\omega_S, \mathbf{a}_S)$. Hence $\omega_J = 2-\omega_S^*$ must be the solution to equation (9.24) since ω_S^* is the solution to equation (9.23). Furthermore, the expected social loss letting only type $J-$voters vote under $\mathbf{a}_J = M - \mathbf{a}_S + 1$,

$$\frac{L}{2}[(1 - F_J(\omega_J^*, \mathbf{a}_J = M - \mathbf{a}_S + 1)) + G_J(\omega_J^*, \mathbf{a}_J = M - \mathbf{a}_S + 1)], \tag{9.25}$$

is equal to the that of letting only type $S-$voters vote,

$$\frac{L}{2}[(1 - F_S(\omega_S^*, \mathbf{a}_S)) + G_S(\omega_S^*, \mathbf{a}_S)]. \tag{9.26}$$

Next, as a benchmark we would like to calculate the expected social loss for two-class voting with $(\mathbf{a}_S, \mathbf{a}_J = M - \mathbf{a}_S + 1)$ as the social choice rules when each class of creditors votes as if their class is the only class that votes. Let $\mathcal{K}$ denote this benchmark loss:

$$\mathcal{K} \equiv \frac{L}{2}\left[(1 - F_S(\omega_S^*, \mathbf{a}_S) F_J(\omega_J^*, \mathbf{a}_J = M - \mathbf{a}_S + 1))\right. \tag{9.27}$$

$$\left. + G_S(\omega_S^*, \mathbf{a}_S) G_J(\omega_J^*, \mathbf{a}_J = M - \mathbf{a}_S + 1)\right] \tag{9.28}$$

$$= \frac{L}{2}\left[(1 - F_S(\omega_S^*, \mathbf{a}_S)) + G_S(\omega_S^*, \mathbf{a}_S)\right]. \tag{9.29}$$

Recall that this expression is equal to 9.26, the expected social loss (per capita) in the equilibrium of the one-class voting game with $\mathbf{a} = M + \mathbf{a}_S$ with type $J-$voters voting for the proposal independently of their signals. Therefore, all we are left to show that there exists social choice rules $(\mathbf{a}_S^{**}, \mathbf{a}_J^{**})$ such that two-class voting results in an expected social loss strictly less than $\mathcal{K}$.

Note that the first order condition for type J voters in a two-class voting game is

$$\frac{F_S f_J}{G_S g_J} = \frac{L+A}{L-A} \tag{9.30}$$

as opposed to $\frac{f_J}{g_J} = \frac{L+A}{L-A}$, the first order condition for one-class voting. Given that $\frac{f_J}{g_J}$ is a decreasing function of ω_J and $\frac{F_S}{G_S} > 1$, the solution to 9.30, ω_J^{**}, must be larger than ω_J^* if we use the same majority rule $\mathbf{a}_J = M - \mathbf{a}_S + 1$. From Lemma 9.6 and Proposition **4.3**, the social choice rule, $\mathbf{a}_J^{**}$, that will induce $\omega_J^{**} \leq \omega_J^*$ must be less than $M - \mathbf{a}_S + 1$. With identical arguments to Lemma 9.7, there exists a M large enough so that there exists an α_J^{**} and hence $\mathbf{a}_J^{**} < \mathbf{a}_J$ yielding $\omega_J^{**} = \omega_J^*$.[26] Therefore, we have

$$\frac{G_J(\omega_J^{**}, \mathbf{a}_J^{**}) - G_J(\omega_J^*, \mathbf{a}_J)}{F_J(\omega_J^{**}, \mathbf{a}_J^{**}) - F_J(\omega_J^*, \mathbf{a}_J)} \equiv \frac{\Delta G_J}{\Delta F_J} = \frac{\sum_{i=\mathbf{a}_J^{**}}^{\mathbf{a}_J-1} G_J^i(\omega_J^*)}{\sum_{i=\mathbf{a}_J^{**}}^{\mathbf{a}_J-1} F_J^i(\omega_J^*)} \equiv \frac{\Delta G_J(\omega_J^*)}{\Delta F_J(\omega_J^*)}. \tag{9.31}$$

Given that $1 - G_J(\omega_J^*, \mathbf{a}_J) = F_S(\omega_S^*, \mathbf{a}_S)$ and $F_J(\omega_J^*, \mathbf{a}_J) = 1 - G_S(\omega_S^*, \mathbf{a}_S)$, we have

$$\frac{\Delta G_J(\omega_J^*)}{\Delta F_J(\omega_J^*)} = \frac{\sum_{i=\mathbf{a}_S+1}^{M-\mathbf{a}_J^{**}+1} F_S^i(\omega_S^*)}{\sum_{i=\mathbf{a}_S+1}^{M-\mathbf{a}_J^{**}+1} G_S^i(\omega_S^*)} \equiv \frac{\Delta F_S(\omega_S^*)}{\Delta G_S(\omega_S^*)}. \tag{9.32}$$

From Lemma 9.1, $\frac{\Delta F_S(\omega_S^*)}{\Delta G_S(\omega_S^*)} < \frac{F_S(\omega_S^*)}{G_S(\omega_S^*)}$, therefore we have $\frac{\Delta G_J}{\Delta F_J} < \frac{F_S(\omega_S^*)}{G_S(\omega_S^*)}$. Arranging terms yields

$$G_S(\omega_S^*)\Delta G_J < F_S(\omega_S^*)\Delta F_J\ . \tag{9.33}$$

[26] Assuming $\alpha_J^{**} M = \mathbf{a}_J^{**}$ is an integer.

On the other hand, the exptected loss for type $S-$voters in a two-class voting with $(\mathbf{a}_S^{**} = \mathbf{a}_S, \mathbf{a}_J^{**})$ is:

$$[\beta_J(\omega_J^{**})(1 - F_S(\omega_S^{**}))(L - A) + (1 - \beta_J(\omega_J^{**}))G_S(\omega_S^{**})(L + A)]. \tag{9.34}$$

This expression must be less than or equal to

$$[\beta_J(\omega_J^{**})(1 - F_S(\omega_S^{*}))(L - A) + (1 - \beta_J(\omega_J^{**}))G_S(\omega_S^{*})(L + A)], \tag{9.35}$$

since otherwise each senior voter will deviate to ω_S^*. Therefore we have

$$(1 - \beta_J(\omega_J^{**}))\Delta G_S(L + A) \leq \beta_J(\omega_J^{**})\Delta F_S(L - A), \tag{9.36}$$

where $\Delta F_S = F_S(\omega_S^{**}) - F_S(\omega_S^*) > 0$ and $\Delta G_S = G_S(\omega_S^{**}) - G_S(\omega_S^*) > 0$. Arranging and simplifying results in

$$\Delta G_S \leq \frac{F_J(\omega_J^{**})}{G_J(\omega_J^{**})}\Delta F_S\frac{(L - A)}{(L + A)}. \tag{9.37}$$

Substituing $F_J(\omega_J^{**}, \mathbf{a}_J^{**}) = F_J(\omega_J^*, \mathbf{a}_J) + \Delta F_J$ and $G_J(\omega_J^{**}, \mathbf{a}_J^{**}) = G_J(\omega_J^*, \mathbf{a}_J) + \Delta G_J$ we have

$$\Delta G_S \leq \frac{F_J(\omega_J^*) + \Delta F_J}{G_J(\omega_J^*) + \Delta G_J}\Delta F_S\frac{(L - A)}{(L + A)}. \tag{9.38}$$

Given that $\frac{(L-A)}{(L+A)} < 1$ we have $\Delta G_S < \frac{F_J(\omega_J^*)+\Delta F_J}{G_J(\omega_J^*)+\Delta G_J}\Delta F_S$. Thus, we have

$$\Delta G_S G_J(\omega_J^*) + \Delta G_J \Delta G_S - (F_J(\omega_J^*)\Delta F_S + \Delta F_J \Delta F_S) < 0. \tag{9.39}$$

Adding this equation to equation 9.33, and adding $1 - F_S(\omega_S^*)F_J(\omega_J^*) + G_S(\omega_S^*)G_J(\omega_J^*)$ to both sides yield

$$(1 - F_S(\omega_S^{**})F_J(\omega_J^{**})) + G_S(\omega_S^{**})G_J(\omega_J^{**}) < (1 - F_S(\omega_S^*)F_J(\omega_J^*)) + G_S(\omega_S^*)G_J(\omega_J^*). \tag{9.40}$$

Multiplying both sides by $\frac{L}{2}$ concludes the proof, since right hand side is equal to $\mathcal{K}$ and the left hand side is the expected social loss in two-class voting. ■

Lemma 9.8. *In all symmetric equilibria in which both types vote responsively we have that (i) either* $\beta_\tau^g > \beta_{0\tau} > \beta_\tau^b$ *and all voters of type* τ *play pure strategies* ($\omega_\tau^g = 1, \omega_\tau^b = 0$)*, or (ii)* $\beta_\tau^g = \beta_{0\tau}$ *and* $\omega_\tau^g < 1, \omega_\tau^b = 0$*, or (iii)* $\beta_\tau^b = \beta_{0\tau}$ *and* $\omega_\tau^g = 1$*,* $\omega_\tau^b > 0$*. There is no equilibrium where* $0 < \omega_\tau^g < 1$ *and* $0 < \omega_\tau^b < 1$*.*

Proof of Lemma 9.8:

Let $\widetilde{\beta}$ stand for the beliefs of a voter (i.e., probability of being in state h) conditional on being pivotal. Then, conditional on observing a good or bad signal, a voter of type τ has beliefs:

$$\beta_\tau^g = \frac{\widetilde{\beta}}{\widetilde{\beta} + (1-\widetilde{\beta})\frac{\varepsilon}{1-\varepsilon}} \tag{9.41}$$

$$\beta_\tau^b = \frac{\widetilde{\beta}}{\widetilde{\beta} + (1-\widetilde{\beta})\frac{1-\varepsilon}{\varepsilon}} \tag{9.42}$$

Note that $\beta_\tau^g > \beta_\tau^b$, since $\varepsilon < \frac{1}{2}$. Therefore, a voter can be indifferent between voting for and against a proposal with at most one signal, good or bad. ■

References

[1] Austen-Smith, David, and Jeffrey S. Banks, 1996, Information Aggregation, Rationality, and the Condorcet Jury Theorem, American Political Science Review 90, pp. 34-45

[2] Chwe, Michael Suk-Young, 1999, Minority Voting Rights Can Maximize Majority Welfare, American Political Science Review 93, no. 1 (March), pp. 85-97

[3] Coughlan, Peter J., 2000 , In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting, American Political Science Review 94, no. 2 (June), pp. 375-393

[4] Dekel, Eddie, and Michele Piccione, 1999, On the Equivalence of Simultaneous and Sequential Binary Elections, Journal of Political Economy, December

[5] DeMarzo, Peter, 1993, Majority Voting and Corporate Control: The Rule of the Dominant Shareholder, Review of Economic Studies 60, no. 204 (July), pp. 713-734

[6] Diermeier, Daniel, and Roger Myerson, 1999, Bicameralism and Its Consequences for the Internal Organization of Legislatures, American Economic Review 89, no. 5 (December), pp. 1182-1196

[7] Easterbrook, Frank H., and Daniel R. Fischel, 1983, Voting in Corporate Law, Journal of Law and Economics 26, no. 2 (June), pp. 395-427

[8] Feddersen, Timothy J., and Wolfgang Pesendorfer, 1996, The Swing Voter's Curse, American Economic Review 86, no. 3 (June), pp. 408-424

[9] ——— , 1996, Two Alternative Elections With Private and Common Values, Mimeo, Northwestern University, (October)

[10] ——— , 1997, Voting Behavior and Information Aggregation in Elections With Private Information, Econometrica 65, no. 5 (September), pp. 1029-1058

[11] ——— , 1998, Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts, American Political Science Review 92, no. 2 (March), pp. 23-35

[12] Klevorick, Alvin K.; Michael Rothschild, and Christopher Winship, 1984, Information Processing and Jury Decision Making, Journal of Public Economics 23, no. 3 (April), pp. 245-278

[13] Ladha, Krishna K., 1992, The Condorcet Jury Theorem, Free Speech, and Correlated Votes, American Journal of Political Science 36, no. 3 (August), pp. 617-634

[14] Maug, Ernst, 1999, How Effective Is Proxy Voting? Information Aggregation and Conflict Resolution in Corporate Voting Contests, Working Paper, Humboldt-Universität Zu Berlin,

[15] McLennan, Andrew, 1998, Consequences of the Condorcet Jury Theorem for Beneficial Information Aggregation by Rational Agents, American Political Science Review 92, (June), pp. 413-418

[16] Myerson, Roger B., 1998, Extended Poisson Games and The Condorcet Jury Theorem, Games and Economic Behavior 25, pp. 111-131

[17] ——— , 1998, Population Uncertainty and Poisson Games, International Journal of Game Theory 27, pp. 375-392

[18] ——— , 1999, Large Poisson Games, Forthcoming, Journal of Economic Theory

[19] Tsebelis, George, and George Money, 1997, Bicameralism, Cambridge (Cambridge University Press)

[20] Wolinsky, Asher, 2000, Eliciting Information From Multiple Experts, Mimeo, Northwestern University, (January)

[21] Yilmaz, Bilge, 1998, Strategic Voting and Proxy Contests, Mimeo, University of Pennsylvania, (November)

[22] Young, H. Peyton, 1988, Condorcet's Theory of Voting, American Political Science Review 82, no. 4 (December), pp. 1231-1244