CHOICE BY SEQUENTIAL PROCEDURES*

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Abstract. In the model of choice by sequential procedures, the successive application of criteria gradually reduces the set of alternatives to a unique element, which is the one actually chosen. We offer a simple property, Independence of One Irrelevant Alternative, and show it is equivalent to choice by sequential procedures. Our property is instrumental for the understanding of choice by sequential procedures, and provides a novel tool with which to study how other behavioral concepts relate to it. We show that the notions of rationalizability by game trees, agenda rationalizability, and choice functions exhibiting a status-quo bias are special cases of choice by sequential procedures.

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1. Introduction

In the model of choice by sequential procedures, a decision-maker (DM) faced with a choice problem, applies a number of criteria in a fixed order of priority, gradually narrowing down the set of alternatives until one is identified as the choice (see Manzini and Mariotti 2007; see also Tversky 1972). The model of choice by sequential procedures is very appealing from a behavioral perspective. It consists of a method that guides the DM through the complex problem of choice. As such, it has attracted a great deal of attention from the psychology, marketing, and management literatures.[1]

For the sake of illustration, imagine that our DM is about to buy a house. She may handle the complexity of the choice problem by first screening out some houses on the basis of location. Although the DM may not have formed a complete ranking over all possible locations, she may prefer some locations over others, and use this criterion to eliminate some houses. Next, the DM may care about the layout of the house, and have some idea of what kinds of layout she dislikes. Hence, from the surviving set of houses, the DM eliminates a

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[1]See, e.g., Tversky 1972, Manrai and Sinha 1989, and Kohli and Jedidi 2007.

further number of houses based on their layout. The DM proceeds in this way, applying additional relevant criteria and gradually shrinking the menu of alternatives, before finally making her choice from the surviving set of alternatives on the basis of price, for example.

This sequential process does not necessarily satisfy the standard axioms of choice, and hence is able to accommodate decisions that are incompatible with the classical model of rational choice. For example, let the set of houses be $\{x, y, z\}$. Suppose that, in terms of location, the only clear comparison for our DM is that z is superior to y; that, in terms of layout, she clearly prefers x to z; and finally that the cheapest house is y. If our DM first applies the location criterion, and follows it with the layout criterion and finally the price criterion, the choices we would observe are y from $\{x, y\}$, z from $\{y, z\}$, and x from $\{x, z\}$ and from $\{x, y, z\}$. The choices of our house buyer are cyclic, and hence cannot be the outcome of the maximization of any preference relation.[2] Therefore, although the choices are perfectly compatible with choice by sequential procedures, they cannot be accommodated within the model of rational choice.

An immediate question is whether all sorts of choice patterns can be accommodated by the sequential application of criteria. A simple example shows that this is in fact not the case. Suppose that we observe that our DM chooses house x from $\{x, y\}$ and from $\{x, z\}$, but chooses house y from $\{x, y, z\}$. It is impossible to reconcile this behavior with choice by sequential procedures. Whatever criterion the DM entertains first, it will lead to a contradiction. That is, the first criterion cannot eliminate house x from $\{x, y, z\}$, because this would imply eliminating x from $\{x, y\}$ or $\{x, z\}$, which is the actual choice from these menus. It obviously cannot eliminate house y from $\{x, y, z\}$. Therefore, the first criterion must eliminate only house z from $\{x, y, z\}$. But then, the choices of house x from $\{x, y\}$ and of house y from $\{x, y, z\}$, which has been reduced to $\{x, y\}$, are incompatible.[3]

It is therefore the case that choice by sequential procedures is able to explain some deviations from the rational model, but not all. For choice by sequential procedures to be a meaningful model of choice, it is necessary to understand what sorts of choice patterns can be rationalized by the sequential application of criteria. Manzini and Mariotti (2007) took some very important first steps in this direction by precisely delimiting the types of

[2] Classic papers documenting systematic cyclic choice behavior include May (1954), Grether and Plott (1979), and Loomes, Starmer and Sugden (1991).

[3] Kalai, Rubinstein, and Spiegler (2002) propose a model where the DM partitions the set of choice problems into different categories and applies one criterion to each category. Clearly, every possible choice pattern can be explained by their model.

revealed choices that are consistent with the application of two or three criteria.[4] However, the general question was left unanswered.

The former wonders about what deviations from the rational model can be explained by choice by sequential procedures. Another crucial question is how choice by sequential procedures relates to other prominent choice models. Given the growing number of alternative choice models that are flourishing in the literature, it is of great importance to impose some structure on the relevant models.

In this paper we offer a property, Independence of One Irrelevant Alternative (IOIA), that addresses the two questions posed above. IOIA can be described as follows. First, from the menu of alternatives A, the DM pinpoints a single pair, which she identifies by following a rule that is totally consistent across menus of alternatives. There are several behavioral interpretations of this rule. For example, the DM may be more used to comparing some pairs of alternatives than others, and may therefore decide to focus on the most familiar pair in every choice problem with which she is faced. Relatedly, pairs of alternatives may differ in terms of the similarity of the alternatives. So, for any choice problem A, the DM may focus on the pair with the most (or the least) similar alternatives. The DM may also perceive the alternatives in a particular physical order and observe that order when identifying the first two alternatives in a choice problem A. Then, having identified a single pair of alternatives from the menu, IOIA requires that the choice from A should not depend on the dominated member of that pair. Thus, IOIA has two parts. It is the result of the interplay of a fully consistent component, the rule identifying the binary problems, and a potentially inconsistent component, the choices in the binary problems.

IOIA delimits what kinds of deviation from the rational model can be explained by the model of choice by sequential procedures. In our first result we show that IOIA is equivalent to the notion of choice by sequential procedures, *regardless of the number of rationales required*. IOIA provides a tool that is easy to use in applications to determine whether observed behavior is or is not consistent with choice by sequential procedures. Moreover, IOIA helps to explain which kinds of deviation from the rational model are compatible with choice by sequential procedures. Interestingly, the fully consistent part of IOIA, that is, the rule identifying the binary problems, is the mechanism through which irrational binary choices spread over choice problems.

Furthermore, IOIA serves as an instrument to link choice by sequential procedures with other prominent choice models in the literature. In particular, we show that the notion of rationalizability by game trees, due to Xu and Zhou (2007), which, in principle, had little to

[4] Salant and Rubinstein (2008) also delimit the case of two rationales, within the framework of a 'limited attention' model.

do with choice by sequential procedures, is a special case of the latter. We also show that agenda rationalizability, a rationalizability notion that is introduced here and is rooted in certain models of choice by ordered elimination, as well as in voting mechanisms based on successive elimination, is also a strict refinement of choice by sequential procedures. Finally, we show that choice functions exhibiting the well-known status-quo bias also are contained in the model of choice by sequential procedures.

The paper is organized as follows. Section 2 introduces the notation and the main definitions to be used thereafter. Section 3 presents our property IOIA and shows that it is equivalent to choice by sequential procedures. Section 4 contains three applications of our property IOIA. Section 5 concludes. The proofs are collected in Appendix A. Appendix B establishes the formal relation between our model of choice by sequential procedures, and other prominent models.

2. Basic Notation and Definitions

Let X be a finite set of $n \geq 2$ objects. A choice function c assigns to every non-empty $A \subseteq X$ a unique element $c(A) \in A$.

Denote by $\mathcal{B}(X)$ the collection of 2-alternative sets (binary problems). A binary selector f is a function that for every choice problem A with at least two alternatives, gives a binary problem in A, i.e. $f(A) \in \mathcal{B}(X)$ with $f(A) \subseteq A$. Given a binary selector f, the direct revealed relation S on $\mathcal{B}(X)$ is defined by $(B_1, B_2) \in S$ if and only if there is a choice problem A such that $B_1 = f(A)$ and $B_2 \subseteq A$. Denote the transitive closure of S by $\overline{S}$. We say that the binary selector f is *consistent* if and only if, for all A, and for all $B_1, B_2 \subseteq A$, $B_1 \neq B_2$, $B_1 \overline{S} B_2$ implies that $B_2 \neq f(A)$. The latter is simply the Strong Axiom over the space of binary problems, where a choice problem A is understood as the collection of binary problems from which it is composed.

Denote by P an acyclic binary relation on X. That is, for any collection $x_1, \ldots, x_r \in X$, with $r > 1$, whenever $(x_i, x_{i+1}) \in P$ for all $i = 1, \ldots, r-1$, it is not true that $(x_r, x_1) \in P$. We will often refer to P as a *rationale*. For any A, $M(A, P)$ refers to the set of maximal elements in A with respect to P, that is, $M(A, P) = \{x \in A : (y, x) \in P \text{ for no } y \in A\}$. Let $M(\emptyset, P)$ be equal to $\emptyset$.

In the classical approach a choice function c is said to be rationalizable if there is a rationale P such that, for any choice problem A, $c(A) = M(A, P)$. Let $M_i^j(A)$ with $i \leq j$ denote the set $M_i^j(A) = M(M(\ldots M(M(A, P_i), P_{i+1}), \ldots, P_{j-1}), P_j)$. That is $M_i^j(A)$ is the set of alternatives surviving from A the sequential application of rationales $P_i, P_{i+1}, \ldots, P_{j-1}, P_j$.

Choice by Sequential Procedures: A choice function c is a choice by sequential procedures whenever there exists a non-empty ordered list $\{P_1, \dots, P_K\}$ of rationales on X such that $c(A) = M_1^K(A)$ for all $A \subseteq X$. In that case we say that $\{P_1, \dots, P_K\}$ sequentially rationalizes c.

This notion of choice by sequential procedures largely follows Manzini and Mariotti (2007). In their original formulation, the only structure that is imposed on the rationales applied by the DM is asymmetry, hence allowing for cycles in the rationales. We believe that the essence of choice by sequential procedures lies in the incompleteness of the criteria, which reflects the crudeness of the binary relations sequentially applied by the DM in order to reach a choice. The use of cyclic criteria may be controversial from a behavioral perspective, since they may appear unrealistic. It seems natural, therefore, to study possibly cyclic choice behavior arising from the sequential application of acyclic binary relations. This is the position we adopt here. In Appendix B we clarify the relation between these two models of sequential choice.[5]

3. Choice by Sequential Procedures and Independence of One Irrelevant Alternative

The classic notion of rationalizability of a choice function c deals with whether or not there exists a rationale that explains choice behavior as the result of maximization. A well-known result establishes that a choice function c is rationalizable if and only if c satisfies Independence of Irrelevant Alternatives (IIA). IIA is a consistency property that forbids any menu-dependency. It states that if an element x is chosen from a set A, it should also be chosen from any subset of A in which x is present.

IIA can be equivalently described as follows. Suppose that, given a menu of alternatives A, the DM pinpoints any binary problem in A, and discards the dominated alternative in the binary problem. It is immediate that if the choices of the DM satisfy IIA, the choice from A will be the same as the choice from A minus the dominated alternative. Moreover, the reverse also applies. Hence, we can write IIA as follows:[6]

Independence of Irrelevant Alternatives (IIA): For any consistent binary selector f and any $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$.

[5]See Bossert, Sprumont and Suzumura (2005) and Ehlers and Sprumont (2008) for a thorough discussion of models of rationalization by a single asymmetric or a single acyclic rationale.

[6]The equivalence between the two statements of IIA is proved in the Appendix, in Lemma A.1.

IIA imposes a great deal of structure on choice by forbidding menu dependencies for every possible consistent way of identifying the binary problems. We offer a property, Independence of One Irrelevant Alternative (IOIA), which constitutes a strict weakening of IIA.

Independence of One Irrelevant Alternative (IOIA): There is a consistent binary selector f such that, for any $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$.

It is immediate that IOIA imposes less structure than IIA. IOIA only requires the DM to be consistent when identifying binary problems in certain ways. For example, the DM must be consistent when focusing on, say, the most salient binary problem for her. Saliency may be determined by the DM's familiarity with the alternatives, or in terms of a particular physical order of presentation of the alternatives, etc. Therefore, contrary to IIA, IOIA allows for menu dependencies when identifying binary problems in other ways.

We show next that IOIA is equivalent to the notion of choice by sequential procedures, regardless of the number of rationales required.

Theorem 3.1. *c is a choice by sequential procedures if and only if c satisfies IOIA.*

The proof of Theorem 3.1 shows that, to assess whether c is a choice by sequential procedures, one simply has to check whether there is a linear order over the binary sets such that, for every choice problem A and for the first binary problem $B \subseteq A$, the choice from A does not depend on the dominated alternative in B. This makes the IOIA property particularly manageable in applications, and easy to use in practice.

IOIA also helps us to understand *how* deviations from rationality emerge in the model of choice by sequential procedures. The sequential procedure has two dimensions. The first is that it addresses the complexity of the choice problem by gradually shrinking the menu of alternatives in sequential steps. The second has to do with the nature of the alternatives that are eliminated in each step. Therefore, deviations from the rational model may emerge from the first dimension, from the second, or from a combination of both. Manzini and Mariotti (2007) show that if behavior in the binary problems is fully consistent so is behavior in larger menus, and vice-versa. The property of No Binary Cycles captures the notion of consistent behavior in the binary problems:

No Binary Cycles: For all $x_1, \ldots, x_{r+1} \in X$, $c(x_j, x_{j+1}) = x_j$, $j = 1, \ldots, r$, implies that $c(x_1, x_{r+1}) = x_1$.

Now, it is easy to reproduce their result by way of our property IOIA.

Lemma 3.2. *c satisfies IOIA and No Binary Cycles if and only if c satisfies IIA.*

IOIA can be understood as the interplay of a fully consistent component, the binary selector f, and a potentially irrational component, choices from binary problems. Now, IOIA together with Lemma 3.2, shows not only that the type of irrational behavior that can be accommodated by choice by sequential procedures starts from cycles in the binary problems, but also that the totally consistent binary selector f is the mechanism through which the irrationality over the binary problems spreads over larger menus of alternatives. Interestingly, if the starting point is that a DM behaves according to choice by sequential procedures, the above lemma shows that to asses whether the DM is rational one only needs to evaluate whether his binary choices are consistent.

4. Applications

4.1. **Rationalizability by Game Trees.** We begin this section by establishing the relation between choice by sequential procedures and a rationalizability notion due to Xu and Zhou (2007): rationalizability by game trees. Xu and Zhou study those choice functions that can be rationalized by extensive games with perfect information. More specifically, we say that $(G; R = (R_1, \ldots, R_K))$ denotes a game tree whenever: (i) G is an extensive game with perfect information that has alternatives X as terminal nodes, such that each alternative in X appears once and only once as a terminal node of G (hence, X and G can be identified), and (ii) every node i in the tree G represents the possible choices of an agent i endowed with a linear order R_i over X. Denote by $G|A$ the reduced tree of G that retains all the branches of G leading to terminal nodes in A, and $SPNE(\Gamma)$ stands for the subgame perfect Nash equilibrium outcome of Γ.

Rationalizability by Game Trees: A choice function c is rationalizable by game trees whenever there exists a game tree $(G; R)$ such that $c(A) = SPNE(G|A; R)$ for all $A \subseteq X$.

The relationship between rationalizability by game trees and choice by sequential procedures is not clear-cut a priori. On the one hand, the game tree structure of the preferences in the former is much richer than the linear structure of the rationales in the latter. On the other hand, the preferences of the players in rationalizability by game trees are imposed to be linear orders, whereas in choice by sequential procedures the rationales are only imposed to be acyclic. The following result establishes a perhaps unexpected relationship between the two notions.

Theorem 4.1. *If c is rationalizable by game trees, c is a choice by sequential procedures. The converse is not necessarily true.*

4.2. **Agenda Rationalizability.** Let us assume that the n elements in X are linearly ordered by $<$. This order may be interpreted as, say, a particular physical presentation of the objects. For any choice problem A in X, write the l elements in A ordered by $<$ as $a(1) < a(2) < \cdots < a(l)$. Consider a tournament T.[7] The DM chooses from A according to the following elimination process. First she chooses between $a(1)$ and $a(2)$ using T, then compares the chosen element from $a(1), a(2)$ with $a(3)$ and makes a new choice according to T. The DM continues in this ordered manner until the surviving element is compared with the last element $a(l)$; this last choice determines the final choice in A. Denote by $e(<, T, A)$ the alternative chosen from A by this process, given the agenda $<$, and the tournament T.

Similar choice-by-ordered-elimination procedures are studied in the choice-theory literature. The models studied in Rubinstein and Salant (2006) and Salant and Rubinstein (2008), for example, include this one as a special case. See also Masatlioglu and Nakajima (2008). But binary choices between alternatives may also be the outcome of majority voting, for example. Therefore voting mechanisms such as those based on successive elimination are also connected to the above (see Dutta, Jackson, and Le Breton, 2002). Consider now the following notion of rationalizability.

Agenda Rationalizability: A choice function c is agenda rationalizable whenever there exists a linear order $<$ over the set of alternatives (an agenda) and a tournament T such that, for every $A \subseteq X$, $c(A) = e(<, T, A)$.

Theorem 4.2 establishes the relation between agenda rationalizability and choice by sequential procedures.

Theorem 4.2. *If c is agenda rationalizable, c is a choice by sequential procedures. The converse is not necessarily true.*

4.3. **Status-Quo Bias.** There is a large literature supporting the view that DMs typically value an alternative more highly when it is regarded as the status quo, than they would otherwise. This is the so-called status-quo bias. Masatlioglu and Ok (2005) provide a theoretical treatment of choice behavior that allows for the presence of a status-quo bias.[8]

[7] A tournament T is a binary relation that is asymmetric and connected.

[8] See Apesteguia and Ballester (2009) for a theoretical treatment of choice behavior dependent on reference points in general, and for references to the empirical and theoretical literature on the status-quo bias in particular.

In Masatlioglu and Ok's setting, a choice problem is a pair (A, x) where A is the set of alternatives, and $x \in A$ or $x = \diamond$. When $x \in A$, the pair (A, x) represents a choice problem with a status quo, while if $x = \diamond$, the choice problem is standard in the sense that it is without a status quo. Thus, choice is defined over the collection of all choice problems (A, x). Masatlioglu and Ok introduce a set of properties for choice behavior that are equivalent to the following status-quo biased choice model. The DM evaluates the alternatives by means of a vector-valued utility function u, in a multi-criteria style. If the DM confronts a choice problem without a status quo, then she simply maximizes an aggregation h of these criteria. If there is a status quo, then the DM compares the status quo with all the alternatives in the set using all the criteria. She will stay with the status quo unless there is an alternative that dominates it in terms of *all* the decision criteria. This represents a marked status-quo bias. If there are alternatives that dominate the status quo by all the criteria, then the DM chooses from them using the same aggregator h as above.

The following definition is a reformulation of Masatlioglu and Ok's Theorem 1 to our setting.

Status-Quo Biased Choice Function: A choice function c is status-quo biased if there exists an element $\overline{x} \in X$, a positive integer q, an injective function $u : X \to \mathbb{R}^q$ and a strictly increasing map $h : u(X) \to \mathbb{R}$ such that:

(1) For all $A \subseteq X$ with $\overline{x} \notin A$, $c(A) = \arg\max_{x \in A} h(u(x))$.
(2) For all $A \subseteq X$ with $\overline{x} \in A$:

$$c(A) = \begin{cases} \overline{x} & \text{if } A \cap \{x \in X : u(x) > u(\overline{x})\} = \emptyset, \\ \displaystyle\arg\max_{y \in A \cap \{x \in X : u(x) > u(\overline{x})\}} h(u(y)) & \text{if } A \cap \{x \in X : u(x) > u(\overline{x})\} \neq \emptyset. \end{cases}$$

Theorem 4.3 shows that status-quo biased choice functions are choice by sequential procedures.[9]

Theorem 4.3. *If c is a status-quo biased choice function, c is a choice by sequential procedures. The converse is not necessarily true.*

[9]It can be proved that the set of status quo biased choice functions is contained in the set of agenda rationalizable choice function, and that the latter is at the same time contained in the set of choice functions that are rationalizable by game trees. In order to illustrate the applicability of our property, IOIA, we have chosen to offer direct links between choice by sequential procedures and these notions of choice.

5. Final Remarks

In this paper we have studied the behavioral structure of choice by sequential procedures and its relationship with other choice models. We have proposed a simple relaxation of the classical IIA property, IOIA, and shown that it is equivalent to the notion of choice by sequential procedures, regardless of the number of rationales required. Further, we have shown that our property does not only allow a better understanding of choice by sequential procedures, but it is useful in the sense that it facilitates investigation of the relationships between different models of choice. We have shown that the notions of rationalizability by game trees, agenda rationalizability, and choice functions exhibiting a status-quo bias are special cases of the model of choice by sequential procedures.

We believe it is crucial to the future development of the field to investigate the links between the relevant models of choice. Manzini and Mariotti (2009) take a further step in this direction. In line with our property IOIA and our Theorem B.1, they show that choice by sequential procedures is equivalent to the model of choice by lexicographic semiorders (see Tversky, 1969). Interestingly, Manzini and Mariotti (2009) build on our paper to offer an alternative property that is also equivalent to choice by sequential procedures.

Our property IOIA suggests promising new lines for future research. Specifically, it would be natural to investigate the nature of the process by which binary problems are identified in certain environments, that is, to determine the essence of the rule f that makes one binary problem stand out from any others. Relatedly, it might be particularly interesting to study the role of marketing strategies or that of a principal in shaping the f of a consumer in a market interaction environment.

Appendix A. Proofs

Lemma A.1. *The following two statements are equivalent:*

(1) *For any $A, B \subseteq X$, if $c(B) \in A \subseteq B$ then $c(A) = c(B)$.*
(2) *For any consistent binary selector f and any $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$.*

Proof of Lemma A.1: (1) $\Rightarrow$ (2). Consider a consistent binary selector f and a set $A \subseteq X$. We first claim that $c(A) \neq x^*$, with $x^* = f(A) \setminus c(f(A))$. If $c(A) \notin f(A)$, it is straightforward that $c(A) \neq x^*$. If $c(A) \in f(A) \subseteq A$ then, by (1), it must be $c(A) = c(f(A))$ and thus, $c(A) \neq x^*$ as desired. Now, since $c(A) \in A \setminus \{x^*\} \subseteq A$, (1) guarantees that $c(A) = c(A \setminus \{x^*\})$.

(2) $\Rightarrow$ (1). Let $A, B \subseteq X$, such that $c(B) \in A \subseteq B$. If $B = A$ the result is obvious. Denote $c(B) = x_1$, the elements in $B \setminus A$ as $x_2, x_3, \ldots, x_k$, with $k > 1$, and finally the rest of elements in X as $x_{k+1}, \ldots, x_n$. Let the binary selector f select, for any $D \subseteq X$ with

$|D| \geq 2$, the first two elements in D according to the previous ordering. It is immediate that f is consistent. Then, by (2) we know that $x_1 = c(B) = c(B \setminus \{x^*\})$, and hence it can only be $x^* = x_2$ and $c(B) = c(B \setminus \{x_2\})$. Iterating this argument it follows that $c(B) = c(B \setminus \{x_2, x_3, \ldots, x_k\}) = c(A)$, as desired. □

Proof of Theorem 3.1: We start by proving that IOIA is a sufficient condition for choice by sequential procedures. IOIA implies that there is a consistent binary selector f such that, for every $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$. Since f satisfies the Strong Axiom, the transitive revealed relation $\overline{S}$ over $\mathcal{B}(X)$ is also antisymmetric. Suppose not. That is, there are binary sets $B_1, B_2, W_1, \ldots, W_r$ such that $B_1 \overline{S} B_2$ and $B_2 S W_1 S \cdots S W_r S B_1$. By definition $B_1 \overline{S} W_r$, and by the Strong Axiom it cannot be that $W_r S B_1$, a contradiction. Now, it is well-known that the asymmetric part of a transitive and antisymmetric binary relation can be extended to a linear order. Denote by $\lhd$ the linear order over $\mathcal{B}(X)$ extended from $\overline{S}$. Consequently, for every A with $|A| \geq 2$, $f(A)$ is the first binary problem contained in A according to $\lhd$. Denote, then, the ordered collection of binary problems with two alternatives according to $\lhd$ by $\{a_i, b_i\}_{i=1}^{n(n-1)/2}$. Without loss of generality, let it be assumed that $a_i = c(a_i, b_i)$, $i = 1, \ldots, n(n-1)/2$. Define $P_i = \{(a_i, b_i)\}$. Clearly, $\{P_i\}_{i=1}^{n(n-1)/2}$ is a collection of rationales.

We prove, by induction over the cardinality of choice problems $A \subseteq X$, that $\{P_i\}_{i=1}^{n(n-1)/2}$ sequentially rationalizes c. It is obvious for the case of $|A| \leq 2$. Supposing that the claim is true for $|A| = t$, we show it to be true for $|A| = t + 1$. By IOIA, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$. By the inductive hypothesis $c(A \setminus \{x^*\}) = M_1^{n(n-1)/2}(A \setminus \{x^*\})$, and by the definition of x^* and the construction of the rationales, $M_1^{n(n-1)/2}(A \setminus \{x^*\}) = M_1^{n(n-1)/2}(A)$. Therefore, $c(A) = M_1^{n(n-1)/2}(A)$, as desired.

In the other direction, we now show that if c is a choice by sequential procedures, then IOIA holds. Let $\{P_1, \ldots, P_K\}$ sequentially rationalize c. First, construct the collection of rationales $\{P'_1, \ldots, P'_K\}$ from $\{P_1, \ldots, P_K\}$, as follows: for all $j = 1, \ldots, K$, $(x, y) \in P'_j$ if and only if $(x, y) \in P_j$ and there is no $i < j$ such that $(x, y) \in P_i$ or $(y, x) \in P_i$. Clearly, $\{P'_1, \ldots, P'_K\}$ is an ordered collection of rationales that sequentially rationalizes c. Assume, without loss of generality, that the constructed collection $\{P'_1, \ldots, P'_K\}$ is composed of non-empty rationales (otherwise, simply remove the empty rationales and re-number them).

Now, consider a rationale P'_j in the constructed collection of rationales $\{P'_1, \ldots, P'_K\}$, that contains more than one pair of alternatives. Since P'_j is acyclic, there is a pair of alternatives (a, b) in P'_j, such that $(b, c) \notin P'_j$ for every $c \in X$. We can split the rationale P'_j into two rationales, $\{(a, b)\}$ and $P'_j \setminus \{(a, b)\}$. We show that for every $A \subseteq X$, $M(A, P'_j) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$. If either a or b is not in A, then clearly

$M(A, P'_j) = M(A, P'_j \setminus \{(a,b)\})$, and since $M(A, \{(a,b)\}) = A$, it follows that $M(A, P'_j \setminus \{(a,b)\}) = M(M(A, \{(a,b)\}), P'_j \setminus \{(a,b)\})$. Thus, $M(A, P'_j) = M(M(A, \{(a,b)\}), P'_j \setminus \{(a,b)\})$. If, on the contrary, $a, b \in A$, then $M(A, \{(a,b)\}) = A \setminus \{b\}$. Given that (a,b) in P'_j and that $(b,c) \notin P'_j$ for every $c \in X$, $M(A, P'_j) = M(A \setminus \{b\}, P'_j)$. Hence, $M(A, P'_j) = M(M(A, \{(a,b)\}), P'_j \setminus \{(a,b)\})$, as desired. It then follows that the ordered collection of rationales $\{P'_1, \ldots, P'_{j-1}, \{(a,b)\}, P'_j \setminus \{(a,b)\}, P'_{j+1}, \ldots, P'_K\}$ also sequentially rationalizes c. By iterating this rationale-splitting process, we end up with a collection of rationales $\{P^*_1, \ldots, P^*_{n(n-1)/2}\}$, each of which contains one pair of alternatives and sequentially rationalizes c.

Construct the binary selector f as follows. For every A with $|A| \geq 2$, $f(A) = \{a, b\}$ if and only if $\{(a,b)\} = P^*_l$ and there is no $\{d, e\} \subseteq A$ with $\{(d,e)\} = P^*_m$ and $1 \leq m < l \leq n(n-1)/2$. It is immediate that f satisfies the Strong Axiom. Given that $\{P^*_1, \ldots, P^*_{n(n-1)/2}\}$ sequentially rationalize c and given the construction of f, it follows that $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$, and hence IOIA holds.□

Proof of Lemma 3.2: The if part is immediate, so let us show the only if part. It is obvious for the case of $|A| \leq 2$. Supposing that the claim is true for $|A| = t$, we show it to be true for $|A| = t+1$. By IOIA there is a consistent binary selector f such that $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$. By the inductive hypothesis $c(A \setminus \{x^*\}) = M(A \setminus \{x^*\}, P)$, where P is a linear order. By definition $x^* \neq c(f(A))$, by the inductive hypothesis $c(f(A)) = M(f(A))$, and given that P is a linear order, it must then be that $M(A \setminus \{x^*\}, P) = M(A, P)$, and we are done.□

Proof of Theorem 4.1: Let c be a choice function rationalizable by game trees. We will prove that c satisfies IOIA and is therefore a choice by sequential procedures. Consider a game tree $(G; R = (R_1, \ldots, R_K))$ that rationalizes c. Suppose, without loss of generality, that players are indexed by a linear order $<$ such that $i < j$ if i is a successor of j in the game tree G. We say that i is resolute on $\{x, y\}$, if R_i determines the outcome over $\{x, y\}$. That is, whenever $SPNE(G|\{x,y\}; R) = SPNE(G|\{x,y\}; (R'_{-i}, R_i))$ for any vector of linear orders of players other than i, R'_{-i}. Define the minimal resolute player in $A \in \mathcal{P}(X)$ by $m(A) = \min\{i \in \{1, \ldots, K\} : i \text{ is resolute over a pair } \{x,y\} \text{ contained in } A\}$.

Consider the following linear order $\lhd$ on $\mathcal{B}(X)$. Let $\{a,b\}, \{d,e\} \in \mathcal{B}(X)$, with $a = c(a,b)$ and $d = c(d,e)$,

$$\{a,b\} \lhd \{d,e\} \Leftrightarrow \begin{cases} m(\{a,b\}) < m(\{d,e\}), \text{ or} \\ m(\{a,b\}) = m(\{d,e\}), (b,e) \in R_{m(\{a,b\})}. \end{cases}$$

We now show that IOIA holds with respect to the constructed linear order $\lhd$. By rationalizability of game trees, $c(A) = SPNE(G|A;R)$, and also $c(A \setminus \{x^*\}) = SPNE(G|(A \setminus \{x^*\});R)$, where x^* is the dominated alternative in the first binary problem in A according to $\lhd$. We show that $SPNE(G|A;R) = SPNE(G|(A \setminus \{x^*\});R)$. By definition of subgame perfect Nash equilibrium, $SPNE(G|A;R) = SPNE(G|(A \setminus \{z\});R)$, where z is every dominated alternative in A according to the minimal resolute player in A. Clearly, by construction, x^* satisfies these conditions, and hence the claim follows.

We now show by way of an example that the inclusion is strict. Let $X = \{1,\ldots,4\}$ and c be such that: $\mathcal{A}_c[1] = \{(1,3,4),(1,3)\}$, $\mathcal{A}_c[2] = \{(1,2,4),\ (1,2),(2,4)\}$, $\mathcal{A}_c[3] = \{(1,2,3,4),(1,2,3),(2,3,4),(2,3),(3,4)\}$ and $\mathcal{A}_c[4] = \{(1,4)\}$, where $A \in \mathcal{A}_c[x] \Rightarrow c(A) = x$. It can be easily checked that IOIA follows with respect to the following linear order $\lhd$ over $\mathcal{B}(X)$: $(2,4) \lhd (2,1) \lhd (3,4) \lhd (3,2) \lhd (1,3) \lhd (4,1)$, and then by Theorem 3.1 c is a choice by sequential procedures. To see that c is not rationalizable by game trees, we show that c does not satisfy the *divergence consistency* property, which Xu and Zhou prove to be a necessary condition for rationalizability by game trees.[10] Note that in c, 1 diverges before 3 and 4, and 3 diverges before 1 and 2. At the same time, we have that $c(1,3) = 1$ but $c(2,4) = 2$, which contradicts divergence consistency. Therefore, c is not rationalizable by game trees and the theorem follows.□

Proof of Theorem 4.2: Suppose that c is agenda rationalizable by the agenda $<$ and the tournament T. Define the binary selector f by $f(A) = \{a(1), a(2)\}$ for every A with $|A| \geq 2$. Clearly, f satisfies the Strong Axiom. By construction, and by agenda rationalizability $x^* = a(1)$ if $(a(2), a(1)) \in T$ and $x^* = a(2)$ if $(a(1), a(2)) \in T$. Hence, it follows that $c(A) = e(<, T, A) = e(<, T, A \setminus \{x^*\}) = c(A \setminus \{x^*\})$. Then, IOIA holds, and by Theorem 3.1 c is a choice by sequential procedures.

Consider now the choice function c defined in the proof of Theorem 4.1. We showed that c is a choice by sequential procedures. To show that c is not agenda rationalizable, note that, since 1 diverges before 3 and 4, any agenda $<$ that rationalizes c should be of the form $3 < 1$ and $4 < 1$. Furthermore, since 3 diverges before 1 and 2, the agenda should also be of the form $1 < 3$ and $2 < 3$. But this is obviously absurd, and therefore, there is no agenda that rationalizes c.□

[10] For any triple x, y, z, x *diverges* before y and z, if $c(x,y) = x, c(y,z) = y$, and $c(z,x) = z$, and $c(x,y,z) = x$. *Divergence consistency*: for any four alternatives x_1, x_2, y_1, y_2, if x_1 diverges before y_1 and y_2, and y_1 diverges before x_1 and x_2, then $c(x_1, y_1) = x_1$ if and only if $c(x_2, y_2) = y_2$.

Proof of Theorem 4.3: Take a status-quo biased choice function c. Define the following two classes of binary problems: $\mathcal{B}_1(X) = \{\{\overline{x}, y\} \in \mathcal{B}(X) :$ it is not true that $u(y) > u(\overline{x})\}$ and $\mathcal{B}_2(X) = \mathcal{B}(X) \setminus \mathcal{B}_1(X)$. Take any linear order $\lhd$ on $\mathcal{B}(X)$ such that for every $\{a, b\} \in \mathcal{B}_1(X)$ and $\{c, d\} \in \mathcal{B}_2(X)$, $\{a, b\} \lhd \{c, d\}$.

Take any A with $|A| \geq 2$ and let x^* denote the dominated alternative in the first binary problem in A according to $\lhd$. In showing that c satisfies IOIA, we distinguish between four cases.

Case 1: $\overline{x} \in A$ and $\{\overline{x}, y\} \in \mathcal{B}_1(X)$ for every $y \in A$. Given that $A \cap \{x \in X : u(x) > u(\overline{x})\} = (A \setminus \{x^*\}) \cap \{x \in X : u(x) > u(\overline{x})\} = \emptyset$, then $c(A) = c(A \setminus \{x^*\}) = \overline{x}$.

Case 2: $\overline{x} \in A$ and $\{\overline{x}, y\} \in \mathcal{B}_2(X)$ for every $y \in A$. Since $A \cap \{x \in X : u(x) > u(\overline{x})\} = A \setminus \{\overline{x}\} \neq \emptyset$, then $c(A) = \underset{y \in A \setminus \{\overline{x}\}}{\arg\max}\, h(u(y))$. The latter being $c(A \setminus \{\overline{x}\})$.

Case 3: $\overline{x} \in A$ and there are two alternatives $y, z \in A$ such that $\{\overline{x}, y\} \in \mathcal{B}_1(X)$ and $\{\overline{x}, z\} \in \mathcal{B}_2(X)$. By construction, $x^* \neq \overline{x}$, and $f(A) = \{\overline{x}, x^*\} \in \mathcal{B}_1(X)$. Then $A \cap \{x \in X : u(x) > u(\overline{x})\} = (A \setminus \{x^*\}) \cap \{x \in X : u(x) > u(\overline{x})\} \neq \emptyset$, then $c(A) = \underset{y \in A \cap \{x \in X : u(x) > u(\overline{x})\}}{\arg\max} h(u(y)) = \underset{y \in (A \setminus \{x^*\}) \cap \{x \in X : u(x) > u(\overline{x})\}}{\arg\max} h(u(y)) = c(A \setminus \{x^*\})$.

Case 4: $\overline{x} \notin A$ then, obviously $c(A) = c(A \setminus \{y\})$ for every $y \neq c(A)$.

To show that the inclusion is strict, consider the example in Theorem 4.1. If $\overline{x} \in \{2, 4\}$, there is a pairwise cycle in $X \setminus \{\overline{x}\}$ which contradicts (1) in the definition of status quo biased choice functions. If $\overline{x} \in \{1, 3\}$, then $3 = c(1, 2, 3) \notin \{1, 2, 3\} \cap \{x \in X : u(x) > u(\overline{x})\} \neq \emptyset$, thus contradicting (2) in the mentioned definition.$\square$

Appendix B. Different Models of Choice by Sequential Procedures

While in this paper we take the rationales to be acyclic, Manzini and Mariotti (2007) impose only asymmetry, and hence allow for cycles in the rationales. The models share the same intuition in that they consider a DM who gradually narrows down the set of alternatives by applying a set of crude criteria in a fixed order. They nevertheless differ in terms of the structure imposed on the rationales. In this section we clarify the relation between these and other models of sequential choice. We first need some definitions.

We say that a binary relation P is asymmetric if, for any $x, y \in X$, whenever $(x, y) \in P$, it is not true that $(y, x) \in P$. We say that P is transitive if, for any $x, y, z \in X$, whenever $(x, y), (y, z) \in P$, then $(x, z) \in P$. We say that P is *simple* whenever P is a singleton set containing a pair of different elements of X, i.e., $P = \{(x, y)\}$ with $x, y \in X$, $x \neq y$. We say that P is α whenever the structure of α lies between the structure of a simple rationale and an acyclic one. Examples of α rationales are (strict) partial orders (transitive and asymmetric) or semiorders (a special case of a transitive and asymmetric binary relation; see Manzini and Mariotti, 2009).

We can now define the notion of choice by sequential procedures on the basis of the structure imposed on the criteria sequentially applied. To this end, take the definition of choice by sequential procedures of section 2 and say that c is a choice by sequential procedures by asymmetric rationales CSP(As), or by acyclic rationales CSP(Ac), or by simple rationales CSP(S), or by α rationales CSP(α) when the ordered collection of rationales is made up of asymmetric rationales, acyclic rationales, simple rationales, and α rationales respectively. Denote by $\mathcal{C}^{\omega}$, with $\omega \in \{$CSP(As),CSP(Ac),CSP(S),CSP(α)$\}$, the class of choice functions that are CSP(As), CSP(Ac), CSP(S), and CSP(α). Finally, denote by $\mathcal{C}^{RSM}$ the set of choice functions that are Rational Shortlist Methods, that is, that are choice by sequential procedures by two asymmetric rationales (Manzini and Mariotti, 2007).

Theorem B.1 shows that it is in fact the case that SR(Ac) and SR(As) are two different notions of sequential choice, and that any model of choice that imposes structure that lies between SR(S) and SR(Ac) is equivalent to SR(S) and SR(Ac).

Theorem B.1. $\mathcal{C}^{RSM} \subset \mathcal{C}^{CSP(S)} = \mathcal{C}^{CSP(\alpha)} = \mathcal{C}^{CSP(Ac)} \subset \mathcal{C}^{CSP(As)}$.

Proof of Theorem B.1: We start by proving that $\mathcal{C}^{RSM} \subseteq \mathcal{C}^{CSP(Ac)}$. Suppose c is an RSM. Then, there exists a pair of asymmetric rationales $\{P_1, P_2\}$ that sequentially rationalizes c. We now construct a collection of acyclic rationales $\{P'_1, \ldots, P'_K\}$ that also sequentially rationalizes c. Let $P'_1 = P_1$ and given $P_2 = \{(a_2^1, b_2^1), \ldots, (a_2^r, b_2^r)\}$, define the following simple rationales $P'_{j+1} = \{(a_2^j, b_2^j)\}$, $1 \le j \le r$. First, we prove that all rationales are acyclic. This is immediate for the simple rationales $P'_2, \ldots, P'_{r+1}$. Suppose, by way of contradiction, that $P'_1 = P_1$ is cyclic. Then, there exists a collection $x_1, \ldots, x_r \in X$, with $r > 1$, such that $(x_i, x_{i+1}) \in P_1$, $i = 1, \ldots, r-1$, and $(x_r, x_1) \in P_1$. Then $M(\{x_1, \ldots, x_r\}, P_1) = \emptyset$, contradicting the fact that $\{P_1, P_2\}$ sequentially rationalizes c. Therefore, all rationales are acyclic.

Second, we show that the collection $\{P'_1, \ldots, P'_{r+1}\}$ sequentially rationalizes c. Take any A. Suppose that $M_1^{r+1}(A)$ contains two or more distinct elements. Take any two such elements $x, y \in M_1^{r+1}(A), x \neq y$. Then, for $j = 1, \ldots, r+1$, it is neither the case that $(x, y) \in P'_j$ nor that $(y, x) \in P'_j$. But then, for $i = 1, 2$ it is neither true that $(x, y) \in P_i$ nor that $(y, x) \in P_i$. This contradicts the fact that $\{P_1, P_2\}$ rationalizes the choice in $\{x, y\}$ and therefore $M_1^{r+1}(A)$ contains at most one element. We now prove that $c(A)$ belongs to $M_1^{r+1}(A)$. Given that c is sequentially rationalized by $\{P_1, P_2\}$, it follows that $c(A) \in M(A, P_1) = M(A, P'_1)$ and, for any $y \in M(A, P'_1)$, it cannot be the case that $(y, c(A)) \in P_2$. Therefore, there is no P'_j, $j = 2, \ldots, r+1$, such that $(y, c(A)) \in P'_j$, and then $c(A) \in M_1^{r+1}(A)$. Hence, $c(A) = M_1^{r+1}(A)$

and $\{P'_1, \ldots, P'_{r+1}\}$ is a collection of acyclic binary relations that rationalizes c. Therefore, it follows that $\mathcal{C}^{RSM} \subseteq \mathcal{C}^{CSP(Ac)}$.

We now show that the inclusion is strict, i.e., $\mathcal{C}^{CSP(Ac)} \not\subseteq \mathcal{C}^{RSM}$. To do so, we define a choice function c_1 that is in $\mathcal{C}^{CSP(Ac)}$, but is not an RSM.

Let $X = \{1, \ldots, 4\}$ and c_1 be such that: $\mathcal{A}_{c_1}[1] = \{(1,3)\}$, $\mathcal{A}_{c_1}[2] = \{(1,2)\}$, $\mathcal{A}_{c_1}[3] = \{(1,2,3,4), (1,2,3), (2,3,4), (2,3), (3,4)\}$, and $\mathcal{A}_{c_1}[4] = \{(1,2,4), (1,3,4), (1,4), (2,4)\}$. Consider the ordered collection of simple rationales $\{P_1, \ldots, P_6\} = \{\{(2,1)\}, \{(1,3)\}, \{(4,1)\}, \{(3,2)\}, \{(4,2)\}, \{(3,4)\}\}$. It is easy to check that this collection sequentially rationalizes c. Note, however, that $4 = c_1(1,2,4) = c_1(1,3,4)$ but $c_1(1,2,3,4) = 3$. This means that c_1 violates the classic Expansion property, and then by Theorem 1 in Manzini and Mariotti (2007), c is not an RSM.[11] Therefore, $\mathcal{C}^{RSM} \subset \mathcal{C}^{CSP(Ac)}$.

We now show that $\mathcal{C}^{CSP(S)} = \mathcal{C}^{CSP(Ac)}$. That $\mathcal{C}^{CSP(Ac)} \supseteq \mathcal{C}^{CSP(S)}$ is immediate, and $\mathcal{C}^{CSP(Ac)} \subseteq \mathcal{C}^{CSP(S)}$ follows from the rationale-splitting argument used in the proof of Theorem 3.1. Now, given that $\mathcal{C}^{CSP(S)} = \mathcal{C}^{CSP(Ac)}$, since an α rationale lies, in terms of structure, between a simple and an acyclic rationale, it follows immediately that $\mathcal{C}^{CSP(S)} = \mathcal{C}^{CSP(\alpha)} = \mathcal{C}^{CSP(Ac)}$.

Lastly, we show that $\mathcal{C}^{CSP(Ac)} \subset \mathcal{C}^{CSP(As)}$. The fact that $\mathcal{C}^{CSP(Ac)} \subseteq \mathcal{C}^{CSP(As)}$ is obvious, so we now prove that the inclusion is strict. Consider the following example.

Let $X = \{1, \ldots, 6\}$ and c_2 be such that: $\mathcal{A}_{c_2}[1] = \{(1,3,4,5), (1,3,4), (1,3,5), (1,4,5), (1,3), (1,5)\}$, $\mathcal{A}_{c_2}[2] = \{(2,3,6), (2,5,6), (2,5), (2,6)\}$, $\mathcal{A}_{c_2}[3] = \{(2,3,4,5,6), (2,3,4,5), (2,3,5,6), (3,4,5,6), (2,3,4), (2,3,5), (3,4,5), (3,5,6), (2,3), (3,4), (3,5)\}$, $\mathcal{A}_{c_2}[4] = \{(2,4,5,6), (1,4,6), (2,4,5), (2,4,6), (1,4), (2,4), (4,6)\}$, $\mathcal{A}_{c_2}[5] = \{(1,3,4,5,6), (1,3,5,6), (1,4,5,6), (1,5,6), (4,5,6), (4,5), (5,6)\}$, $\mathcal{A}_{c_2}[6] = \{(1,3,4,6), (2,3,4,6), (1,3,6), (3,4,6), (1,6), (3,6)\}$, and $c_2(A) = c_2(A \setminus \{2\})$ whenever $\{1,2\} \subseteq A$.

We first show that $c \in \mathcal{C}^{CSP(As)}$. Consider for instance $P_1 = \{(1,2)\}$, $P_2 = \{(1,3), (3,4), (4,2), (2,5), (5,6), (6,1)\}$, $P_3 = \{(5,4), (1,5), (2,6), (6,3), (3,5), (4,6)\}$ and $P_4 = \{(4,1), (3,2)\}$. One can verify that all choices are sequentially rationalized by this ordered collection of asymmetric rationales.

Suppose that c_2 is sequentially rationalized by the ordered collection of simple rationales $\{P_1, \ldots, P_K\}$ with $P_i = \{(a_i, b_i)\}$, $i = 1, \ldots, K$. Let T be the smallest positive integer such that $P_T \neq \{(1,2)\}$ and $P_T \neq \{(2,1)\}$. T is well-defined since the collection of simple rationales must rationalize the choice in set $\{1,3\}$, for instance.

We now show that for every A with $\{a_T, b_T\} \subseteq A$ and $\{1,2\} \not\subseteq A$, it must be that $c_2(A) = c_2(A \setminus \{b_T\})$. If $T = 1$, it is immediate that $c_2(A) = M_1^K(A) = M_1^K(A \setminus \{b_1\}) = c_2(A \setminus \{b_1\})$.

[11] Expansion: For all $A, B \subseteq X$, $x = c(A) = c(B) \Rightarrow x = c(A \cup B)$.

If $T > 1$, $\cup_{i<T}\{a_i, b_i\} = \{1, 2\}$, and then it follows that $c_2(A) = M_1^K(A) = M_T^K(A) = M_T^K(A \setminus \{b_T\}) = M_1^K(A \setminus \{b_T\}) = c_2(A \setminus \{b_T\})$. Consider now the following cases:

- $c_2(\{1, 3, 4, 5, 6\}) = 5 \neq 6 = c_2(\{1, 3, 4, 5, 6\} \setminus \{5\})$ implies $(a_T, b_T) \notin \{(1, 5), (3, 5)\}$.
- $c_2(\{1, 3, 4, 5, 6\}) = 5 \neq 3 = c_2(\{1, 3, 4, 5, 6\} \setminus \{1\})$ implies $(a_T, b_T) \notin \{(4, 1), (6, 1)\}$.
- $c_2(\{1, 3, 4, 5, 6\}) = 5 \neq 1 = c_2(\{1, 3, 4, 5, 6\} \setminus \{6\})$ implies $(a_T, b_T) \notin \{(4, 6), (5, 6)\}$.
- $c_2(\{1, 3, 4, 6\}) = 6 \neq 4 = c_2(\{1, 3, 4, 6\} \setminus \{3\})$ implies $(a_T, b_T) \notin \{(1, 3), (6, 3)\}$.
- $c_2(\{2, 3, 4, 6\}) = 6 \neq 3 = c_2(\{2, 3, 4, 6\} \setminus \{6\})$ implies $(a_T, b_T) \neq \{(2, 6)\}$.
- $c_2(\{2, 3, 4, 6\}) = 6 \neq 2 = c_2(\{2, 3, 4, 6\} \setminus \{4\})$ implies $(a_T, b_T) \neq (3, 4)$.
- $c_2(\{2, 3, 4, 5, 6\}) = 3 \neq 6 = c_2(\{2, 3, 4, 5, 6\} \setminus \{5\})$ implies $(a_T, b_T) \neq (2, 5)$.
- $c_2(\{2, 3, 6\}) = 2 \neq 6 = c_2(\{2, 3, 6\} \setminus \{2\})$ implies $(a_T, b_T) \neq (3, 2)$.
- $c_2(\{2, 4, 5\}) = 4 \neq 5 = c_2(\{2, 4, 5\} \setminus \{2\})$ implies $(a_T, b_T) \neq (4, 2)$.
- $c_2(\{2, 4, 5\}) = 4 \neq 2 = c_2(\{2, 4, 5\} \setminus \{4\}$ implies $(a_T, b_T) \neq (5, 4)$.

Thus, $(a_T, b_T) \notin \{(1, 2), (1, 5), (3, 5), (4, 1), (6, 1), (4, 6), (5, 6), (1, 3), (6, 3), (2, 6), (3, 4), (2, 5), (3, 2), (4, 2), (5, 4)\}$. Given the binary choices in c_2, (a_T, b_T) is then a pair such that $b_T = c(\{a_T, b_T\})$. However, $M_1^K(\{a_T, b_T\}) = M_T^K(\{a_T, b_T\}) = a_T$ leading to a contradiction with the fact that the simple rationales sequentially rationalize c_2. Therefore, no collection of simple rationales can sequentially rationalize c_2. Since $c_2 \notin \mathcal{C}^{CSP(S)}$ implies $c_2 \notin \mathcal{C}^{CSP(Ac)}$, it follows that $c_2 \in \mathcal{C}^{CSP(As)} \setminus \mathcal{C}^{CSP(Ac)}$, which concludes the proof of the theorem.□

Having shown that CSP(As) differs from CSP(Ac), it is not difficult to prove that a property along the lines of IOIA is equivalent to CSP(As). The only modification one needs to make is to allow f to pinpoint from any choice problem A more than one binary problem. That is, to define f as a correspondence.

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