Contractually Stable Networks*

(very preliminary version)

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Abstract

The aim of this paper is to develop a theoretical framework that allows us to study which bilateral links and coalition structures are going to emerge at equilibrium. We define the notion of *coalitional network* to represent a network and a coalition structure, where the network specifies the nature of the relationship each individual has with his coalition members and with individuals outside his coalition. This new framework forces us to redefine key notions of theory of networks, value and allocation rules, and to introduce a new solution concept: contractual stability.

The idea of contractual stability is that adding or deleting links needs the consent of coalition partners. Moreover, the formation of new coalition structures needs the consent of original coalition partners. We also show that the Myerson value has a corresponding allocation rule in the context of coalitional networks and we propose a characterization of this allocation rule for coalitional networks.

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1 Introduction

The organization of individual agents into networks and groups has an important role in the determination of the outcome of many social and economic interactions. For instance, networks of personal contacts are important in obtaining information about job opportunities. Goods can be traded and exchanged through networks, rather than markets, of buyers and sellers. Networks also play important roles in providing mutual insurance especially in developing countries.[1] Partioning of societies into groups is also important in many contexts, such as the provision of public goods and formation of alliances, cartels and federations. The understanding of how and why such networks and groups form and the precise way in which they affect outcomes of social and economic interactions has been apprehended separately by the coalition theory and network theory.

However, a limit of both theories is that it cannot incorporate the existence of bilateral agreements among agents belonging to different coalitions. For instance, the formation of associations of firms, like R&D joint ventures or groups of firms adopting common standards, in an oligopolistic industry has been analyzed by Bloch (1995). He determines endogenously the structure of associations that emerges from a noncooperative sequential game of association formation. Goyal and Moraga-González (2001) and Goyal and Joshi (2003) have analyzed the incentives for R&D collaboration between horizontally related firms by considering that collaboration links are bilateral and are embedded within a broader network of similar links with other firms. But, firms A and B may decide to form an R&D joint venture while firms B and C sign a bilateral R&D agreement. Which are the architecture of the resulting collaboration network and the structure of associations that are likely to emerge?

It is very common that firms having some common interest within an industry regroup themselves into an industry association. For instance, a trading company in a developing country is like a sellers' association in that it represents a group of domestic sellers. The trading company specializes in forming links with international buyers and is thus able to match such buyers with the sellers that it represents. Until now, the literature on network formation has studied the endogenous formation of networks between input suppliers and manufacturers (Kranton and Minehart, 2000) and between manufacturers and retailers (Mauleon-Sempere-Vannetelbosch, 2005), when such industry associations do not form. What is the architecture of stable coalitional networks of distribution when both manufacturers and retailers decide not only the bilateral links they want to establish among them but also the trade associations they want to form?

Goyal and Joshi (2005) and Furusawa and Konishi (2006) have investigated the for-

[1] Jackson (2003, 2005) provides surveys of models of network formation.

mation of free trade agreements as a network formation game. Yi (1996) has studied the incentives of countries to form regional free trade associations and customs unions, and the strategic stability of particular trading regimes. All these previous papers have considered that countries participate either in the formation of bilateral free trade agreements or in the formation of customs unions or multilateral free trade agreements, but not in the formation of both types of agreements at the same time. This is a strong restriction indeed since many countries are involved in both types of agreements at the same time (Mexico belongs to NAFTA, and at the same time it has a bilateral free trade agreement with the European Community, while United States has not specific trade agreements with the European Community). What are the incentives of different countries to form bilateral free trade agreements and/or customs unions?

The aim of this paper is to develop a theoretical framework that allows us to study which bilateral links and coalition structures are going to emerge at equilibrium. We define the notion of *coalitional network* to represent a network and a coalition structure, where the network specifies the nature of the relationship each individual has with his coalition members and with individuals outside his coalition. This new framework forces us to redefine key notions of theory of networks, value and allocation rules, and to introduce a new solution concept : contractual stability.

The idea of contractually stability is that adding or deleting a link needs the consent of coalition partners. For instance, in the context of R&D alliances, firms may decide to have a common laboratory with some partners, while developing bilateral R&D agreements with other partners. The signing of a bilateral R&D agreement may need the consent of those partners within the common laboratory or joint venture. Moreover, the formation of new coalition structures needs the consent of original coalition partners. Thus, once a coalition has been formed, we ask for the consent of all coalition partners in order to add or delete links that affect some coalition partners. The consent of all coalition partners is also needed in order to modify the existing coalition.[2]

The Myerson value is an allocation rule defined in the context of cooperative games with communication structures and is a variation on the Shapley value (see Myerson (1977)). This rule was subsequently referred to as the Myerson value. The Myerson value has a corresponding allocation rule in the context of network games as well. We show that the Myerson value has also a corresponding allocation rule in the context of coalitional networks and we propose a characterization of this allocation rule.

[2]Although other forms of consent could be used, such as weighted or simple majority, we assume in the paper that unanimity is required in order to modify the existing coalition and the existing network that affects the coalition partners. This decision rule is used in the European Community whenever a new bilateral Free Trade Agreement is proposed by a country member.

The paper is organized as follows. In Section 2 we introduce the notion of coalitional network and we define the concept of contractual stability. In Section 3 we give a public good example to illustrate the new stability concept. In Section 4 we characterize the Myerson value for coalitional networks.

2 Coalitional networks

Players

Let $N = \{1, ..., n\}$ be the finite set of players who are connected in some network relationship and who may form coalitions.

Networks

The network relationships are reciprocal and the network is thus modeled as a non-directed graph. Players are the nodes in the graph and links indicate bilateral relationships between players. Thus, a network g is simply a list of which pairs of players are linked to each other. If we are considering a pair of players i and j, then $\{i, j\} \in g$ indicates that i and j are linked under the network g. For simplicity, we write ij to represent the link $\{i, j\}$, and so $ij \in g$ indicates that i and j are linked under the network g. Let g^N be the set of all subsets of N of size 2. $\mathbb{G} = \{g \mid g \subset g^N\}$ denotes the set of all possible networks or graphs on N, with g^N being the complete network. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network obtained by deleting link ij from an existing network g is denoted $g - ij$. For any network g, let $N(g) = \{i \mid \exists j$ such that $ij \in g\}$ be the set of players who have at least one link in the network g. Let $n(g) = \#N(g)$ be the number of players involved in g. Let $L_i(g)$ be the set of links that player i is involved in, so that $L_i(g) = \{ij \mid \exists j$ such that $ij \in g\}$, and let $l_i(g) = \#L_i(g)$. Thus, the number of links in g is given by $\frac{1}{2}\sum_i l_i(g)$.

Coalitions and coalitional networks

The players are forming coalitions. Let $\mathbb{P}$ be the finite set of coalition structures. A coalition structure $P = \{S_1, S_2, ..., S_m\}$ is a partition of the player set N, $S_a \cap S_b = \varnothing$ for $a \neq b$, $\cup_{a=1}^m S_a = N$ and $S_a \neq \varnothing$ for $a = 1, ..., m$. Let $\#S_a$ be the cardinality of coalition S_a. Let $S(i) \in P$ be the coalition whose player i belongs. A coalitional network (g, P) consists of a network $g \in \mathbb{G}$ and a coalition structure $P \in \mathbb{P}$. It describes how players are connected in some network relationship and to which coalitions they belong. For instance, if $N = \{1, 2, 3\}$, then $(g, P) = (\{12, 23\}, \{\{1, 3\}, \{2\}\})$ is the coalitional network in which there is a link between players 1 and 2 and a link between players 2 and 3 but no link between players 1 and 3, and players 1 and 3 are in the same coalition while player 2 is alone.

Let

$$g|_{P,S} = \{ij : ij \in g \text{ and } i \in S,\, j \in S \wedge ij \in g \text{ and } i \in S' \text{ and } j \in S',\, S' \in P,\, S' \cap S \neq \varnothing\}.$$

Thus, $g|_{P,S}$ is the network found deleting all links except those that are between players in S and those that are between players belonging to $S' \in P$ with $S' \cap S \neq \varnothing$. In other words, $g|_{P,S}$ represents the links in g at the interior of all coalitions $S' \in P$ that include some members of S and that, therefore, are controlled by the members of coalition S. In the example above, take coalition $S = \{1, 3\}$, then $g|_{P,S} = \{\emptyset\}$ because the link $13 \notin g$. With coalition $S = \{2, 3\}$, $g|_{P,S} = \{2, 3\}$. Notice that the link $12 \in g$ does not belong to $g|_{P,S}$ because players 1 and 2 are not in the same coalition $S', S' \in P, S' \cap S \neq \emptyset$.

For any coalitional network (g, P), let $N(g, P)$ be the set of players who have at least one link in the network g or that belong to a coalition $S \in P$ such that at least one member of S has a link in the network g. That is, $N(g, P) = \{i \mid \exists j \text{ such that } ij \in g\} \cup \{i \mid i \in S,$ $S \in P \text{ and } \exists j \in S \text{ such that } jk \in g \text{ for some } k\}$.

Paths and components

A path in a network $g \in \mathbb{G}$ between players i and j is a sequence of players $i_1, ..., i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, ..., K-1\}$ with $i_1 = i$ and $i_K = j$. A component of a coalitional network (g, P) is a nonempty coalitional subnetwork (g', Q), with $g' \subset g$ and $Q = \{S_k : S_k \in P\} \subset P$, such that

- $g' = \{ij : ij \in S, \forall S \in Q \wedge ij : i \in S, j \in S' \neq S, \text{for any } S \in Q, S' \in Q\}$
- for all $S \in Q$ there exists $ij \in g'$ with $i \in S$ such that either $j \in S$ or $j \in S' \neq S$, $S' \in Q$.

A component (g', Q) of the coalitional network (g, P) is a nonempty subnetwork g' of g and the coalitions in P that contain at least one player with a link in g'. That is (g', Q) represents a subnetwork g' connecting the players in $N(g', Q)$ and the coalitions in P that are "connected" by the links in g'.
The set of components of (g, P) is denoted as $C(g, P)$. Note that under this definition of a component, a coalition whose members have no links is not considered a component.[3]
Let $\Pi(g, P)$ denote the partition of N induced by (g, P). That is, $S \in \Pi(g, P)$ if and only if (i) there exists $(g', Q) \in C(g, P)$ such that $S = N(g', Q)$, or (ii) $S \in P$ such that for all

[3]However this is not necessary. One could extend the definition to consider a coalition with no link at all also as a component

$i \in S, i \notin N(g, P)$.

Partition value functions and allocation rules

Different coalitional network configurations lead to different values of overall production or overall utility to players. These various possible valuations are represented via a partition value function. A *partition value function* is a function $v : \mathbb{G} \times \mathbb{P} \to \mathbb{R}$. For simplicity, in what follows we maintain the normalization that $v(\varnothing, \{\{1\}, \{2\}, ..., \{n\}\}) = 0$. The set of all possible partition value functions is denoted $\mathcal{V}$. A partition value function only keeps track of how the total societal value varies across different coalitional networks. The calculation of partition value is a richer object than a partition function in a partition game and/or a value function in a network game, as it allows the value generated to depend both on the coalition structure and on the network structure.

A partition value function v is *component additive* if $\sum_{(g',Q)\in C(g,P)} v(g', Q) = v(g, P)$. Component additivity is a condition that rules out externalities across components but still allows them within components.[4]

A coalitional network (g, P) is efficient relative to a partition value function v if $v(g, P) \geq v(g', P')$ for all $g' \in \mathbb{G}$ and all $p' \in \mathbb{P}$. A coalitional network is efficient if it gives the maximum value over all possible (g, P).

We also wish to keep track of how that value is allocated or distributed among the players forming a network and a coalition structure. An *allocation rule* is a function $Y : \mathbb{G} \times \mathbb{P} \times \mathcal{V} \to \mathbb{R}^N$ such that

$$\sum_{i\in N} Y_i(g, P, v) = v(g, P) \text{ for all } v, g \text{ and } P. \tag{1}$$

It is important to note that an allocation rule depends on g, P and v. This allows an allocation rule to take full account of a player i's role in the network and in the coalition structure. This includes not only what the network configuration and coalition structure are, but also and how the value generated depends on the overall network and coalition structure.

Contractual Stability

A simple way to analyze the coalitional networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that, no coalition benefits from altering the coalitional network.

A coalitional network (g', P') is obtainable from (g, P) via S, $S \subseteq N$ if

(i) $ij \in g'$ and $ij \notin g$ implies $\{i, j\} \subset S$, and

[4] Note that the definition of component additive partition value function does not imply that $v(\emptyset, S)$ be zero for any S if we allow coalitions without link to be components.

(ii) $ij \notin g'$ and $ij \in g$ implies $\{i,j\} \cap S \neq \emptyset$, and

(iii) $\{S_a \setminus (S_a \cap S) : S_a \in P\} = \{S'_a \in P' : S'_a \subset N \setminus S\}$, and

(iv) $\exists \{S'_1, S'_2, ..., S'_m\} \subset P'$ such that $\cup_{a=1}^{m} S'_a = S$.

The first condition asks that any new links that are added can only be between players inside S. Condition (ii) requires that there must be at least one player belonging to S for the deletion of a link.[5] Condition (iii) embodies the assumption that no simultaneous deviations are possible. So if players in S deviate leaving their coalition in P, non-deviating players do not move. Condition (iv) allows deviating players in S to form one or several coalitions in the new coalitional structure P'. Non-deviating players do not belong to those new coalitions.

Definition 1 *A coalitional network (g, P) is Contractually Stable or C-Stable with respect to partition value function v and allocation rule Y if for any $S \subset N$, (g', P') obtainable from (g, P) via S and $i \in S$ such that $Y_i(g', v, P') > Y_i(g, v, P)$, there exists $k \in S(j)$ with $S(j) \in P$ and $j \in S$ such that $Y_k(g', v, P') < Y_k(g, v, P)$.*

The move from a coalitional network (g, P) to any obtainable coalitional network (g', P') needs the consent of every deviating player and the consent of every member of the initial coalitions of the deviating players. A coalitional network is C-stable if any deviating player or any member of the former coalitions of the deviating players suffers from the deviation.

When a coalitional network (g, P) is not C-stable it is said to be defeated by some obtainable coalitional network (g', P').

Efficiency

In evaluating societal welfare, we may take various perspectives. A coalitional network (g, P) is *Pareto efficient* relative to v and Y if there does not exist any $(g', P') \in \mathbb{G} \times \mathbb{P}$ such that $Y_i(g', P', v) \geq Y_i(g, P, v)$ for all i with strict inequality for some i. This definition of efficiency of a coalitional network (g, P) takes Y as fixed, and hence can be thought of as applying to situations where no intervention is possible. A coalitional network (g, P) is *strongly efficient* relative to v if $v(g, P) \geq v(g', P')$ for all $g' \in \mathbb{G}$ and $P' \in \mathbb{P}$. This is a strong notion of efficiency as it takes the perspective that value is fully transferable.

[5] These first two conditions have been introduced first by Jackson and van den Nouweland (2005) to define the netwoks obtainable from a given network by a coalition S.

3 The Myerson value for coalitional networks

A partition value function v is *component additive* if $\sum_{(g',Q)\in C(g,P)} v(g',Q) = v(g,P)$. Component additivity is a condition that rules out externalities across components but still allows them within components.[6]

An allocation rule Y is *component balanced* if for each component additive $v \in \mathcal{V}$, $g \in \mathbb{G}$, $P \in \mathbb{P}$, and $S \in \Pi(g,P)$

$$\sum_{i\in S} Y_i(g,P,v) = v(g|_{P,S}, P.) \tag{2}$$

Component balance requires that the value of a given component of a network be allocated to the members of that component in cases in which the value of the component is independent of how other components are organized.[7]

An allocation rule satisfies *equal bargaining power* if for any component additive v, $g \in \mathbb{G}$ and $P \in \mathbb{P}$, we have:

$$Y_i(g,P,v) - Y_i(g-ij,P,v) = Y_j(g,P,v) - Y_j(g-ij,P,v), \text{ for all } ij \text{ in } g. \tag{3}$$

The equal bargaining power condition states that two players i and j linked directly together face the same loss in payoff when the link between them is deleted from the set of links. Myerson (1977) was first to show, in the context of communication games, that the equal bargaining power condition (3) leads to a component balanced allocation rule that is a variation on the Shapley value. This rule was subsequently referred to as the Myerson value. The Myerson value also has a corresponding allocation rule in the context of network games as well, as shown by Jackson and Wolinsky (1996). Here, we show that the Myerson value has also a corresponding allocation rule in the context of coalitional networks. That allocation rule is expressed as follows.

$$Y_i^{MV}(g,P,v) = \sum_{S\subset N\setminus\{i\}} \left[v(g|_{P,S\cup\{i\}},P) - v(g|_{P,S},P)\right] \cdot \left[\frac{\#S!(n-\#S-1)!}{n!}\right].$$

Theorem 1 *Y satisfies component balance and equal bargaining power if and only if $Y(g,P,v) = Y^{MV}(g,P,v)$ for all $g \in \mathbb{G}$, $P \in \mathbb{P}$, and any component-additive v.*

Proof.

We first show that there exists only one allocation rule satisfying equal bargaining power for a given coalitional game. Take $Y^1 : \mathbb{G}\times\mathbb{P}\times\mathcal{V} \to \mathbb{R}^N$ and $Y^2 : \mathbb{G}\times\mathbb{P}\times\mathcal{V} \to \mathbb{R}^N$

[6] Note that the definition of component additivity does not imply that $v(\emptyset, S)$ be zero for any S if we allow coalitions without link to be components.

[7] When $S \in \Pi(g,P)$ and $S = N(g',Q)$, then $C(p|_{P,S}),P) = \{(g',Q)\}$.

satisfying (2) and (3) and suppose that they are different. Let (g, P) be a coalitional network such that g contains a minimum number of links such that $Y^1(g, P) \neq Y^2(g, P)$. Let $y^1 = Y^1(g, P)$ and $y^2 = Y^2(g, P)$ so that $y^1 \neq y^2$. By the minimality of g, if ij is any link of g, then $Y^1(g - ij, P) = Y^2(g - ij, P)$. Hence (3) yields

$$y_i^1 - y_j^2 = Y_i^1(g - ij, P) - Y_j^1(g - ij, P) = Y_i^2(g - ij, P) - Y_j^2(g - ij, P) = y_i^2 - y_j^2.$$

Transposing, we deduce

$$y_i^1 - y_i^2 = y_j^1 - y_j^2$$

whenever i and j are linked, and so also, they are in the same connected component S of (g, P). Thus we may write $y_i^1 - y_i^2 = d_S(g, P)$, where $d_S(g, P)$ depends only on S, g and P only and not on i. But by (2) we have $\sum_{i \in S} y_i^1(g, P) = \sum_{i \in S} y_i^2(g, P)$. Hence $\sum_{i \in S} y_i^1 - y_i^2 = 0 = |S| d_S(g, P)$ and so $d_S(g, P) = 0$. Hence $y^1 = y^2$, a contradiction. That is there can be at most one allocation rule satisfying equal bargaining power in a given coalitional network.

It only remains to show that $Y(g, P) = Y^{MV}(g, P)$ satisfies (2) and (3).

We first show component balance.

Select any (g, P). For each $S \in \Pi(g, P)$, define u^S to be a characteristic function game such that

$$u^S(T) = \sum_{(g', Q) \in C(g|_{P,(T \cap S)}, P)} v(g', Q), \forall\, T \subseteq N.$$

Now, any two players connected in $C(g|_{P,T}, P)$ are also connected in N, so

$$C(g|_{P,T}, P) = \bigcup_{S \in \Pi(g,P)} C(g|_{P,(T \cap S)}, P).$$

Therefore, $v(g, P) = \sum_{S \in \Pi(g,P)} u^S$.

But S is a carrier for u^S, because $u^S(T) = u^S(T \cap S)$. By the carrier axiom of Shapley (1953), for any $S \in \Pi(g, P)$ and $T \in \Pi(g, P)$,

$$\sum_{i \in S} Y^{MV}(u^T) = \begin{cases} u^T(N) & \text{if } S = T \\ 0 & \text{if } S \cap T = \emptyset. \end{cases}$$

Thus, by linearity of Y^{MV}, if $S \in \Pi(g, P)$, then

$$\sum_{i \in S} Y_i^{MV}(g, P, v) = \sum_{T \in \Pi(g,P)} \sum_{i \in S} Y_i^{MV}(u^T) = u^S(N) = \sum_{(g', Q) \in C(g|_{P,S}, P)} v(g', Q) = v(g|_{P,S}, P).$$

To show that (3) holds, take any $g \in \mathbb{G}$, $P \in \mathbb{P}$ and any $ij \in g$. Let $w = v(g, P) - v(g - ij, P)$. Observe that $C(g|_{P,S}, P) = C(g - ij|_{P,S}, P)$ if $\{i, j\} \subsetneqq S \bigcup S'$, $S' \in P$, $S' \bigcap S \neq \emptyset$. So if $i \notin S \bigcup S'$ or $j \notin S \bigcup S'$ we get :

$$w(S) = \sum_{(g', Q) \in C(g|_{P,S}, P)} v(g', Q) - \sum_{(g', Q) \in C(g - ij|_{P,S}, P)} v(g', Q) = 0.$$

Only coalitions with nonzero wealth in w are coalitions such that i and j are in S', $S' \in P$, $S \bigcap S' \neq \emptyset$. So by the symmetry axiom of Shapley, it follows that $Y_i^{MV}(w) = Y_j^{MV}(w)$. By linearity of Y^{MV}, $Y_i^{MV}(g|_{P,S}, P, v) - Y_i^{MV}(g - ij|_{P,S}, P, v) = Y_i^{MV}(w) = Y_j^{MV}(w) = Y_j^{MV}(g|_{P,S}, P, v) - Y_j^{MV}(g - ij|_{P,S}, P, v)$.

■

4 Conclusions

We have developed a theoretical framework that allows us to study which bilateral links and coalition structures are going to emerge at equilibrium. We have introduced the notion of *coalitional network* to represent a network and a coalition structure, where the network specifies the nature of the relationship each individual has with his coalition members and with individuals outside his coalition. In order to predict the coalitional networks that are going to emerge at equilibrium a new solution concept has been proposed: contractual stability.

The idea of contractual stability is that adding or deleting links needs the consent of coalition partners. Moreover, the formation of new coalition structures needs the consent of original coalition partners. We have also shown that the Myerson value has a corresponding allocation rule in the context of coalitional networks and we have proposed a characterization of this allocation rule for coalitional networks.

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