Anti-Malthus: Conflict and the Evolution of Societies☆

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Abstract

The Malthusian theory of evolution disregards a pervasive fact about human societies: they expand through conflict. When this is taken account of the long-run favors not a large population at the level of subsistence, nor yet institutions that maximize welfare or per capita output, but rather institutions that maximize free resources. These free resources are the output available to society after deducting the payments necessary for subsistence and for the incentives needed to induce production, and the other claims to production such as transfer payments and resources absorbed by elites. We develop the evolutionary underpinnings of this model, and examine the implications of free resource maximization for the evolution of societies in several applications. Since free resources are increasing both in per capita income and population, evolution will favor large rich societies. We will show how technological improvement is likely to increase per capita output as well as increase population, and how economically inefficient institutions such as bureaucracy arise.

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1. Introduction

> no possible form of society [can] prevent the almost constant action of misery upon a great part of mankind
>
> There are some men, even in the highest rank, who are prevented from marrying by the idea of the expenses that they must retrench
>
> Malthus [48]

We are all familiar with the caricature of the Malthusian theory of population: population grows until it is checked by disease and starvation. In the long-run we are all at the boundary of subsistence, on the margin between life and death. And while we may seem to have escaped for a time, perhaps ultimately the rapidly growing developing countries will overwhelm the gradually shrinking rich developed world and sink us all back into misery.

Malthus was more subtle in his thinking than this caricature. While he wrote of positive checks on population such as disease and starvation, he also wrote of preventative checks such as delayed marriage. The purpose of this paper is to present an alternative to simple Malthusian theory, a theory driven by societies that compete over resources. The basic idea is straightforward. Imagine two societies side by side. One is a society of unchecked breeders, of subsistence farmers living on the edge of starvation, their population limited only by the lack of any additional food to feed extra hungry mouths. Next door is a society with high property requirements for marriage and strong penalties for out-of-wedlock birth - a social arrangement quite common in history, and one that Malthus acknowledges, although he is skeptical that such arrangements can persist in the long-run. This non-Malthusian society naturally has output well in excess of subsistence.

Each social arrangement is incentive compatible. Let us then ask the key question: Which will dominate in the long-run? What happens when a disciplined and rich society turns its covetous eye towards the land of their more numerous but poorer neighbors? How indeed are the wretched poor - for whom to take even an hour away from toil in the fields is to starve - to be able to defend themselves from well-fed and well-armed intruders? The question answers itself.

In our setting we measure the ability of a society to impose itself on its neighbors and to resist encroachment by what we call *free resources.* This is the output available to society after the payments necessary for subsistence and for the incentives needed to induce production are made and after other claims such as transfer payments and resources absorbed by elites are paid.

Our goal in this paper is to examine a theory of evolution in the presence of conflict between societies. We adopt the theoretical approach pioneered by Kandori, Mailath and Rob [43] and Young [58]. We suppose that people live on different plots of land. Our model starts with the incentives of the individual, and with the possibility of random changes in behavior - mutations if you wish - that may drive social arrangements on a particular plot of land to various incentive compatible equilibria. In the presence of multiple Nash equilibria - a situation we will argue is ubiquitous in a social setting - will not group selection operate in favor of the more efficient Nash equilibria? If people are free to choose which plot of land to live on, Ely [32] showed this is exactly the case.[3] But we do not believe that historically people have generally moved from one location to another through a kind of voluntary immigration into the arms of welcoming neighbors. Rather people and institutions have more often spread through invasion - most often in the form of physical conquest, but also through means such as proselytizers and missionaries, or just exploration of new territory.[4]

We consider, then, a setting in which societies are formed of plots of land with similar social institutions, and these institutions are spread through invasion. Formally we examine a conflict resolution function similar to those

[3]Ely uses a model similar to the one used here, but similar results using more biologically oriented models have been around for some time. For example Aoki [1] uses a migration model to study efficiency, while more recently Rogers, Deshpande and Feldman [54] use a migration model to show how unequal resources can lead to long-run inequality.

[4]Note that our model is not incompatible with the voluntary spread of ideas. Although in our examples we take free resources to be oriented towards conflict, the theory requires only that free resources increase the chance of "disrupting" neighbors. For example, it may be that the reason that a group chooses to join a neighboring society is not because they are enslaved by invaders, but rather that they are envious of their neighbors higher standard of living. In this case free resources would simply be per capita income. However, at least for pre-historic times, Bowles [20] estimates of the mortality due to conflict is evidence that conflict is an important force.

studied in the economic literature on warfare.[5] What matters in our context are free resources - the resources that are available for conquest and for defense against potential conquerors. We imagine also that what matters are the overall resources of a society: wars are not fought by individual villages on particular plots of land, but by entire societies that share a common set of institutions.

In this setting we show that in the long-run it is the incentive compatible (Nash equilibrium) institutions that generates the most free-resources that will emerge in the long-run. Interestingly what matter most is not the ability of such a society to conquer its neighbors, but rather its ability to defend itself against encroachers.

The result yields a positive theory of population size: as long as there are incentive compatible institutions that control population growth, the equilibrium population is the one that maximizes total free resources. This is inconsistent with growing so large as to reach subsistence, as such a society generates no free resources. It is equally inconsistent with maximizing per capita output, since this requires a very tiny society that generates many free resources per person, but very few in total.[6] Rather the long-run population is at an intermediate level, greater than that which maximizes per capita income, but less than subsistence.

We then examine the impact of technological change in a population setting and uncover very non-Malthusian results. Malthus predicts that the benefits of technological change will in the long-run be dissipated entirely in increased population with no increase in per capita output, which remains at subsistence. When there is relatively strong diminishing returns on plots of land, maximization of free resources implies that improved technology results primarily in increased per capita output. However, depending on the underlying returns to population size, technological change can also result in diminished per capita output in some parameter range. The Malthusian case of per capita output independent of technology will only occur as a non-

[5]See, for example, Garfinkel and Skaperdas [37], Hausken [39] and Hirshleifer [40]. An important focus of this literature has been in figuring out how shares are determined by conflict resolution function.

[6]Maximizing free resources is clearly not the same as maximizing per capita output. Anticipating some notation, if total output is a function $Y(z)$ of population size z and B is subsistence level of per capita output, free resources in a simple population model are $Y(z) - zB = z[Y(z)/z - B]$.

generic accident. For simple and plausible cases, continued technological improvement first lowers then raises per capita output. This theory is very much more in accord with the evidence than Malthusian theory.[7]

Maximization of free resources leads more broadly to a positive theory of the state: it has implications for institutions other than those that govern population size. It does not imply, as does, for example, the theory of Ely, economic efficiency. In a setting of moral hazard, we show how maximization of free resources can lead to inefficiently low levels of output (Section 6).[8] In a setting of hidden endowments, we show how an inefficient bureaucracy will generate more free resources than a more efficient libertarian state (Section 7).

The idea that evolution can lead to both cooperation and inefficiency is scarcely new, nor is the idea that evolutionary pressure may be driven by conflict. There is a long literature on group selection in evolution: there may be positive assortative matching as discussed by Bergstrom [9]. Or there can be noise that leads to a trade-off between incentive constraints and group welfare as in the work of Price [51, 52]. Yet another approach is through differential extinction as in Boorman and Levitt [16]. Conflict, as opposed to migration, as a source of evolutionary pressure is examined in Bowles [19], who shows how intergroup competition can lead to the evolution of altruism. Bowles, Choi and Hopfensitz [23] and Choi and Bowles [24] study in group altruism versus out group hostility in a model driven by conflict . Rowthorn and Seabright [55] explain a drop in welfare during the neolithic transition as arising from the greater difficulty of defending agricultural resources.

More broadly, there is a great deal of work on the evolution of preferences as well as of institutions: for example Blume and Easley [12], Dekel, Ely and Yilankaya [27], Alger and Weibull [6], Levine et al [45] or Bottazzi and Dindo [17]. Some of this work is focused more on biological evolution than social evolution. As Bisin [10] and Bisin and Topa [11] point out the two are not the same.

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[7]This theory of population size of a given geographical extent should be compared to the theory of Alesina and Spolaore [5] who examine the optimal geographical extent of a nation.

[8]There are many other channels through which evolution can lead to inefficiency. For examples Bowles [18] discusses how inefficiency can arise in a Kandori, Mailath and Rob [43] and Young [58] type of setting with groups when they are of different sizes or have different memory lengths.

This paper is driven by somewhat different goals than earlier work. We are interested in an environment that can encompass relatively general games and strategy spaces; in an environment where individual incentives matter a great deal; and in an environment where the selection between the resulting equilibria are driven by conflict over resources ("land"). By employing the stochastic tools of by Kandori, Mailath and Rob [43] and Young [58] we are able with relatively weak assumptions to characterize stochastically stable states - the "typical" states of the system - as those among the incentive compatible states that feature large societies maximizing free resources.

2. The Economic Environment

Time lasts forever $t = 1, \ldots$. There are J identical plots of land $j = 1, \ldots J$. In each period objective circumstances on a particular plot of land are described by one of a finite set of *capital structures* $\omega_t^j \in \Omega$. The capital structure ω_t^j is a broader notion than that of the capital stock (which is ordinarily a one dimension aggregate): it includes private and public capital and infrastructure as well as population, human capital, and knowledge capital. As we will allow only finitely possible different investments and do not contemplate a model with unlimited growth, it is reasonable also to restrict attention to a finite number of possible capital structures. On each plot of land there are N potential players or player roles $i = 1, \ldots N$. We say "potential players" or "player roles" since we allow for the possibility of varying population sizes, and so not all players may get to play for all ω_t^j. For terminological simplicity, however, we will simply say "players." Each player i on each plot of land j chooses one of a finite number of actions $a_t^{ij} \in A^i$. Actions describe production and consumption as well as reproduction decisions, but also political actions concerning institutions and power structure. We use $a_t^j \in A$ and $a_t^{-ij} \in A^{-i}$ for profiles of actions on a particular plot of land j. Since the set of possible actions is independent of the capital structure, an action may be a contingent response. For example, it may have the form "have many children if population is low and have few children if the population is high." In addition every potential player i takes an action regardless of whether or not that player gets to play. So an action also has the contingent interpretation "take this action if the opportunity to play arises." We will consider specific examples shortly.

Players are assumed to be myopic, caring only about the immediate consequences of actions taken in the current period on the plot of land on which they reside. Put differently we consider relatively long periods encompassing the horizon of the players. The preferences of players are described by a utility function $u^i(a_t^j, \omega_t^j)$. We refer to the game on a particular plot of land with a particular ω_t^j induced by these utility functions during a particular period as the *stage game*.

Actions, however, also have future consequences, although by assumption players do not care about these consequences. One consequence of actions taken on a particular plot of land j is that it determines the capital structure on that plot next period, according to $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$. The other set of future consequences of actions involve interactions between plots of land. Interactions between plots, as well as behavior, depend in part on a noise parameter $\epsilon \geq 0$. Subsequently we will be considering limits as $\epsilon \to 0$. Depending on players play on the various plots there is a possibility each period t that a single plot of land k is *disrupted* to an action profile $a_{t+1}^j \in A$ the following period. This disruption may have the form of conquest, that is the new profile that k is forced to play may be the same as that of a "conqueror," but it is a more general concept: for the theory it matters only that the plot that is disrupted be forced to adopt a different action profile than the one hitherto used. For example, the result of conquest may not be that the conquered adopt the customs of the conquerors, but rather than the conquered fall into anarchy. Let $a_t = (a_t^j)^{j=1,\dots J}$ denote the profile of actions over players and plots, and $\omega_t = (\omega_t^j)^{j=1,\dots,J}$. The probability that plot k is disrupted to action a_{t+1}^j (which it will play at $t+1$) is given by the *conflict resolution function* $\pi^k(a_{t+1}^j, a_t, \omega_t)[\epsilon] \geq 0$ where since at most one plot can be disrupted $\sum_{k=1}^J \sum_{a_{t+1}^j \neq a_t^j} \pi^k(a_{t+1}^j, a_t, \omega_t)[\epsilon] \leq 1$. In fact we assume that this inequality is strict, so that there is a strictly positive probability that no disruption occurs, and that $\pi^k(a_{t+1}^j, a_t, \omega_t)[\epsilon] > 0$ when $\epsilon > 0$.

That completes the basic description of the physical setting of the model; however we do not consider completely arbitrary functions g, π^k. We turn next to assumptions about the setting.

2.1. Dynamics of the Capital Structure

The evolution of the capital structure g is assumed to have two properties. First, every stock should be reachable from every other and if players maintain the same action profile for a sufficient number of periods then a steady state is reached:

Assumption 1. *[Irreducibility and Steady State] For every pair ω_1^j, ω^j there is a sequence of action profiles $a_1^j, a_2^j \ldots a_\tau^j$ and a corresponding sequence $\omega_2^j, \omega_3^j \ldots \omega_{\tau+1}^j$ such that $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$ with $\omega_{\tau+1}^j = \omega^j$. There is a number T such that for any initial capital structure ω_1^j and any action profile a^j there is an $\omega^j(a^j)$ such that if $\omega_{t+1}^j = g(a^j, \omega_t^j)$ then for $t \geq T$ we have $\omega_t^j = \omega^j(a^j)$.*

Of particular interest are steady states that are Nash equilibria.

Definition 1. A (pure) *steady state Nash equilibrium on a plot j* is a pair consisting of a capital structure ω_t^j and an action profile a_t^j such that $\omega_t^j = g(a_t^j, \omega_t^j)$, (that is, a steady state) and such that a_t^{ij} is a best-response to a_t^{-ij} at ω_t^j, that is, the profile a_t^j is a Nash equilibrium at ω_t^j.

Note that there is no guarantee that steady state Nash equilibria exist: we will subsequently limit attention to situations where they do.

Before describing our assumptions about conflict resolution, let us illustrate this setup with an example - we will consider variations on this example with additional economic content later in the paper.

Example 1. In the *Malthusian Game* we suppose that all players have identical actions $A^i = \{1, 2, \ldots, N\}$ which represent a desired target population. It might seem more natural to consider the choice of action to be the number of children, but this is inflexible, since players might choose to have (for example) many children if the population is low, and few children if the population is large. The capital structure ω_t^j is simply the current population size, that is $\Omega = \{1, 2, \ldots, N\}$. Utility - as this is a Malthusian model - is taken to be a (linear) increasing function of the target population: $u^i(a_t^j, \omega_t^j) = a_t^{ij}$.

The future consequences of actions for a particular plot are given by $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$. Although only ω_t^j players are alive at time t we assume that the particular players who are born in that period are chosen randomly. Hence each player has an equal probability of playing. A simple model of the change in population is then to assume that population increase is determined by the average targets of the population. If players are chosen to play randomly this would itself be random, so we take the mean of these average targets over choices of players - in practice since players have equal probability of playing, it means we take the average of the target populations of the players living on a plot $\bar{a}_t^j = \sum_{i=1}^N a_t^{ij}/N$. We then assume that population increases $g(a_t^j, \omega_t^j) = \omega_t^j + 1$ when the average target is sufficiently greater than the current population $\bar{a}_t^j \geq \omega_t^j + 1/2$, declines $g(a_t^j, \omega_t^j) = \omega_t^j - 1$ when the average target is sufficiently below the current population $\bar{a}_t^j < \omega_t^j - 1/2$, and is stationary $g(a_t^j, \omega_t^j) = \omega_t^j$ otherwise, that is, when $\omega_t^j - 1/2 \le \bar{a}_t^j < \omega_t^j + 1/2$.

Observe that the function g is chosen to satisfy Assumption 1 [Irreducibility and Steady State]. Given a target ω^j if all players choose the action $a^{ij} = \omega^j$ then the target is hit in at most $N-1$ periods. On the other hand, for any action profile a^j there is a unique integer $\omega^j(a^j)$ satisfying $\bar{a}^j - 1/2 \le \omega^j(a^j) < \bar{a}^j + 1/2$ and if players play a^j then the steady state $\omega^j(a^j)$ is reached in at most $T = N-1$ periods as well.

The stage game played on each plot with utility function $u^i(a_t^j, \omega_t^j) = a_t^{ij}$ has the strictly dominant strategy for each player of playing $a_t^{ij} = N$, and if players play this way after at most $N-1$ period the population rises to the maximum possible. In particular on each plot j there is a unique steady state Nash equilibrium at $\omega_t^j = N$ and $a_t^{ij} = N$. That is of course the Malthusian result.

Remark 1. Notice that the second part of Assumption 1 [Irreducibility and Steady State] rules out the possibility that when players play a fixed profile this results in a cycle. The Malthus example makes clear how this might happen: if we assumed that population grows when the target is higher than current population and conversely, then when $\bar{a}^j$ is not an integer, ω_t^j would cycle back and forth between the population below and above $\bar{a}^j$. This is inconvenient without being terribly interesting, and we rule it out by requiring that there be a sufficient gap between the target and current population before population grows or declines.

2.2. Histories and Player Behavior

The behavior of players depends on the history of past events as well as their current utility function. Let H denote the set of L-length sequences of action profiles and capital structure in all plots. At the beginning of a period the *state* is $s_t \in S^* \equiv H^1 \times \dots H^J \times \{0, 1, 2, \dots J\} \times A$, that is a list of what has happened on each plot for the previous L periods $\tau = t-L+1, t-L+2, \dots t$, plus an indicator of which plot has been disrupted and the action to which it was disrupted. So an element s_t of the state space S^* has $J+2$ coordinates: the first J are histories of the actions and capital structures, $s_t^j = h_t^j$, $j = 1, \dots J$ where $h_t^j = (a_\tau^j, \omega_\tau^j)_{\tau=t-L+1}^t$; coordinate $s_t^{J+1} \in \{0, 1, 2, \dots J\}$ denotes the disrupted plot, where $s_t^{J+1} = 0$ is used to mean that no plot has been disrupted; and the last coordinate indicates the new action (if any), so $s_t^{J+2} \in A$. The stochastic process on which the paper is focused will be defined to be Markov on this state space, and we assume that there is a given initial condition s_1.

We now describe how the action profile on each plot j is determined at time t. If a plot was disrupted, that is $j = s_{t-1}^{J+1} > 0$, then players on that plot play $a_t^j = s_t^{J+2}$. Otherwise play is stochastic, each player plays independently, and play depends only on the history at that plot: we denote by $B^i(s_{t-1}^j)$ the probability distribution over A^i played by player i at time t on plot j.

For each player we distinguish two types of states:

Definition 2. A *quiet state* s_t for player i on plot j is a state in which neither the action profiles nor the capital structure has changed on that plot, $(a_{t-L+1}^j, \omega_{t-L+1}^j) = (a_{t-L+2}^j, \omega_{t-L+2}^j) = \cdots = (a_t^j, \omega_t^j)$, and for which a_t^{ij} is a best response to $a_t^{-i,j}, \omega_t^j$. We call a_t^{ij} the *status quo response.* Any state state for player i on plot j other than a quiet state is a *noisy state.*

In other words, in a quiet state, nothing has changed and player i has been doing the "right thing" for at least L periods. In this case, we assume that if not disrupted, the player continues to play the same way; otherwise there is some chance (appreciable in the sense of being independent of ϵ) of picking any other action:

Assumption 2. *If s_{t-1} is a quiet state where a_t^{ij} is the status quo response, then $B^i(s_{t-1}^j)(a_t^{ij}) = 1$. If s_{t-1} is a noisy state for player i on plot j then $B(s_{t-1}^j)(a_t^{ij}) > 0$ for all $a_t^{ij} \in A^i$.*

Definition 3. A state s_t is a *Nash state* if every plot of land is in a steady state Nash equilibrium and it is quiet for every player in every plot.

Notice that if a state is Nash then all plots are quiet, and hence unless there is a disruption, the next state will be the same as the current state. On the other hand a disrupted plot begins a possibly long epoch of turmoil which however, with positive probability, will end with the plot entering an existing society, which will then have free resources increased by the new addition. The process of evolution of societies is thus viewed as more flexible and general than a military conquest followed by submission of a loser. Societies are introduced formally in the next section.

Remark 2. This dynamic is a simplified version of Foster and Young [33] - it is a simple and relatively plausible model. It has the implication that in the absence of conflict each plot will be absorbed in some Nash equilibrium, and that all of these equilibria have some chance of occurring.

3. Societies and Conflict

The central idea of the paper is that conflict resolution depends in an important way on two things: the desire of players to expand into neighboring plots of land, and their ability to do so. Note that expansion may have many forms and motivations: two examples are conquest through warfare or conversion, but others are the desire to explore new territory, contact other societies and mix with them, propose values and possibly learn from outside communities. The Roman empire is a strong example of the first type of expansion; more modern expansions have often involved religious conversion - for example, the sending of religious missionaries, although this has often occurred in the context of warfare, for example the conversion of the South and Central American Indian populations to Christianity through a combination of conquest and missionary activity. Equally relevant is influence through exchange of goods spurred by explorations (think of Marco Polo), or the more modern culture spreading through the sale of goods ranging from Coca-Cola to television sets. Or going to the other extreme, we may think of the "curiosity", that is the expansionism, of the primitive hunters-gatherers.

There are two ideas we wish to capture. First, a prospective invader would find it much easier to conquer, say, Singapore, than, for example, Shanghai. The reason is that China, while per capita a poorer society than Singapore, has a much larger and more capable military. In other words, plots of land are organized into larger societies, and the ability of a society to defend itself - or to conquer other societies - depends at least in part on the aggregate resources of that society, not merely the resources of individual plots of land. To capture this idea we must model how plots of land are formed into larger societies, and this ultimately depends on how individuals feel about allegiance to a larger society. Second, societies differ in their inclination to export their ideas and social norms. Regardless of the form of expansion, expansionary institutions are not universal - an insular society is not likely to expand.[9] Religions such as Christianity and Islam have historically been expansionary trying actively to convert nonbelievers. By contrast since the diaspora Judaism has been relatively insular in this respect, and the same has been true of other groups such as the Old Believers in Czarist Russia.

We now first discuss willingness to expand. The attitudes of a plot of

[9]Our notion of expansionism is connected to Aoki, Lehmann and Feldman [2011]'s theory of the transmission of innovations.

land towards expansion and their willingness to belong to a larger society must be a consequence of the actions taken by individuals on that plot of land. Formally, we assume that each action profile also generates a single integer value $\chi(a_t^j) \in \mathbb{Z}$ on plot j (for both simplicity and conceptual reasons independent of the ω_t^j). We consider three possible attitudes towards expansion and social organization, represented by positive, negative and the zero value of $\chi(a_t^j)$. One possibility is that a plot of land may not wish to belong to a larger society, or may be unable to agree on doing so. We refer to such a plot using such an action profile as *isolated*, and represent this by assigning $\chi(a_t^j) = 0$.

For action profiles that constitute a willingness by the plot to belong to a larger society, the question arises as to which society the plot wishes to belong. At a minimum, we require that a plot using a particular profile a_t^j and wishing to belong to a larger society agrees that it is willing to join with other plots using the same profile. However, we also allow multiculturalism, that is, a plot may agree to be allied in a single society with other plots that use different profile - the European Union springs to mind as an example of such a society. Of course various complicated possibilities can occur: one plot playing $a_t^j = A$ may be willing to ally only with plots playing B, while a plot playing B may be willing to ally with either A or C. As our goal is not to understand the details of coalition formation, we assume this problem away: we assume that profiles are partitioned into subsets (societies), with the members of an element of the partition agreeing that they are willing to ally themselves with any other profile in the same subset. Formally, values of $\chi(a_t^j) \neq 0$ are taken to represent different elements of the partition, that is, different potential societies. All plots j with a common non-zero value x of $\chi(a_t^j)$ are taken to belong to the corresponding society, which will then represented by that integer x.

As we indicated, societies may or may not be expansionary. We use positive values of $\chi(a_t^j)$ for those societies that are expansionary, and negative values for those that are not. Since we are interested in settings with many Nash equilibria, we assume that at least one is in fact expansionary (that is willing to belong to an expansionary society):

Assumption 3. *There is at least one steady state Nash equilibrium which is expansionary, that is has* $\chi(a_t^j) > 0$.[10]

[10]Note that whether or not a society is expansionary plays no role in the determination

3.1. Regularity

We pause for a moment to introduce the concept of regularity which will be used extensively in the sequel. When we introduce the dynamics of disruption, the system as a whole will be a Markov process parametrized by a positive ϵ. We will characterize the long-run behavior of the Markov process when ϵ is small, that is, in the limit as $\epsilon \to 0$. This is known, for example from Young [58] to depend on the rate at which probabilities go to zero with ϵ.

Definition 4. Let $Q[\epsilon]$ be a function of ϵ. We say that Q is *regular* if $r[Q] \equiv \lim_{\epsilon\to 0} \log Q[\epsilon]/\log \epsilon$ exists and $r[Q] = 0$ implies $\lim_{\epsilon\to 0} Q[\epsilon] > 0$. For a regular Q we call $r[Q]$ the *resistance of* Q.

Remark 3. Notice that probability decreasing faster ("lower probability") means higher resistance. We shall presently introduce probability of disruption, and in that context resistance will be interpreted as a "resistance" to being disrupted.

3.2. Conflict Resolution and Free Resources

We now turn to the "ability to expand" aspect mentioned above and relate expansion to free resources. We begin by describing how the organization of plots into societies and the actions taken on those plots results in the disruption of plots of land through conflict between different societies. This is represented formally by the conflict resolution function, now described in greater detail.

First we define the probability of society x being disrupted, denoted by $\Pi(x, a_t, \omega_t)[\epsilon]$, as the probability that one of its plots is disrupted to an alternative action. Note the ϵ parameter. In the case $x \neq 0$ this is given by

$$\Pi(x, a_t, \omega_t)[\epsilon] = \sum_{k|\chi(a_t^k)=x} \sum_{a_{t+1}^j \neq a_t^k} \pi^k(a_{t+1}^j, a_t, \omega_t)[\epsilon],$$

and for an isolated society playing a_t^k by

$$\Pi(a_t^k, a_t, \omega_t)[\epsilon] = \sum_{a_{t+1}^j \neq a_t^k} \pi^k(a_{t+1}^j, a_t, \omega_t)[\epsilon].$$

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of Nash equilibrium.

We assume regularity of this disruption function:

Assumption 4. *The function* $\Pi(x, a_t, \omega_t)[\epsilon]$ *is regular, and normalized so that* $r[\Pi(x, a_t, \omega_t)] \leq 1$.

Notice that the upper bound on resistance is normalized to one.

As we said, the ability to expand depends not only on the desire to do so, but also on the resources available. So we assume that the action profile and capital structure generate a strictly positive value $f(a_t^j, \omega_t^j) > 0$ called *free resources* which can be thought of as total output on plot j in excess of subsistence, incentive and other voluntary payments. Free resources are the resources a society can exploit to expand. They depend on actions via production decisions and institutions in place, and on the capital structure which includes population but also non-military, for instance transportation, infrastructure which is relevant for expansion potential.

Example 2. [Malthus Example 1 continued] In the Malthusian model let the function $Y(z)$ be a non-negative function of the non-negative real argument z, where Y represents aggregate output as a function of the population (possibly fractional) on a plot of land. We assume that this function is strictly concave, differentiable, and strictly increasing. With one more person than the maximum, per capita output is $Y(N+1)/(N+1)$. We assume that the subsistence level B which we think of as the level of basic needs required to survive satisfies $Y(N)/N > B > Y(N+1)/(N+1)$.[11] This means that while the population can grow to drive per capita output to nearly the subsistence level, it cannot quite drive it all the way there. Free resources then are given by $f(a_t^j, \omega_t^j) = Y(\omega_t^j) - \omega_t^j B$, that is the output that is not required for subsistence. Notice by construction this is strictly positive.

As we indicated, in resolving conflict the central idea is that it is not free resources on a particular plot of land that is critical, but rather some measure of the total free resources available to a country or society. It would seem most natural to define the aggregate free resources $F(x, a_t, \omega_t)$ available to a society $x \neq 0$ as the sum of free resources belonging to the plots of that society

$$\sum_{\chi(a_t^j)=x} f(a_t^j, \omega_t^j).$$

[11] This is consistent with a standard definition of subsistence as the lowest level of consumption below which the population must decline.

However, it may be that aggregate free resources grow less than linearly with the number of plots, so we allow more general forms of aggregation. For example two plots each with a unit of free resources may be weaker than a single plot with two units of free resources if not all the units can be mobilized for joint operations or there are other coordination problems between the plots. Denote by $J(x)$ the number of plots in society x, and by $\bar{f}(x, a_t, \omega_t)$ the average free resources per plot, in case $x \neq 0$ given by $\sum_{\chi(a_t^j)=x} f(a_t^j, \omega_t^j)/J(x)$. We we will take aggregate resources available to society x as given by a function of average free resources per plot and the fraction of plots controlled by the society

$$F(x, a_t, \omega_t) = \Phi(\bar{f}(x, a_t, \omega_t), J(x)/J).$$

On Φ we make the following

Assumption 5. *The function* $\Phi(\overline{f}, \phi)$ *is continuous and differentiable in the interior, with strictly positive partial derivatives and* $\lim \Phi_{\phi \to 0} = 0$.

So more free resources coming from any plot old or new always increase the society's free resources. Note that if a society x is not present in a_t then the corresponding aggregate free resources F are zero.

3.3. Disruption, Expansionism and Free Resources

We are now in a position to state our three assumptions relating disruption probability Π to free resources. The first assumption is that comparing two societies, resistance to disruption is lower for the one with fewer free resources, and indeed resistance to disruption when there is an expansionary society with at least as many free resources is zero. Let $E(x)$ denote whether x is expansionary or not, that is, $E = 1$ if $x > 0$, and $E = 0$ otherwise.

Assumption 6. *[Monotonicity] If* $F(x, a_t, \omega_t) \leq F(x', a_t, \omega_t)$ *then* $r[\Pi(x, a_t, \omega_t)] \leq r[\Pi(x', a_t, \omega_t)]$*, and* $r[\Pi(x, a_t, \omega_t)] = 0$ *if* $E(x') = 1$*. Moreover, if* a_{t+1} *differs from* a_t *solely in that society* x *has lost a single plot of land, then* $r[\Pi(x, a_{t+1}, \omega_t)] \leq r[\Pi(x, a_t, \omega_t)]$.

The first part says roughly that if two societies coexist in the sense that they are part of the same a_t, ω_t then the one with more free resources has at least the same resistance as the one with fewer free resources. The second part strengthens this to say that an expansionary society with at least as many free resources as a rival in fact has an appreciable chance of disrupting it. This rules out the possibility of there simultaneously being multiple expansionary societies for a substantial length of time, and enables us to use an analysis akin to Ellison [31]'s method of the radius. Without it, the analysis is more akin to his method of the co-radius, and we have neither been able to establish the result nor provide a counter-example in that case. The third part says that losing land does not increase resistance.

Our next assumption on Π specifies how resistance depends directly on the ratio of free resources when there are only two societies. Say that a_t is *binary* if there are only two societies, which we denote as x and x'.

Assumption 7. *[Ratio] If a_t is binary then*

$$r[\Pi(x, a_t, \omega_t)] = q(F(x', a_t, \omega_t)/F(x, a_t, \omega_t), E(x')),$$

where q is non-increasing and left continuous in the first argument, $q(0, E) = q(\phi, 0) = 1$ and there exists $\phi > 0$ such that $q(\phi, 1) > 0$.

In other words, resistance in the binary case depends monotonically on free resources and whether or not the rival society is expansionary. Moreover $q(0, E) = 1$ says that when the opponent has zero free resources resistance is at the highest possible level - recall that we have assumed that resistance is always bounded above by one. In addition $q(\phi, 0) = 1$ asserts that a plot that is not expansionist always generates the same maximal resistance regardless of how many free resources it has available. Notice that the assumption $q(0, E) = 1$ applies to *mutations* - actions that are not currently being used. In this setup the chance of a mutation entering the population is the same (in resistance terms) for all mutations - the free resources associated with the mutant action profile become available for initiating or defending against disruption only after it enters the population - that is, the period after the mutation takes place. This follows from our assumption that the societies corresponding to action profiles that are not currently in use have zero free resources. The idea is that mutants need a period to get organized.

Observe that Assumption 6 implies that $\bar{\phi} = \inf\{\phi | q(\phi, 1) = 0\} \leq 1$, since eventually if an expansionary society has enough free resources, it has an appreciable chance of disrupting a rival plot of land. Note that because $r[q(\phi, 1)]$ is left rather than right continuous we must use the inf here, and because we have assumed explicitly that there is some value of $\phi > 0$ for which the resistance is strictly positive, we know that $\bar{\phi} > 0$. Looking at what this means in terms of probability, we see that this zero up to $\bar{\phi}$ after which it becomes strictly positive. That is, in the limiting case a sufficiently small society has no chance at all of disrupting a plot from a larger one.

The last assumption on Π states that disruption is not more likely when opponents are divided. Let $\Upsilon(a_t)$ denote all the societies in a_t, that is the values of $x \neq 0$ in the range of χ plus the different values of a_t^j that correspond to isolated societies, that is with $\chi(a_t^j) = 0$.

Assumption 8. *[Divided Opponents] If a_t is binary, $\tilde{a}_t$ has $F(x, a_t, \omega_t) = F(x, \tilde{a}_t, \omega_t)$ and $\sum_{x' \in \Upsilon(a_t) \backslash x} F(x', a_t, \omega_t) \geq \sum_{x' \in \Upsilon(\tilde{a}_t) \backslash x} F(x', \tilde{a}_t, \omega_t)$ then $r[\Pi(x, a_t, \omega_t)] \leq r[\Pi(x, \tilde{a}_t, \omega_t)]$.*

4. Dynamics and Stochastically Stable States

The dynamics of the stage game and of disruption together with the behavioral rules of the players induce a Markov process $M^*(\epsilon, J)$ on the state space S^*. We are interested in this process, but primarily in the limit of this process as $\epsilon \to 0$.

Notice that for $\epsilon > 0$ every combination of actions on plots has positive probability because there is positive probability of disruption, hence every sequence of L combinations of profiles has positive probability. However, some ω^j sequences may be unreachable, so such states must be transient. Let S be the set of non-transient states. Since the states not in S are transient let us assume that the initial condition is in S so that we restrict attention to $M(\epsilon, J)$, the process on S.

Theorem 1. *For $\epsilon > 0$ the process $M[\epsilon, J]$ is aperiodic and irreducible and hence has a unique invariant distribution $m[\epsilon, J]$.*

Proof. This follows from the fact that every combination of actions on every plot has positive probability and the definition of the state space $S \subseteq S^*$. □

We denote by $S[0, J]$ the ergodic classes of $M[0, J]$. We refer to steady state Nash equilibria and to societies as expansionary if the corresponding action profile is.

Proposition 1. *$\sigma \in S[0, J]$ if and only σ is a singleton, that is, $\sigma = \{s_t\}$, if s_t is a Nash state, and if s_t has either no expansionary society, or a single expansionary society such that all other societies (if any) have positive resistance to disruption.*

Proof. Follows directly from the definitions. See Appendix I. □

Hereafter we simply write $s_t \in S[0, J]$. By Proposition 1 there are three types of Nash states in $S[0, J]$. There are *monolithic states* consisting of a single expansionary society; there are *mixed states* consisting of a single expansionary society and at least one non-expansionary society, and there are *non-expansionary states* in which there is no expansionary society. For monolithic states $s_t \in S[0, J]$ there is a unique per plot average level of free resources. Of particular importance will turn out to be those monolithic states that have the greatest free resources among all monolithic states. We call those *maximum free resource monolithic states.* We denote the corresponding maximum per-plot free resources by $f^{\max}$.

We use the following Theorem from Young [58]:

Theorem 2. $m = \lim_{\epsilon \to 0} m[\epsilon, J]$ *exists and* $m(s_t, J) > 0$ *implies* $s_t \in S[0, J]$.

Let $S[m, J] \subseteq S[0, J]$ to be the set of states that have positive probability in the limit (that is $s_t \in S[m, J]$ iff $m(s_t, J) > 0$). These are called the *stochastically stable states.* Let $f_{\min}(J)$ denote the least per plot free resources of any stochastically stable monolithic steady state, and recall that $f^{\max}$ is the maximum per plot free resources of any monolithic steady state. The central result of the paper is

Theorem 3. *[Main Theorem] If s_t is a maximum free resource monolithic state then it is stochastically stable. As to the converse: For J large enough every stochastically stable state is monolithic, and* $\lim_{J \to \infty} f_{\min}(J) \to f^{\max}$.

Proof. Follows from least resistance tree arguments detailed in Theorem 5 and Corollary 2 in Appendix I. □

Remark 4. The most important implication of the Theorem is that for large J the stochastically stable states approximately maximize free resources among Nash states. It is worth indicating how the stochastically stable states relate to the dynamics of the Markov process for $\epsilon > 0$. It is important to understand that the system does not in any sense converge asymptotically to the stochastically stable state. Rather the expected length of time the system spends at that state is roughly proportional to $1/\epsilon$ raised to the power of the least resistance of leaving the state.[12] The system is genuinely random: dis-

[12] This is shown by Ellison [31] who refers to this least resistance as the radius of the state.

ruptions can and do occur. Suppose the system is currently in a stochastically stable state. Sooner or later there will be enough unlucky coincidences to disrupt it and the system will fluctuate randomly for some period of time as there is an appreciable probability that individuals will change their behavior. Eventually the system will settle down to some other steady state, not necessarily the stochastically stable one. However that steady state will also eventually be disrupted, more fluctuations will occur, then another steady state will be reached. At some point another stochastically stable state will be reached. The key point is that the amount of time spent at steady states is high relative to the amount of time the system spends fluctuating randomly, and the amount of time spent at the stochastically stable states is high relative to the amount of time spent during fluctuations and at steady states that are not stochastically stable.

Remark 5. *(Relation to Literature on Group Evolution)* The novelty of our approach lies in the fact that we study group evolution as evolution of Nash equilibria. Existing literature in the area mainly focuses on the interplay between individual and group evolutionary selection: individual behavior which increases fitness of a group, typically some form of "generosity", may be harmful for individual fitness. This is the case both in the Haystack Model as in Maynard Smith [49] or Richerson and Boyd [53] and in Bowles' model of conflict and evolution (Bowles [20]). The equilibrium dimension in the group selection literature is missing.

In relation to this trade-off our result may be interpreted as saying that evolution, favoring expansionism, favors generosity, which may be seen as a necessary condition for expansionism; but also that given generosity, it favors large groups maximizing free resources, which are needed to survive competition between groups.

5. Social Norms, Population Games and Growth

In our Malthusian example all the elaborate evolutionary arguments are irrelevant because there is just one Nash equilibrium: population is maximized subject to the subsistence constraint. This gives the Malthusian conclusion that population expands until everyone is at the subsistence level and any technological improvement in the long-run raises population but not per capita output. However, this Malthusian conclusion was not actually Malthus's conclusion as he correctly recognized that there are stable social norms - late marriage for example - that lead to per capita output above subsistence.

In real societies, long before the advent of birth control, population was controlled - largely, of course, by abstinence from intercourse. It is easy to imagine a stable social norm - a Nash equilibrium - that achieves this result: women are limited to a certain number of children, and anyone who attempts to violate the norm is put to death along with her children. In practice societies often used methods not so different than this. Marriage was limited and delayed through requirements of substantial accumulation of capital or side-payments as a prerequisite to get married, and unwed mothers were severely punished, in many cases through capital punishment. This seems to be understood by demographic historians such as Bacci [8].

There is, of course, a big debate about whether per capita income was above subsistence prior to the industrial revolution. Some estimates of per capita income such as those of Clark [26] suggest not. However, a close reading of the literature reveals serious problems. The most central problem is that at best what is computed is median per capita income - that is, the typical income of a poor person. Of course the upper classes consume considerably more than subsistence so the mean must also be above subsistence. A typical example of this problem is in the classical and much cited Ladurie [44] study of Languedoc peasants in France. Ignored in this study are the facts that the nobles live above subsistence; that the entire area made substantial payments to the King - and indeed the ability of France to conduct continual wars throughout this period indicates that substantial free resources were available. More serious students of historical per capita GDP such as Maddison [47] point out the Malthusian bias implicit in these studies of particular groups of poor individuals as an indication of per capita GDP.

From a game theoretic point of view, modeling the fact that there are many social norms is no longer an open problem. The folk theorem points to the existence of many social norms. Although the basic theorem involves an infinitely repeated game, with discounting the game will end in finite time with probability one. Moreover, there are folk theorems for games with overlapping generations of players as in Kandori [42], for finite horizon games where the stage game has multiple Nash equilibria as in Benoit and Krishna [30], and for one-shot self-referential games Levine and Pesendorfer [46]. As this literature is well developed we will adopt a simple two stage approach to get at the issue of population that maximizes free resources.

Imagine we are given a stage game with strategy spaces $\tilde{A}^i$ and utility functions $\tilde{u}^i(a_t, \omega_t) \geq 0$. We define an elaborated game by adding a second stage in which players choose a vector of N 0's or 1's, where if the ith component is zero this interpreted as "shun player i." We are given a threshold $N-1 \geq \tilde{N} > 0$ and define utility of player i in the elaborated game as $\Pi \leq 0$ if she is shunned by $\tilde{N}$ or more players in the second stage, and $\tilde{u}^i(a_t, \omega_t)$ otherwise. The interpretation is that a player who is shunned still receives consumption of B but has reduced actual utility due to shunning. Strategies in the elaborated games are a choice of first period action in $\tilde{A}^i$ and a contingent response to the profile in the first period. A moment of reflection shows that

Proposition 2. *If* $\Pi < 0$ *every profile in* $\tilde{A}$ *is a Nash equilibrium enforced by the strategy of shunning anyone who does not play the target action in the first period.*

Notice that this equilibrium relies on perfect observability and involves a great deal of indifference - it exploits the fact that nobody suffers from shunning (only from being shunned), and even if they did, that no shunning occurs on the equilibrium path. However, the issue of indifference is relatively minor. It is true that adding a cost of shunning and noise so that shunning occurs on the equilibrium path would destroy this folk theorem result; but it can equally be restored by adding an infinite sequence of punishment rounds, or by making the game self-referential as in the model of Levine and Pesendorfer [46]. The key fact is that in theory and in practice strict social norms are self-referential in the sense that following the norm includes punishing those that violate it. This has been made explicit in the repeated as well as the static setting, so we will not pursue the matter here, taking the elaborated game as a simple folk theorem model.

The issue of perfect observability is rather more serious. Whether a "folk-like" theorem holds in practice depends on how rapidly information emerges relative to how patient people are. In the case of population games, since the number of children a woman has is relatively public, assuming perfect observability makes sense. In subsequent sections we will consider the possibility of private information: this will result both in the failure of the folk theorem as well as resulting in the possibility that shunning may occur in equilibrium.

Example 3. This is simply the elaborated form of the earlier Malthusian example, including the assumption that all profiles are expansionary. Hence by Proposition 2 and Theorem 3 for sufficiently large J the unique stochastically stable state maximizes free resources. Our goal is now to investigate how population size depends on technology in this model.

Recall that output $Y(z)$ is a differentiable, strictly concave and strictly increasing function of population z. The subsistence level satisfies $Y(N)/N > B \geq Y(N+1)/(N+1)$. We now generalize this model to allow for neutral technological change, so that output is given by $AY(z)$ where $A \geq 1$. As we vary A we hold fixed the subsistence level B so that the potential population N is allowed to depend on A. We assume that all resources not used for subsistence are free. Hence free resources are given by $f(z) = AY(z) - zB$. We will explore the (unique since $Y(z)$ is strictly concave) real value of z that maximizes free resources; since the problem is concave, the optimal integer valued population must be one of the adjoining grid points.

First consider the Malthusian case - that is, the game without elaboration. Here we consider the unique value of z that satisfies $AY(z)/z = B$. This is strictly increasing in A and gives the usual Malthusian result: technological change in the long-run leaves per capita income unchanged and leads merely to an increase in population. The situation in the elaborated game is given by

Proposition 3. *The value of z^* that maximizes free resources is strictly increasing in A. Per capita output increases if and only if*

$$-\frac{zY''(z^*)}{Y'(z^*)} > 1 - \frac{zY'(z^*)}{Y(z^*)}.$$

Proof. We want the derivative of $AY(z)/z$ with respect to A, where z is a function of A defined implicitly by the equation $AY'(z) = B$. From the latter condition we get $0 = Y'dA + AY''dz$ whence $dz/dA = -Y'/AY'' > 0$. Computing $d(AY(z)/z)/dA$ results in the last condition. □

Corollary 1. *Let again z^* be free resource maximizing, as a function of A. In the Cobb-Douglas case $Y(z) = z^\alpha$ per capita output is independent of A. In the logarithmic case $Y(z) = \log(a+z)$, $a > 0$ per capita output is increasing for sufficiently large A, while for large enough a it is decreasing for small A and increasing for large A.*

Proof. In the Cobb-Douglas case we have

$$-\frac{zY"(z)}{Y'(z)} = 1 - \frac{zY'(z)}{Y(z)} = 1 - \alpha,$$

so this case is completely neutral, just as in the Malthus case.

In the logarithmic case $Y'(z) = 1/(a+z)$, $Y"(z) = -1/(a+z)^2$. The condition in Proposition 3 can be simplified to

$$\frac{z}{a+z} > 1 - \frac{z}{(a+z)\log(a+z)}$$

which is equivalent to $\log(a+z) < z/a$ because $z \geq 1$. Now $\log(a+z) < \log a + z/a$ for all $z > 0$, so the above inequality is satisfied for all $z \geq 1$ if $a \leq 1$. For $a > 1$ it is true for z large enough (the RHS goes to ∞ as $z \to \infty$), so it is satisfied for big enough A. Looking at $z = 1$ we get

$$a \log(1+a) < 1$$

which clearly fails for big enough a. Hence for large a per-capita income first goes down then up. □

It is sometimes claimed that farming societies were worse off than hunter gatherers, while of course industrial societies are much better off. The case of logarithmic output for large a provides one possible theory of why this might be. Concerning differential effects of technological progress on population size and income per capita, Ashraf and Galor [7] elaborate evidence of increase in both population and income per capita in the last two thousand years, and estimate that technological progress in this period has had more impact on population size than on income per capita.

Of course there remains the question of whether we should imagine that technology is more like that of Cobb-Douglas or of the logarithmic form in population size. It seems compelling that only so many people can fit on a particular plot of land before production becomes impossible due to overcrowding. In this case it is easy to see why per capita output must increase with technological improvement: once the upper bound on population is reached there is no point in adding more people regardless of the state of technology. The only way to take advantage of improved technology to get more free resources is through increased per capita output. In other words, we expect that returns to population drops to zero as population grows. While we have not yet reached the unfortunate state of affairs in which production is impossible due to overcrowding, this argument does indicate some reason to think that returns to population diminish rather quickly as population on a plot of land grows. It suggests that the more rapidly decreasing returns of the logarithmic model may make more sense than the rather slowly decreasing returns of the Cobb-Douglas model.[13]

6. Choice of Technology Games: What are Free Resources?

We now wish to look more closely at free resource maximization, in particular we would like to study how it differs from efficiency. If we think of free resources as what is left over after agents receive subsistence payments, we can think of this question as the obverse of "what is the subsistence level." Historically, the subsistence level meant "the physical requirements to survive and reproduce." In the hands of modern economic historians such as Clark [26] "subsistence" has become an elastic concept meaning the "some socially determined level of per capita income above which population decreases and below which it increases." This is somewhat awkward as the cross-sectional evidence is clear that rich countries reproduce as much lower rates than poor ones.

While "subsistence" has always led a difficult conceptual existence, the notion of free resources is considerably more clear, and we analyze examples in this and the next section. The key thing is that free resources are (part of) what is left over after people have been paid what is required to get those resources produced in the first place.[14] In particular, incentive payments without which production will not take place cannot be part of free resources. With private information, which is the case we cover in the present section, the corresponding informational rents are not part of free resources. But there are other examples: on a different vein, Weightman [57] reports how British workers in the 19th century consumed roast beef. It was a luxury, but it made them stronger, better workers than on the continent where diet was poorer; so presumably it made them better soldiers as well. In general a diet above subsistence may increase free resources because it increases the ability of workers to produce output. Hence the payments that enable this improved diets are cannot be part of free resources. Conversely, luxuries that

[13]The mechanism here is not dissimilar to that discussed in Hansen and Prescott [38]: there it is the exhaustion of land that forces a change to a capital based technology that increases per capita income.

[14]A society may also choose to voluntarily use part of output for other purposes that prevent those resources from being "free," but such a society will not maximize free resources. Hence we focus on those payments that cannot be part of free resources.

people were entitled to during peace may be given up during time of war, increasing free resources and fighting potential.

With a single composite output, free resource maximization for a society is similar in many respects to profit maximization for a firm. As we shall see, the primary difference is that while participation constraints in standard principal agent model are formulated in terms of utility, in this setting the corresponding notion of the subsistence constraint is formulated in terms of consumption. Incidentally, this connection between profit maximization and free resource maximization may explain the historical importance of what can only be described as profit maximizing monarchies.

We will now hold the population fixed, and consider the choice of different production technologies. Following the repeated game literature such as Fudenberg Levine and Maskin [36] we can map an underlying mechanism design problem into a game by adding a stage in which people vote for the preferred mechanism. Either everyone gets some very low level of utility because they disagree, or if they all agree, then the agreed upon mechanism is implemented. In such a game every incentive compatible mechanism is a Nash equilibrium. Hence we may focus on the issue of which incentive compatible mechanism maximizes free resources, as this will be the one chosen by evolutionary forces in the long-run. In particular, the interesting Nash equilibria can be determined through the revelation principle.

In this section we consider a simple and relatively standard principal-agent model of effort provision. There is a representative agent with linear utility. We continue to have available the shunning technology used in the population model of Section 5. Consequently both output and utility depend both on effort and on whether or not the agent is shunned. With the unobserved effort level $e \in \{0, 1\}$ the agent produces observed low output $B + y$ with probability $1 - \pi_e$ and high output $B + Y$ with probability π_e when not shunned, and $B + y_S$ and $B + Y_S$ respectively when shunned. Effort increases the probability of higher output, $y < Y$, $y_S < Y_S$ and $\pi_0 < \pi_1$; and for a given level of effort there is less output if shunned: $y_S < y$, $Y_S < Y$. Notice that the realization of the observed state of output is not affected by shunning, but the actual amount of output in that state is. We continue to denote utility if shunned by $\Pi \leq 0$.

Finally, each agent enjoys utility from a fraction $\rho \in [0, 1]$ of per-capita free resources, which we denote by f. The idea here is that in "normal" times the fraction ρ may even be 1, although during the brief periods of war those free resources are no longer used for consumption.

The income of the agent and probability of being shunned depend on the observed output of that agent. We denote then the incentive payments and probability of shunning by w, W and p, P in low and high output states; also, different wages w, w_S and W, W_S may be paid conditional on whether shunning takes place. [15]

A key element of the analysis is the subsistence constraint. *Ex post* different individuals are paid differently, so they will not generally reproduce at the same rate. The steady state population can be maintained, for example, when those who are well paid reproduce at a high rate, even though those who are poorly paid are unable to reproduce at all. A simple model of this, consistent with risk neutrality, is to assume that the subsistence constraint holds in expected value terms. That is, given the expected wage

$$\bar{W} = (1 - \pi_e)[(1 - p)w + pw_S] + \pi_e[(1 - P)W + PW_S],$$

the subsistence constraint is $\bar{W} \geq B$. This boils down to assuming that the rate of reproduction is proportional to income. Note that the subsistence constraint is akin to the participation constraint in the standard principal-agent problem - the difference is that subsistence is formulated in terms of consumption while participation is formulated in terms of utility. In other words, a participation constraint would have to take account the cost of being shunned, but the subsistence constraint does not.

The production function for expected output and free resources are respectively

$$\bar{Y}_e = B + (1 - \pi_e)[y - p(y - y_S)] + \pi_e[Y - P(Y - Y_S)]$$

and

$$f_e = B+(1-\pi_e)[y-p(y-y_S)-(1-p)w-pw_S]+\pi_e[Y-P(Y-Y_S)-(1-P)W-PW_S],$$

so that

$$f_e = \bar{Y}_e - \bar{W}.$$

[15] For a similar idea of punishment in a principal-agent model, whose solution has similar qualitative features, see Acemoglu and Wolitzky [3]. That paper and the surrounding literature focus on profit maximization of firms, although conceptually different than free resource maximization, has many of the same implications.

Social feasibility requires $\bar{W} \leq \bar{Y}_e$, or equivalently $f_e \geq 0$.

On the consumer side, expected utility is

$$u_e = -e + \rho f_e + (1 - \pi_e)[w - p(w - \Pi)] + \pi_e[W - P(W - \Pi)]$$

This enables us to compute the incentive compatibility constraint as

$$(1-\pi_1)[(1-p)w + p\Pi] + \pi_1[(1-P)W + P\Pi] - 1 \\ \geq (1-\pi_0)[(1-p)w + p\Pi] + \pi_0[(1-P)W + P\Pi],$$

which setting $\Delta \equiv (\pi_1 - \pi_0)^{-1}$ may be written as[16]

$$(1-P)W - (1-p)w - (p-P)\Pi \geq \Delta.$$

Our next result compares the maximization of utility and free resources, subject to the relevant constraints.

Proposition 4. *Under utility maximization*

1. shunning never occurs (so there is no output loss)

2. effort is induced if and only if the expected output gain is no smaller that effort cost (that is iff $(\pi_1 - \pi_0)(Y - y) \geq 1$*)*

3. all output is paid out in wages and there are no free resources.

Free resource maximization on the other hand

1. may entail shunning

2. may entail either under- or over-provision of effort

3. the shunning probability p *is lower the higher is productivity (as measured by* $\pi_1 - \pi_0$*)*

4. under some parameter configurations it is also the case that harsher shunning (larger absolute Π*) implies effort is provided for a wider range of expected output gains* $(\pi_1 - \pi_0)(Y - y)$.

Proof. A standard computation. See Appendix II. □

Remark 6. If we think of technological change as leading to higher productivity $\pi_1 - \pi_0$ then the third results on free resource maximization implies that

[16] Notice that we do not include the term ρf_e in the incentive compatibility constraint, having in mind a representative agent in a continuum, so that aggregate output (and free resources) are not appreciably affected by individual actions.

technological improvement should lead to a reduction in punishments and increase in incentive payments. At one time slavery with the concomitant use of punishments - such as whipping - to provide incentives was common. It has since passed out of favor: one possible explanation is that increased productivity has led to a situation where punishments are no longer useful in maximizing free resources.

7. Choice of Technology Games: Theory of the Bureaucratic State

The maximization of free resources gives rise to a positive theory of government. There are two widely used theories of government: one is the theory of the kleptocratic state widely prevalent in libertarian thought. In this view the government is a thief that has succeeded in establishing a monopoly over thievery. The more widespread view is that of a benevolent government that serves to provide public goods that are subject to severe free rider problems, and perhaps to provide greater allocational fairness. The maximization of free resources is an alternative to both of these views: here government provides a public good in the form of free resources, but the theory provides a definite objective for the government, with the provision of public goods other than free resources only occurring either because it increases free resources or because it is an unavoidable product of freeing resources. In some respects this theory is closest in spirit to Thompson and Hickson [56]'s theory of the vital organs of the state. In this section we examine a simple model of the bureaucratic state designed to explore these ideas.

We start by observing that bureaucracy is not present only in the public sector, but also among profit maximizing firms. IBM, a successful and 100 year old firm is renowned for its bureaucracy, for example. At the level of the government, we must remark on the enormous success of the Chinese bureaucratic system that persisted over 2 millennia from roughly the end of the warring states period in 221 BC to the communist take-over in 1949 AD. This system was widely imitated: even today in the United Kingdom senior civil servants are referred to as "mandarins."

Our view is that bureaucracy is functional because it provides monitoring, enabling informational rents to be converted to profits (in the case of firms) or free resources (in the case of governments). So we examine a simple economy in which production (or endowments) are individually produced - but are private information to that individual. Hence, in the simplest case, there are no free resources, as no individual would be willing to advertise the resources they have available. We refer to this as the *libertarian paradise.* In this world to obtain free resources requires the monitoring of individual production by bureaucrats.

Each individual then produces a random output divided into two component, observable and unobservable. Free resources are the observable part plus the minimum in the support of the unobservable part, which we take to be zero. Specifically, an individual produces unobserved output y drawn from a positive continuous density on $[0, \infty]$. Everyone is risk neutral, and as always the subsistence level is B. To avoid triviality, we assume $Ey > B$. In the libertarian paradise in which everyone consumes their own output utility is Ey, and there are no free resources.

We now introduce *commissars* who monitor other players (including each other) converting unobservable output into observable output. In doing so they may reduce output by interfering with the people they are monitoring. Hence an individual can be either monitored or unmonitored. A monitored individual produces observable output y_S that is (weakly) stochastically dominated by y.

Commissars must be paid the same amount as everyone else (that is the per capita expected income of non-commissars), otherwise they will refuse to monitor and rejoin the producing classes. Hence the costs of commissars are that they produce no output, they must be paid, and in the process of converting unobservable output to observable, they reduce it. However, they are needed if there are to be free resources. We shall show that there is a number of commissars bigger than zero and less than everyone when free resources are maximized.

We assume that each commissar can monitor one other commissar, plus κ other individuals. Let ψ denote the fraction of the population who are commissars: this together with the payment w to monitored individuals are the "mechanism design parameters" that will be selected by evolution to maximize free resources. Let $\bar{W}$ be the expected income. The subsistence constraint is again $\bar{W} \geq B$.

There are ψN commissars in the population of N, and $N(1-\psi)$ producers. Commissar 1 monitors commissar 2, ..., commissar ψN monitors 1. Each commissar also monitors κ producers, with a total of $\kappa N \psi$. Passing to fractions: there is a fraction ψ of commissars and $1 - \psi$ of producers; the fraction of monitored producers is $\kappa N\psi/(N(1-\psi)) = \kappa\psi/(1-\psi)$. The condition that this fraction/probability not to exceed 1 is $\psi \leq 1/(1+\kappa)$. That completes the description of the environment.

With this setup the expected income of a producer is

$$\bar{W} = \frac{\kappa\psi}{1-\psi}w + \left(1 - \frac{\kappa\psi}{1-\psi}\right)Ey = Ey - (Ey - w)\frac{\kappa\psi}{1-\psi}.$$

To compute free resources observe that as noted, commissars must be paid $\bar{W}$, else they will rejoin the population rather than monitoring. A fraction $\psi\kappa$ of the population is made of monitored producers, who produce Ey_S and are paid w. Finally, $(1 - \psi(\kappa + 1))$ of the population is unmonitored. This gives per capita free resources as

$$f = \kappa\psi(Ey_s - w) - \psi\bar{W}.$$

We also want to introduce, in an admittedly crude form, the possibility of bureaucratic mission creep. That is, just as individual behavior is restricted by incentive constraints, so large powerful organizations are hard to control. Hence to get free resources a powerful bureaucracy may be necessary, but the bureaucrats have incentives of their own; there is self-selection among bureaucrats for the "right kind of personality" and so forth, and there is the possibility of corruption, rent-seeking and political influence. It may not be possible to design a bureaucracy that just maximizes free resources and does not interfere in daily life and engage in redistribution. To capture these kind of "organizational constraints" we introduce the idea that the higher the fraction of commissars the higher their cost. So we consider the possibility that as ψ grows bigger, the number of producers each commissar can monitor decreases. As an alternative to the κ constant case above we then consider the *creeping bureaucracy case* in which the number monitored is given by

$$\kappa(\psi) = \kappa(1 - \psi).$$

In this case the monitored fraction of producers is

$$\frac{\kappa(\psi)\psi}{1-\psi} = \kappa\psi,$$

and the condition that this fraction be less than 1 is $\psi \leq 1/\kappa$. Expected income is

$$\bar{W} = Ey - (Ey - w)\kappa(\psi)\psi/(1-\psi) = Ey - (Ey - w)\kappa\psi = (1 - \kappa\psi)Ey + \kappa\psi w,$$

and per capita free resources can be simplified to

$$f = \psi\{\kappa(1-\psi)Ey_s - (1-\kappa\psi)Ey - \kappa w\}$$

Proposition 5. *If $Ey_s > Ey/2$ and $\kappa > 1$ then maximization of free resources calls for a positive fraction of commissars. This fraction is higher with creeping bureaucracy. The fraction of monitored producers is the same regardless of whether or not bureaucracy is creeping.*

Proof. In Appendix III. □

Remark 7. Modern societies are extremely efficient at monitoring. In historical times it was difficult to collect much tax revenue, and often monarchies fell back on grants of monopoly rights as alternatives to taxes. Now even 50% or more of GDP can be collected. Even as individual incomes have increased enormously due to informational rents, the fraction available as free resources has gone up because the same technological change makes transactions easier to monitor.

8. Conclusion

Readers of grand theories of history such as those of McNeil [50], Cipolla [25], or Diamond [28] will not find surprising the idea that ideas are spread by the conquest of the less advanced by the more advanced. Missing from these accounts, however, is the notion that it is free resources above and beyond subsistence and incentive payments that matter for the long-term success of societies. In essence, the conclusion of our theory is that evolution favors large expansionary societies made strong by availability of free resources. This is also what historical evidence shows, from old China to the Romans, to modern England and the contemporary United States.

It would be amazing indeed if a simple theory with single scalar variable "free resources" could explain all of history. Missing is any account of the geographical barriers that in practice have prevented a single monolithic society from covering the entire globe. Indeed, England, fast behind her water barrier, continually favored the weaker side in continental Europe to prevent a monolithic society from arising there. Perhaps the history of Asia would have been very different if Japan had been geographically capable of playing a similar role in China. Geography may also be intertwined with strategical considerations: the small Kingdom of Sardinia was located between the great powers of Austria and France who could have easily conquered it, but they never did because a small buffer at their borders made attack by one another harder.[17]

Indeed, geography plays a role in a variety of ways: technology matters of course - water barriers matter much less to societies that have boats - and even less if they have airplanes. The libertarian success stories favored by Milton Friedman in Singapore and Hong Kong were also protected - in the case of Hong Kong by the British military, and in the case of Singapore by a water barrier. One aspect of the theory worthy of future exploration is the idea that small geographically protected areas are likely to have a broader range of social arrangements - both efficient and inefficient - being protected from conquest and disruption by neighbors.[18]

Missing is also a explicit analysis of the role of institutions and their evolution with extraction of free resources in view. For example, Hoffman and Rosenthal [41] argue that the transition from absolute to constitutional monarchy in Europe was determined by the higher tax revenue to be employed for military purposes which a parliament could generate. Shaping of internal legislation also is linked to what we have called resistance: It is assumed in the paper that resistance to barbarian hordes is fixed at 1, but in reality steps are often taken to minimize or prevent internal upheavals, see for example the case of electoral franchise discussed in Acemoglu and Robinson [4].

On the other hand, there are a variety of historical episodes that may be interesting to explore through the lens of free resources. For example, at the beginning of the cold war, technology favored assembly line manufacturing which is relatively amenable to central planning, and so the Soviet Union, a system that excelled at appropriating a high fraction of resources as free, was able to compete successfully with the United States. By contrast as technology changed to favor greater decentralization, it is likely that the enormous growth of GDP in the United States relative to the Soviet Union made it impossible for the Soviet Union to continue to compete, despite its ability to appropriate a very high fraction of total resources. In a similar way, the development of firearms at the end of the medieval period favored

[17] A recent empirical paper on the relation between warfare and institutions in the Italian *Risorgimento* is Dincecco, Federico and Vindigni [29].

[18] The wide range of (admittedly very primitive) social arrangements in New Guinea may be a case in point.

moderately skilled mass armies over small highly trained armies of specialists. Hence to generate large free resources, higher per capita income was needed. The ultimate failure of poorly trained peasants to resist moderately trained lower middle class soldiers was seen in the early 20th Century in the defeat of Russia first by Japan, and eventually by Germany which effectively ended the Russian Empire at the battle of Tannenberg.

We should acknowledge also that while conflict is an important force in the spread (and disruption) of institutions and ideas, voluntary movement of the type discuss by Ely [32] exists as well and provides a force away from free resource maximization and towards efficiency. This suggests a more refined theory in which both free resources and efficiency matter, with the relative strengths of the two depending on the relative importance of ideas spreading through conquest versus voluntary movement.

In summary, the notion of evolution through contacts and conflicts between societies leads to a simple and in our view plausible model of stochastically stable states that maximize free resources. Implications of the theory range from determining the level of population to the type of technologies and institutions we may expect to find.

Appendix I: Theorems About Stochastic Stability

Proposition. *[Proposition 1 in text]* $\sigma \in S[0, J]$ *if and only* σ *is a singleton, that is,* $\sigma = \{s_t\}$*, if* s_t *is a Nash state, and if* s_t *has either no expansionary society, or a single expansionary society such that all other societies (if any) have positive resistance to disruption.*

Proof. First we observe that the s_t as described are absorbing states of the Markov chain, hence certainly in $S[0, J]$. This is trivial, since by assumption no disruption is possible at these states. To prove the theorem it is sufficient to show that from any other initial condition there is a positive probability of reaching one of these absorbing states. This rules out existence of other ergodic classes.

We show that one of these absorbing states has positive probability of being reached from any initial condition. First notice that there is a positive probability that for $T + L + 1$ periods no plot is disrupted. During such a period a quiet plot remains quiet. In a plot j in which some player is not quiet there is a positive probability that all players on that plot will not be quiet the following period. There is then a positive probability that for the next $T+L$ periods all players will play a steady state Nash equilibrium profile and the plot will become quiet. Since this is true of all plots and there are finitely many of them, there is a positive probability that after $T+L+1$ periods the state will be a Nash state.

Suppose we begin in a Nash state which is not one of the described absorbing states. Then there is some expansionary society x that has the most free resources among all expansionary societies (there may be more than one such). If there is more than one expansionary society, one of them has free resources relative to some other of at least 1, and hence by Assumption 6 it has positive probability of becoming the sole expansionary society. Hence multiple expansionary societies are transitory.

If there is no other expansionary society, by assumption one of them has positive probability of being disrupted. Subsequently the disrupted plot has positive probability of joining society x and so there is positive probability of moving to a steady state Nash equilibrium where x has one more plot. By the second part of Assumption 6 we can repeat the process (still with positive probability) until the absorbing state in which $J(x) = J$ is reached. □

To prove the main theorem, we will now apply a method of Friedlin and Wentzell [34] described in Young [58] to analyze the case $\epsilon > 0$ and the limit as $\epsilon \to 0$. We use the characterization of stochastically stable states given by Young [58]. Let $\mathcal{T}$ be a tree whose nodes are the set $S[0, J]$ with any set of edges. We denote by $D(s)$ the unique node from s in the direction of the root. An s-tree is a tree whose root is s, denoted $\mathcal{T}(s)$. For any two points $s_0, s_t \in S[0, J]$ we define the resistance as follows. First, a path from s_0 to s_t is a sequence of points $s_0, \ldots, s_t \in S$, where the transition from s_τ to $s_{\tau+1}$ has positive probability for $\epsilon > 0$. The resistance of the path is the sum of resistances between points in the path $\sum_{\tau=0}^{t-1} r(s_\tau, s_{\tau+1})$. The resistance $r(s_0, s_t)$ is the least resistance of any path from s_0 to s_t. The resistance $r(\mathcal{T}(s_t))$ of the s_t-tree $\mathcal{T}(s_t)$ is the sum over non-root nodes s_τ of $r(s_\tau, D(s_\tau))$. Finally, the resistance $r(s_t)$ is the least resistance of all the s_t-trees. The following Theorem is proved in Young [58].

Theorem 4. *s_t is a stochastically stable state if and only if $s_t \in S[0, J]$ and $r(s_t) = \min_{s_\tau \in S[0,J]} r(s_\tau)$.*

We can provide a lower bound on the resistance of the trees on $S[0, J]$. For any state $s_t \in S[0, J]$ define the least resistance states $LR(s_t) \subseteq S[0, J]$ to be the collection of states $s_\tau \in S[0, J]$ such that $r(s_t, s_\tau) \leq r(s_t, \tilde{s}_\tau)$ for all $\tilde{s}_\tau \in S[0, J]$. Let $lr(s_t)$ be the corresponding least resistance. This is equivalent to Ellison [31]'s notion of the radius of a state. Define $mlr \equiv \max_{s_t \in S[0,J]} lr(s_t)$.

Lemma 1. *Suppose $\bar{s}_t$ has $lr(\bar{s}_t) = mlr$. Then for any s_t-tree $\mathcal{T}(s_t)$ we have $r(\mathcal{T}(s_t)) \geq \sum_{s_\tau \in S[0,J] \setminus \bar{s}_t} lr(s_\tau)$.*

Proof. For any s_t-tree $\mathcal{T}(s_t)$ we have $r(\mathcal{T}(s_t)) = \sum_{s_\tau \in S[0,J] \setminus s_t} r(s_\tau, D(s_\tau)) \geq \sum_{s_\tau \in S[0,J] \setminus s_t} lr(s_\tau)$. That is, the best you can possibly do with s_t as root is to have a tree where all edges are least-resistance. So the best you can possibly do with any root is

$$\min_{s_t} \sum_{s_\tau \in S[0,J] \setminus s_t} lr(s_\tau).$$

The sum to minimize here is the sum of the least resistances of all states except one, the root. Clearly the minimum is achieved if you leave the highest term out. This gives a lower bound on the resistance of any tree. □

The bound established in the Lemma is not generally a useful one, but in the current setting we shall show that there is a tree that achieves this bound. Such a tree is necessarily a least resistance tree.

The central theorem of the paper, Theorem 3 in text, itself has two parts, which we cover in the next result and the corollary which follows.

Theorem 5. *If s_t is a maximum free resource monolithic state then it is stochastically stable.*

Proof. We show that it is possible to build a tree with a root s_t such that $lr(s_t) = mlr$ and where all edges are least resistance. This achieves the lower bound by Lemma 1, so must be a least resistance tree.

We show that the lower bound $\min_{s_t} \sum_{s_\tau \in S[0,J] \setminus s_t} lr(s_\tau)$ is achieved by constructing a tree that achieves it. We begin by giving a partial characterization of $LR(s_t)$. In a non-expansionary state, it requires a resistance of exactly 1 to go anywhere else in $S[0, J]$. Indeed, with a resistance of 1 a plot is disrupted, and with zero resistance (that is, positive probability) it moves to any steady state Nash equilibrium. If that steady state Nash equilibrium is expansionary and there is zero resistance to a monolithic state, we move there; otherwise we move to either another mixed or non-expansionary state in which exactly one plot has changed. Regardless, the least resistance is 1.

To leave mixed states it is least resistance to have a plot from the non-expansionary society with the least free resources to be disrupted (Assumption 6). This has resistance of no greater than 1.

From a monolithic state with society x, to go anywhere else at least enough of its plots must be disrupted that the initial society falls below the threshold $\bar{\phi}$. Let us pick a pair (a_t^k, ω_t^k) that maximizes free resources among all expansionary profiles and call it *the barbarian horde*, say with $x(a_t^k) = x'$. Notice that this is not required to be either a steady state or a Nash equilibrium. There are two cases depending on whether there is a barbarian horde that is *not* a steady state Nash equilibrium. If there is, then it follows from Assumption 8 that the quickest way for a monolithic state to fall is for there to be a single plot disrupted by this barbarian horde, which then continues disrupting and aggregating a_t^k-playing plots until $\Phi(\bar{f}(x', a_t, \omega_t), J(x')/J) \geq \bar{\phi}\Phi(\bar{f}(x, a_t, \omega_t), J(x)/J)$, at which point the remaining transitions have no resistance. Then the state may then move with zero resistance to any element of $S[0, J]$. [19]

In case all barbarian hordes are steady state Nash equilibria, then a least resistance path out of a monolithic steady state with society x is to move to a monolithic steady state with society x' that maximizes free resources. The argument is similar to the previous case, but we can no longer conclude that at the end the state can move to any element of $S[0, J]$ with zero resistance, only that the state can move with zero resistance to that particular x'.

The conclusion is that maximum free resource monolithic states have maximum least resistance, that is, they achieve the bound $lr(s_t) = mlr$. This follows from the fact that they have the largest least resistance of any monolithic state, and monolithic states all have least resistance of at least 1 (to get out of a monolithic state requires at least one mutation, and every mutation has resistance one by Assumption 6), while non-expansionary and mixed states have least resistance no greater than 1. Hence maximum free

[19] Note that the newly disrupted plots of land must be "conquered" in the sense of "joining the barbarian horde and playing a_t^k". However this is a zero resistance event. Since the disrupted plot is not quiet, it has positive probability of repeatedly playing the action of the barbarian horde. The barbarian horde also has positive probability of continuing to play the same action. Hence with positive probability after some period of time the new and old plots are all in the same steady state capital structure. After this they may coincidentally follow the same sequence of action profiles to achieve any particular target ω_t^k, and the following period all switch to a_t^k.

resource Nash states are obvious candidates for roots of least resistance trees. We now construct such a tree that achieves the lower bound.

Pick a maximum free resource monolithic state s_t with corresponding society x. Build an s_t-tree with least resistance edges as follows. Each non-expansionary state has one plot of the least resistance society taken over by x; this connection has the least resistance of 1. Each mixed society has one plot of the least resistant non-expansionary society taken over by the expansionary society; this also is least resistance and has resistance no greater than 1. Each monolithic society moves to s_t in the least costly way described above. So any non-expansionary state becomes a mixed state (or monolithic if the profile played by x on a single plot is already over the threshold), every mixed state increases the size of the expansionary society until it become monolithic, and finally every monolithic society goes to the root. □

Corollary 2. *For J large enough every stochastically stable state is monolithic, and* $\lim_{J\to\infty} f_{\min}(J) \to f^{\max}$.

Proof. It follows from the proof of Theorem 5 that if s_t is the root of a least resistance tree, the lower bound must be achieved, and this is possible only if $lr(s_t) = mlr$. We first show that for large J monolithic states have $lr(s_t) > 1$, hence (since the other states have have $lr(s_t) \leq 1$) only them can satisfy satisfy $lr(s_t) = mlr$, hence only those can be stochastically stable. The claim $lr(s_t) > 1$ amounts to asserting that for large J resistance of a monolithic state x to disruption by a barbarian horde x' is positive for at least the first two plots; but as J grows large $2/J \to 0$, so (using last limit in Assumption 7) the ratio of free resources $\phi = F(x', a_t, \omega_t)/F(x, a_t, \omega_t) \to 0$, and for small ϕ resistance is positive by Assumption 5.

For the last assertion of the theorem observe that the possibility that a monolithic state with fewer free resources than $f^{\max}$ have $lr(s_t) = mlr$ can be only due to the round-off error caused by the discrete size of the plots (which makes the barbarian horde jump above the threshold $\bar{\phi}$ in a certain number of steps); but as J grows large this error goes to zero because each conquered plot makes ϕ move less. From this the result follows. □

Appendix II: Principal-Agent

[Since this is a minor variation on the standard Principal-Agent problem, this appendix is a candidate for a web appendix.]

Proposition. *[Proposition 4 in text] Under utility maximization*

1. shunning never occurs (so there is no output loss)

2. effort is induced if and only if the expected output gain is no smaller that effort cost (that is iff $(\pi_1 - \pi_0)(Y - y) \geq 1$*)*

3. all output is paid out in wages and there are no free resources.

Free resource maximization on the other hand

1. may entail shunning

2. may entail either under- or over-provision of effort

3. the shunning probability p *is lower the higher is productivity (as measured by* $\pi_1 - \pi_0$*)*

4. under some parameter configurations it is also the case that harsher shunning (larger absolute Π*) implies effort is provided for a wider range of expected output gains* $(\pi_1 - \pi_0)(Y - y)$*.*

We first reduce the problems via preliminary Lemmas, then prove the result.

Lemma 2. *Both free resource and utility maximization have a solution with* $w_S = W_S = 0$*.*

Proof. Given a solution define $\tilde{w} = w, \tilde{w}_S = 0$, $\tilde{W} = (1-\pi_e)pw_S/(1-P)\pi_e + [(1-P)W + PW_S]/(1-P), \tilde{W}_S = 0$. This corresponds to using the proceeds from setting $\tilde{w}_S = \tilde{W}_S = 0$ entirely to increase W leaving w unchanged. Since it leaves $\bar{W}$ unchanged, this preserves both social feasibility and the subsistence constraint, while also leaving free resources unchanged. Moreover since $\tilde{w} = w, \tilde{W} \geq W$ utility is not decreased and incentive compatibility is preserved. This gives a solution with punishment wages equal to zero. □

When $w_S = W_S = 0$

$$\bar{W} = (1 - \pi_e)(1 - p)w + \pi_e(1 - P)W$$
$$u_e = -e + \rho f_e + \bar{W} + [(1 - \pi_e)p + \pi_e P]\Pi.$$

Lemma 3. *Both free resource and utility maximization have a solution with* $w = 0$*.*

Proof. Given a solution with $w_S = W_S = 0$ define $\tilde{w} = 0$, $\tilde{W} = (1 - \pi_e)(1 - p)w/\pi_e(1 - P) + W$. This leaves $\bar{W}$ unchanged, so preserves both social feasibility and the subsistence constraint, while also leaving free resources and utility unchanged. Moreover since $\tilde{w} \leq w, \tilde{W} \geq W$ incentive compatibility is preserved. This gives a solution with wages for low output equal to zero. □

This gives the further simplification for expected wage and incentive constraint

$$\bar{W} = \pi_e(1-P)W, \quad (1-P)W - (p-P)\Pi \geq \Delta.$$

Lemma 4. *Both free resource and utility maximization have a solution with $P = 0$.*

Proof. From $\bar{W} = \pi_e(1-P)W$ we may choose $\tilde{P} = 0$, $\tilde{W} = (1-P)W$ which holds fixed $\bar{W}$ hence social feasibility, utility and free resources. Moreover incentive compatibility continues to hold since $\Pi \leq 0$. □

To summarize, we now choose two numbers p, W and whether or not to provide effort. We must satisfy subsistence and social feasibility

$$B \leq \bar{W} = \pi_e W \leq \bar{Y}_e,$$

and if we wish effort to be provided, incentive compatibility

$$W - p\Pi \geq \Delta.$$

The free resource objective is $f_e = \bar{Y}_e - \pi_e W$ and the utility maximizing objective is

$$u_e = -e + \rho\bar{Y}_e + (1-\rho)\pi_e W + (1-\pi_e)p\Pi.$$

We can now prove the Proposition.

Proof of Proposition. In the utility maximization problem the objective is increasing in W, in the free resource problem it is decreasing in W. Hence for utility maximization we must have $W = \bar{Y}_e/\pi_e$, while for free resources $W = \max\{B/\pi_1, \Delta + p\Pi\}$ if effort is to be induced, otherwise $W = \pi_0 B$. Note that expected output is now

$$\bar{Y}_e = B + [(1-\pi_e)y + \pi_e Y] - (1-\pi_e)p(y - y_S) = \bar{y}_e - p(1-\pi_e)(y - y_S),$$

where $\bar{y}_e = B + [(1-\pi_e)y + \pi_e Y]$ is output in normal (no shunning) conditions.

Utility maximization case: To induce no effort of course $p = 0$, so $\max u_0 = \bar{y}_0$. To induce effort the incentive constraint must hold. But observe that $\bar{Y}_1 \geq B + \pi_1 Y + (1-\pi_1)y_S \geq \pi_1(Y - y_S) \geq \pi_1(Y - y) = \pi_1\Delta(\bar{y}_1 - \bar{y}_0)$, the last equality from $\bar{y}_1 - \bar{y}_0 = (\pi_1 - \pi_0)(Y - y) = \Delta^{-1}(Y - y)$. Hence if $\bar{y}_1 - \bar{y}_0 \geq 1$ then $\bar{Y}_1/\pi_1 \geq \Delta$, whence for $W = \bar{Y}_e/\pi_e$ incentive compatibility is satisfied with $p = 0$, which is therefore optimal (Although this may seem a bit convoluted the underlying idea is simple. If we pay output $w = B + y, W = B + Y$ the effort decision is completely internalized. Hence effort will be made if and only if $\bar{y}_1 - \bar{y}_0 \geq 1$. Here we pay everything in good state and nothing in the bad state, and since there is risk neutrality that works as well.

In conclusion, it is never optimal to punish for utility maximization. And for both values of e one has $\max u_e = \bar{y}_e - e$, so effort is induced if and only if $\bar{y}_1 - \bar{y}_0 \geq 1$. Finally, since $W = \bar{Y}_e/\pi_e$ we have $f_e = \bar{Y}_e - \pi_e W = 0$.

Free resource case: Conditional on no effort, $W = B$ and $p = 0$ so $\max f_0 = (1 - \pi_0)y + \pi_0 Y$. Conditional on effort:

$$\begin{aligned} f_1 &= \bar{Y}_1 - \pi_1 W = \bar{y}_1 - (1 - \pi_1)p(y - y_S) - \pi_1 \max\{B/\pi_1, \Delta + p\Pi\} \\ &= \bar{y}_1 - (1 - \pi_1)p(y - y_S) - \max\{B, \pi_1(\Delta + p\Pi)\}. \end{aligned}$$

Suppose $B > \pi_1/(\pi_1 - \pi_0) = \pi_1\Delta$. Then $p = 0$ and $W = B/\pi_1$, so $\max f_1 = (1 - \pi_1)y + \pi_1 Y > (1 - \pi_0)y + \pi_0 Y = \max f_0$ and effort is always induced. In this case there is too much effort. The intuition here is that large B implies large wage payments anyway, and at that point it is worth doing it in the form of incentive payments to get more output that adds to free resources.

Suppose from now on $B < \pi_1\Delta$. There are two possibilities for Π: Case (a) $\Pi \geq -(1 - \pi_1)(y - y_S)/\pi_1$, and case (b) $\Pi < -(1 - \pi_1)(y - y_S)/\pi_1$.

In case (a) the objective function is decreasing in p so the optimum is $p = 0$ and $W = \Delta$, with effort if and only if $\bar{y}_1 - \bar{y}_0 \geq \pi_1\Delta - B$. Here we have under-provision of effort if $\pi_1\Delta - B < 1$, and over-provision if $\pi_1\Delta - B > 1$. In the latter case informational rents are so high that it does not pay to induce effort even though it is efficient to do so.

In case (b) optimal policy depends on the sign of $(\Delta - B/\pi_1)/(-\Pi) - 1$, so we distinguish: Case (b1): $\Pi < -(1 - \pi_1)(y - y_S)/\pi_1 < B/\pi_1 - \Delta$; Case (b2): $B/\pi_1 - \Delta < \Pi < -(1 - \pi_1)(y - y_S)/\pi_1$. Consider (b1). From $\Pi < -(1 - \pi_1)(y - y_S)/\pi_1$ it then follows that $(\Delta - B/\pi_1)/(-\Pi) < 1$. $(\Delta - B/\pi_1)/(-\Pi) < 1$. The optimum p is given by $\pi_1(\Delta + p\Pi) = B$, that is $p = (\Delta - B/\pi_1)/(-\Pi) > 0$: shunning arises in equilibrium. The optimum wage is $W = B/\pi_1$, and the condition for effort is $\bar{y}_1 - \bar{y}_0 \geq (1 - \pi_1)p(y - y_S)$.

Consider now (b2). Here $(\Delta - B/\pi_1)/(-\Pi) > 1$, so the optimum is $p = 1$, $W = \Delta + \Pi$ and effort provision if and only if $\bar{y}_1 - \bar{y}_0 \geq (1 - \pi_1)(y - y_S) + \pi_1[\Delta + \Pi] - B$.

From inspection of all cases we see that the shunning probability p is non-decreasing in $\Delta = (\pi_1 - \pi_0)^{-1}$, strictly in case (b2).

Finally, take parameters satisfying $-(1-\pi_1)(y-y_S)/\pi_1 < B/\pi_1 - \Delta < 0$, equivalently

$$B < \frac{\pi_1}{\pi_1 - \pi_0} < B + (1-\pi_1)(y-y_S).$$

Then if $\Pi < -(1-\pi_1)(y-y_S)/\pi_1$ it is case (b1), while if $\Pi \geq -(1-\pi_1)(y-y_S)/\pi_1$ it is case (a). In case (b1) condition for effort is $\bar{y}_1 - \bar{y}_0 \geq (1-\pi_1)(y-y_S)(\Delta - B/\pi_1)/(-\Pi)$, in case (a) $\bar{y}_1 - \bar{y}_0 \geq \pi_1\Delta - B$. To finish the proof it suffices to check that if $\Pi < -(1-\pi_1)(y-y_S)/\pi_1$ then $(1-\pi_1)(y-y_S)(\Delta - B/\pi_1)/(-\Pi) < \pi_1\Delta - B$. □

Appendix III: Bureaucracy

Proposition. *[Proposition 5 in text] If $Ey_s > Ey/2$ and $\kappa > 1$ then maximization of free resources calls for a positive fraction of commissars. This fraction is higher with creeping bureaucracy. The fraction of monitored producers is the same regardless of whether or not bureaucracy is creeping.*

Proof. We shall show that in both cases $\kappa(\psi) = \kappa$ and $\kappa(\psi) = \kappa(1-\psi)$ the fraction of monitored producers is the same, namely

$$\frac{\kappa(\psi)\psi}{1-\phi\psi} = 1 - \frac{B}{Ey}.$$

From this the results are immediate.

We start with the constant κ case. Obviously to maximize free resources for any value of ψ one has to minimize w; so the subsistence constraint binds:

$$Ey - (Ey - w)\kappa\psi/(1-\psi) = B,$$

whence $w = \max\{0, Ey - (1-\psi)(Ey - B)/\kappa\psi\}$.For small [respectively large] ψ it is $w = 0$ [respectively $w > 0$]. There are two possibilities.

Case1: $w = 0$. Then $\bar{W} = Ey(1 - \kappa\psi/(1-\psi))$ and we find $f = \kappa\psi Ey_s - Ey[1-(1+\kappa)\psi]\dfrac{\psi}{1-\psi}$. This gives

$$f' = \kappa Ey_s - \frac{Ey}{(1-\psi)^2}[1 - \psi(1+\kappa)(2-\psi)]$$

Case 2: $w = Ey - (1-\psi)(Ey - B)/\kappa\psi$ so $\bar{W} = B$ and $f = \kappa\psi Ey_s + Ey[1-(1+\kappa)\psi] - B$ in this case f' turn out to be negative over the whole relevant range:

$$f' = \kappa E y_s - (1+\kappa)Ey < 0.$$

We therefore conclude that the optimum must be where $w = 0$. Hence case 1 is the only relevant case. We re-write the expression for f' in that case:

$$f' = \kappa E y_s - Ey + Ey[1 - \frac{1-(1+\kappa)\psi(2-\psi)}{(1-\psi)^2}]$$

Note that the expression in square brackets is positive because $1-(1+\kappa)\psi(2-\psi) = (1-\psi)^2 - \kappa\psi(2-\psi) < (1-\psi)^2$. Since the assumptions imply $\kappa E y_s - Ey \geq 0$, we then have $f' > 0$, from which it follows that the optimal ψ is at the upper bound in the relevant range, i.e. such that exactly $Ey = (1-\psi)(Ey-B)/\kappa\psi$. Thus at the optimum one has

$$\frac{\kappa\psi}{1-\psi} = 1 - \frac{B}{Ey}$$

We now turn to the creeping bureaucracy case. From the expressions for $\bar{W}$ and f we see that again subsistence constraint binds, so

$$w = \max\{0, [B-(1-\kappa\psi)Ey]/\kappa\psi\}.$$

And again we look at the two possible cases.

Case 1: $w = 0$. Then $\bar{W} = (1-\kappa\psi)Ey$ and $f = \psi\{\kappa(1-\psi)Ey_s - (1-\kappa\psi)Ey\}$so $f' = \kappa E y_s - Ey + 2\kappa\psi(Ey - Ey_s)$

Case 2: $w = [B-(1-\kappa\psi)Ey]/\kappa\psi$. Then $\bar{W} = (1-\kappa\psi)Ey + B - (1-\kappa\psi)Ey = B$ and $f = \kappa\psi(1-\psi)Ey_s - \psi(1-\kappa\psi)Ey - B + (1-\kappa\psi)Ey$. So $f' = -Ey - [Ey - Ey_s]\kappa(1-2\psi) < 0$ whence again it must be $w = 0$ at optimum. And again since $\kappa E y_s - Ey > 0$ the f' for that case is positive whence optimum is at the boundary of the relevant range, that is with $(1-\kappa\psi)Ey = B$. Therefore

$$\kappa\psi = 1 - \frac{B}{Ey},$$

and since here $\kappa\psi = \kappa(\psi)\psi/(1-\psi)$ that completes the proof. □

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