Entertaining Malthus: Bread, Circuses and Economic Growth☆

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Abstract

We augment a simple Malthusian model by allowing agents to consume something besides food. Whereas food (in our case: bread) is both enjoyable and necessary for survival, the other good, circuses, is pure entertainment: it has absolutely no impact on survival, but does enhance the quality of life. With this very simple modification, we show that, whereas food supply may remain at subsistence, technological change will have a great impact on the consumption of everything else. Most strikingly, though the population remains bound by a Malthusian constraint, sustained improvements to living standards can no longer be ruled out. We also argue that our model better fits the known historical facts–the widely-held belief that growth prior to the Industrial Revolution was flat is based largely on historical measures of food production. Although food supply throughout history may have been (and for most individuals still is) at or near subsistence levels technological advancements have brought greater convenience and comfort to wealthy and poor alike, a possibility that Malthus himself conceded in his later (and often ignored) work. Therefore, our model not only describes historical growth patterns and demographic transitions better than traditional "Malthusian" models, but is also more in line with Malthus' later thinking.
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1. Introduction

There are few theories with so little evidence that have captured the imagination of researchers as has that of Malthus. Roughly, the theory asserts that for per-capita income above some subsistence level, population increases and below that level, it declines. Consequently, population converges to a steady state at the subsistence level. Even the obvious fact that countries with high per-capita income have much lower (or even declining) rates of population growth, while countries with low per-capita income have much higher rates of population growth, has not shaken the faith of the Malthusians. Nor has the demographic transition and the enormous increase in per-capita consumption that has taken place since the Industrial Revolution. Rather, it is asserted that the modern period is somehow different, while from the dawn of the human race until the beginning of the Industrial Revolution, there was very little change in per-capita consumption.[1] Nor do facts—not to speak of common sense—indicating that, for example, per-capita consumption was considerably higher at the height of the Roman hegemony than was enjoyed by primitive groups in New Guinea in the early 20th Century, seem to carry any weight in this debate. Rather, facts are warped to suit the theory.

The claim that pre-Industrial Revolution consumption was static seems to be based largely on an analysis of median food consumption. No doubt this has hovered near subsistence for most people through most of history—and unfortunately does not seem to have changed much in modern times. Modern data are analyzed with averages and not medians, and a moment of reflection will show that most of our current consumption is not of food, and that historically average consumption includes considerable non-food consumption—ranging from decent dwellings and entertainment in Roman times, to the many luxuries enjoyed by royalty and nobility throughout history.

Our goal in this paper is to show that, even if we accept the Malthusian assumption that population increases above some subsistence level and declines below that level, Malthusian conclusions about consumption do not hold. This is true even in a one-sector model, where we show that if the elasticity of substitution between labor and land is greater than one, the long-run steady state will result in consumption above subsistence. Although such a model seems to fit the data better than the more traditional models presuming unit elasticity, it can be argued that the implication that, as population grows to infinity, per-capita output remains bounded away from zero, is not particularly plausible.

This leads us to the central contribution of this paper: if there is something to consume besides food, even if food consumption may in the long-run be kept at subsistence, technology change can

[1] For example, Jones (1999) writes "For thousands of years, the average standard of living seems to have risen very little, despite increases in the level of technology and large increases in the level of the population." Or for example Hansen and Prescott (2002), who begin with the assertion that "Prior to 1800 living standards in world economies were roughly constant over the very long run..." Or Galor and Weil (1999, 2000), who assert "This Malthusian framework accurately characterized the evolution of population and output per capita for most of human history. For thousands of years, the standard of living was roughly constant, and it did not differ greatly across countries."

have a great impact on everything else, possibly driving continuous growth in living standards. Our key innovation is to allow agents bound by a Malthusian constraint to consume a good that improves the quality of life, but has absolutely no bearing, for good or ill, on population growth or the survival of the species. In our model, bread provides enjoyment and also determines the level of the population, whereas circuses can only enhance utility. With this very simple addition, we show that the standard Malthusian constraint no longer guarantees that per-capita income is kept low. Not only does this model seem more consistent with the basic facts as known, it seems to provide a better foundation for understanding how the Industrial Revolution led to the demographic transition than existing "Malthusian" models. It also provides what may be a fruitful set of ideas for historical and anthropological examination of consumption and welfare across time and space.

A careful reading of Malthus suggests that he himself (eventually) found the assumption of food being the most important, if not the only, determinant of the "condition" of society to be restrictive. Numerous studies have highlighted the following passage from his earliest work—the first edition of *An Essay on the Principle of Population.*

This natural inequality of the two powers of population, and of production in the earth, and that great law of our nature which must constantly keep their effects equal, form the great difficulty that to me appears insurmountable in the way to the perfectibility of society. All other arguments are of slight and subordinate consideration in comparison of this. ... And it appears, therefore, to be decisive against the possible existence of a society, all the members of which, should live in ease, happiness, and comparative leisure; and feel no anxiety about providing the means of subsistence for themselves and families.

Few studies, however, mention his later acknowledgement of the potential role played by the non-food sector. In the following passage, taken from the sixth edition of his essay, Malthus claims that wealth can indeed bring benefits to the poor that are not limited to promoting survival or subsistence, but might instead involve technologies and goods that effectively raise the quality of life. He writes:

On an attentive review, then, of the effects of increasing wealth on the condition of the poor, it appears that, although such an increase does not imply a proportionate increase of the funds for the maintenance of labour, yet it brings with it advantages to the lower classes of society which may fully counterbalance the disadvantages with which it is attended; and, strictly speaking, the good or bad condition of the poor is not necessarily connected with any particular stage in the progress of society to its full complement of wealth. A rapid increase of wealth indeed, whether it consists principally in additions to the means of subsistence or to the stock of conveniences and comforts, will always, ceteris paribus, have a favourable effect on the poor; but the influence even of this cause is greatly modified and altered by other circumstances, and nothing but the union of individual prudence with the skill and industry which produce wealth can permanently secure to the lower classes *of society that share of it which it is, on every account, so desirable that they should possess.*

Here, Malthus' only substantial concern is that an increase in wealth, if it arises through manufacturing, might entail worse working conditions. Poor working conditions in the manufacturing sectors were certainly widespread in Malthus' time. Never-the-less, it can hardly be considered a reasonable fundamental assumption that any wealth derived from a non-food sector can only hurt the working poor since it may be accompanied by a deterioration of working conditions.

2. Previous Research on Historical Growth Patterns

A variety of models have been proposed to explain stagnant pre-1760 economic growth followed by explosive post-Industrial Revolution growth, an empirical pattern that is largely taken for granted. A summary of these papers is presented in Table 1. For each paper, the table includes: assumptions on preferences and technology, historical evidence on the abrupt shift in growth patterns to which the paper appeals and main findings. The table is meant to make it apparent that nearly all of these papers are motivated by an identical interpretation of a limited set of empirical sources claiming that growth prior to 1760 was negligible. They also rely on similar theoretical devices to generate their conclusions about historical consumption patterns. In particular, common features of these papers include: (i) Cobb-Douglas production technology (ii) a single consumption good and (iii) a reliance on data from sources like Maddison (1982) on food production to infer stagnant pre-1760 economic growth.[2] As our analysis proceeds, we challenge the sensibility of each of these features. In particular, we will propose a more reasonable reading of historical data, which will guide us in relaxing overly-restrictive modeling assumptions, thereby generating decidedly non-Malthusian conclusions.

In most of the models summarized in Table 1, economic growth is driven by endogenous technological advancements. For example, Arifovic et al. (1997) present a model where, through an adaptive learning process, and beyond a threshold of accumulated human capital, an economy moves from a low-growth to a high-growth equilibrium. Becker and Barro (1988), Lucas (2002) and Razin and Ben-Zion (1975) employ dynastic utility functions to examine how endogenous fertility patterns interact with changes in technology. Another set of papers studies population growth as it relates to innovation and specialization (Kremer, 1993; Galor and Weil, 2000; Goodfriend and McDermott, 1995; Lee, 1980, 1988)). Still others hypothesize that jumps in growth can be explained by a switch to technologies requiring less land as relative returns to available technologies shift (Hansen and

[2]For example Arifovic et al. (1997) who cite Maddison (1982) to conclude "Prior to Industrialization, all of today's highly developed economies experienced very long periods, *epochs*, of relatively low and stagnant growth in per capital income...While these data are highly aggregated and necessarily involve some guesswork, few economists would question the picture they paint." Goodfriend and McDermott (1995) make a similar assertion: "Maddison, presents population and per capita growth rates since 500 AD. Although the numbers are highly aggregated both across countries and over time and are obviously imprecise, they tell a dramatic story. For the thousand years following the fall of Rome, there was little net progress in population and none in per capita product." Other sources that are often used as evidence of stagnant growth include McEvedy et al. (1978), Clark (1998) and Parente and Prescott (1993), among others.

Prescott, 2002). Finally, several researchers examine how institutions determine growth through their effect on returns to innovation and inventions (Baumol (1990); Jones (1990)).[3]

A few other studies of historical growth tell more nuanced stories that go beyond the contention that everything prior to 1760 was flat. A key example is Acemoglu and Zilibotti (1997), who develop a model emphasizing the high variability of output during early stages of development. In their setup, output growth is slow, but also subject to randomness.[4] They explain this with a model where capital projects are few and subject to indivisibilities, which limits risk-spreading and encourages investment in safer projects that are less productive.

The vast majority of papers that study Malthusian dynamics and that we list in Table 1 assume a single consumption good. While a single composite good is a useful and perhaps fairly innocuous assumption in many economic models, it turns out to be problematic once a Malthusian constraint is imposed. Expounding upon why this is problematic is the main goal of this paper and the reasoning will be discussed at length below. The reasoning turns on the notion that imposing that every good, including non-food items, be vital for survival leads to an erroneous representation of reality. Looking at Table 1, a key exception to the single composite good is the possibility that parents have preferences over children (or over children's consumption). Preferences over offspring, however, do not really constitute a meaningful exception as they are also wrapped up with population growth and, once again, fail to capture growth in consumption goods that increase utility without directly promoting the survival of the species.

From the two-sector point of view, a few papers are more similar to ours. One example is Hansen and Prescott (2002). Their Malthus technology uses a fixed factor (land) while the Solow technology does not. Their motivation for the Malthusian technology is that its output has a considerable share of food in it and therefore requires land while the Solow production function results in mostly non-farm output (or "factory" output) and therefore requires very little land. The Solow technology, which produces non-farm output, appears similar to our circus sector technology. The biggest difference here is that while they divide the sectors production-wise—land-using or not, we divide the sectors consumption-wise—bread and circus have different demographic effects. Essentially, there is only one good in their economy. Their setup is troublesome since it implicitly assumes that individuals consider food and non-food items to be substitutable reproductively on a one-to-one basis. Given that their population growth function depends only on consumption and not consumption of food, the model violates the basic Malthusian premise of population growth being kept in check by the availability of food.[5]

A related paper, Yang and Zhu (2013), allows the Solow technology to govern both non-agricultural production, which never uses land, and farm production, which does initially use land.

[3]Another set of papers test empirical implications of the Malthusian story. See, for example, (Galor, 2010).

[4]Related, Voigtländer and Voth (2006) emphasize randomness in explaining why the Industrial Revolution began in Great Britain versus, say, France.

[5]It should be noted that Hansen and Prescott (2002) is not the only paper we have discussed in which a model is proposed that is not really Malthusian. In Acemoglu and Zilibotti (1997) and Arifovic et al. (1997), land is not a factor of production, which strictly speaking may be seen as violating a key Malthusian assumption.

After the industry develops, however, capital replaces land in farm production.[6] In another related paper, Voigtländer and Voth (2013), high growth is achieved by a major shock to population, the Black Death, which raised wages and also induced demand in goods produced in cities. As a result, wages in cities rose, which attracted new citizens. As death rates in urban centers were higher, population growth was kept in check. If we read closely, however, it becomes clear that these latter two papers also assume away the possibility that one good can only affect the quality of life without influencing survival. In Yang and Zhu (2013), total consumption drives population growth; in Voigtländer and Voth (2013), preferences are designed so that agents derive utility from non-food items only after a certain subsistence level has been reached. As we will show in our model, divorcing the consumption of at least one good from survival so that it can only be consumed to enhance the quality of life leads to very different implications about long-run growth. The implications of the differences between these two-sector models and our model will be revisited when we discuss our main results.

Table 1: Economic Literature on Historical Growth

Paper	Technology	Preferences	Evidence	Findings
Acemoglu and Zilbotti (1997)	$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$	$\mathrm{E}_t U_t = \log(c_t)$ $\beta \int_0^1 \log(c_{t+1}^j)\mathrm{dj}$ where E_t is the expectations operator and j is the state.	Braudel and Kochan (1973); Braudel (1982); North and Thomas (1973); De Vries (1976)	Early stages of growth are characterized by volatility, not just stagnation.
Arifovic, Bullard, and Duffy (1997)	$f(k(t)) = k(t)^{\alpha}$ where k is the capital-to-effective-labor ratio and t in parentheses denotes real time.	$U_t = \ln c_t(t) + \ln c_t(t+1)$ where subscript t denotes generation and t in parentheses denotes real time.	Maddison (1982) (table 1.2); Summers and Heston (1991)	Transition from stagnation to growth is a long, endogenous process.

Table 1 - continued on next page

[6] Related, Voigtländer and Voth (2012), argue that a new Malthusian equilibrium arises prior to the Industrial Revolution where population is lower and wealth is higher. This occurs since land abundance induced by the Black Death led to a shift towards land-intensive agricultural production in which women were employed. Higher female employment raised the opportunity cost of marriage and child-bearing, which lowered population and increased wealth.

Table 1 - continued from previous page

Paper	Technology	Preferences	Evidence	Findings
Ashraf and Galor (2011)	$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$	$U_t = (c_t)^{1-\gamma}(n_t)^{\gamma}$	McEvedy, Jones et al. (1978); Ramankutty et al. (2002); Michalopoulos (2012); Putterman (2008); Peregrine (2003)	Empirical analysis: until 1500 CE, superior productivity increased population and left living standards unchanged.
Becker and Barro (1998)	NA	$U_0 = v(c_0, n_0) +$ $a\psi(U_1, n_0)$, where U_0 is parents' utility, v is period utility and U_1 is child's utility.	NA	Implications of dynastic utility and parent altruism: long-term interest rates, child survival rates and altruism increased fertility and altruism; rate of technological progress decreases it.
Galor and Weil (2000)	$Y_t =$ $L_t^{\alpha}(A_t T)^{1-\alpha}$	$U_t =$ $c_t^{1-\gamma}(w_{t+1}, n_t, h_{t+1})^{\gamma}$ where $h$is human capital per child.	Maddison (1982); Lee (1980); Chao (1986)	A unified model to capture population output and technology change from Malthusian to modern growth.
Goodfriend and McDermott (1995)	$Y =$ $(e_Y hN)^{1-\alpha} \times$ $\int_0^M [x(i)]^{\alpha} di$ where e_Y is the fraction of time devoted to prod. of final goods, M is the measure of intermediate goods and $x(i)$ is the quantity of good i.	$\int_0^{\infty} n(t) \text{ln} c(t) e^{-\rho t} dt$ where n, here, is family size, t in parentheses is calendar time and ρ governs discounting of future consumption.	Maddison (1982)	Population growth leads to increases in specialization, which eventually activates a learning technology that initiates industrial growth.

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Paper	Technology	Preferences	Evidence	Findings
Hansen and Prescott (2002)	$Y_{1t} = A_{1t}K_{1t}^{\alpha}N_{1t}^{\eta}\times L_{1t}^{1-\alpha-\eta}$ and $Y_{2t} = A_{2t}K_{2t}^{\theta}N_{2t}^{1-\theta}$ where subscripts denote production sectors for the same consumption good.	$U = \log c_{1t} + \beta \log c_{2,t+1}, \ldots$ for consumption of goods 1 and 2.	Clark (1998); Wrigley (1997); Maddison (1991)	Stagnant to growing living standards occurs as profit-maximizing firms begin employing a less land-intensive production process.
Jones (1999)	$Y_t = A_t^{\alpha}L_{Yt}^{\eta}T_t^{1-\eta}\epsilon_t$ where ϵ_t is a shock.	$u = (1-\mu)\frac{\tilde{c}_t^{1-\gamma}}{\gamma} + \mu\frac{\tilde{n}_t^{1-\psi}}{1-\psi}$. where the tilde indicates deviations from the mean.	Maddison (1982); Lee (1980); Jevons (1896); Schoenhof (1903)	The single most important factor in the transition to modern growth is increased compensation (as a a fraction of output) to inventors.
Kremer (1993)	$Y = Ap^{\alpha}T^{1-\alpha}$ where p is population.	Single good; preferences left unspecified.	McEvedy, Jones et al. (1978); Deevey (1960); United Nations (various years)	Population spurs technology, which (according to Malthus) promotes population growth. Hence, high populations lead to population growth.
Lee (1980)	$A = \mu_0(\mu_1(T^{-\alpha} + (1-\mu_1 N_A^{-\alpha})^{-1/\alpha}$	Single good; preferences left unspecified.	Russel (1958); Phelps-Brown and Hopkins (1955); Kerridge (1953)	Short-run negative effects of population growth. Population swings explained by exogenous mortality rates, not as a response to technological change.

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Paper	Technology	Preferences	Evidence	Findings
Lee (1988)	$Y = N^{1-\alpha}R^{\eta}$ where R detnotes resources.	Single good; preferences left unspecified.	Russel (1958); Phelps-Brown and Hopkins (1955); Kerridge (1953)	Population-induced technological change and Malthusian growth generate accelerating growth.
Lucas (2002)	$f(x, l, k) = Ax^{\alpha}l^{1-\alpha-\eta}k^{\eta}$, where x and z are land and capital per-capita.	$u_t = f(c_t, n_t, u_{t+1})$ where subsctipt t denotes a generation.	Johnson (1997); Kremer (1993); Maddison (various); McEvedy, Jones, et al (1978); Parente and Prescott (1993); Pritchett (1997)	Endogenous capital accumulation cannot alone explain the Industrial Revolution. Fertility and changes in returns to human capital are also important.
Razin and Ben-Zion (1975)	$K_{t+1} = F(K_t - C_t, L_t)$ where F is linear and homogenous; $F = r(K-C)^{\alpha}L^{1-\alpha}$	$V = U_t(c_t, \lambda_t U_{t+1}(c_{t+1}, \lambda_{t+1}, U_{t+2}(\dots))),$ $t = 0, 1, \dots,$ where λ is per capita births.	NA	Increases in capital productivity can lower the rate of population growth if current generations care about future generations' well being.
Voigtlander and Voth (2013)	$Y_1 = A_1 N_1^{\alpha} L^{1-\alpha}$ $Y_2 = A_2 N_2$ where subscipts are sectors: 1 is food; 2 is other goods.	$u = (c_1 - \underline{c})^{\gamma} c_2^{1-\gamma}$ if $c_1 > \underline{c}$ $u = \phi(c_1 - \underline{c})$ if $c_1 \leq \underline{c}$	Clark (various); Maddison (various)	Population shocks like the Black Death can lead to high-income steady states.

Table 1 - continued on next page

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Paper	Technology	Preferences	Evidence	Findings
Yang and Zhu (2013)	$Y_{1t}^{\text{OLD}} = T_t^{1-\alpha}(A_{2t}L_{2t})^{\alpha}$ $Y_{1t}^{\text{NEW}} = [T_t^{1-\alpha}(A_{2t}L_{2t})^{\alpha}]^{1-\eta} \times X_t^{\eta}$ $Y_{2t} = A_{2t}L_{t2}$ where subscripts are sectors: 1 is food; 2 is other goods. food is produced with either OLD or NEW technology, where NEW takes an "industry supplied" good X_t.	Food consumption is constant. Remaining income is spent on non-food items.	Clark (various); Maddison (2003); Wrigley (various)	Agricultural modernization ignites the transition to modern growth.

Equations are from the corresponding paper, where some notation has been altered to facilitate comparison across studies. The following notation holds in all equations (unless otherwise indicated): Y: output; K: capital, L: labor; T: land, A: technology (where subscripts indicate factor-specific technology); U: utility; c: consumption; n: number of children; w: wage; h: human capital per worker; and N: number of workers in the economy. t-subscripts indicate time. Where subscripts represent something other than time (e.g., generations, states or sectors) they are identified as such. Unless otherwise indicated, lower-case letters are per-capita units. Finally, α, η and θ are production parameters between 0 and 1. γ and ψ are consumption parameters between 0 and 1.

Table 2: Literature From History on Pre-Modern Growth

Paper	Evidence	Findings
Finley (1965)	Various	Some evidence of technological progress in the ancient world.
Malanima (2011)	Data on Northern Italian agricultural production and urbanization	Evidence that pre-Industrial Revolution growth in Northern and Central Italy was stagnant.
Pritchett (1997)	Maddison (various)	Post-1870 divergence in slow-growing economics.
van Zanden et. al. (2012)	Own data set	Evidence of pre-Industrial Revolution persistent growth in Holland.

The rest of the paper is organized as follows. In Section 3, we study a one-sector CES Malthusian model, and show how the Cobb-Douglas function has misguided economists into Malthusianism. Section 4 discusses how historical data on food have been inappropriately used to make broad statements about consumption in general. The section proceeds to relax the assumption that human beings only consume food, presenting the two-sector Malthusian model, which delivers an even heavier blow to Malthusian theory. Section 5 concludes.

3. One Sector CES

3.1. How Economists Have Been Deceived by Cobb-Douglas Production Functions

Cobb-Douglas production functions have long been conventional in Malthusian models. Among all the papers in Table 1, Lee (1980) is the only exception, adopting a CES framework. This convention is not innocuous since Cobb-Douglas functions presume a unit elasticity of substitution between land and labor (denoted: $\epsilon = 1$) which turns out to be crucial for the Malthusian constancy of living standards. We show that, even if there is only one sector, income per-capita can follow a trend if we relax the unit elasticity: $\epsilon \neq 1$.[7] Therefore, for Malthusian theory to hold, a theorist would have to justify two crucial assumptions. The first is the reducibility of food and non-food sectors into a single sector, which we address in Section 4. The second is the knife-edge unit elasticity of substitution between land and labor. Both being implausible, we reject conventional Malthusian theory. The rejection leaves two possibilities: That living standards were not constant or alternatively that they were constant, but for reasons other than the Malthusian constraint. This paper investigates the first possibility.[8]

3.2. The Model

In this section, we study a one-sector Malthusian model. People produce goods X with a simple constant returns CES technology with elasticity $\epsilon = 1/\sigma$ between land L and labor N:

$$X = A(N^{1-\sigma} + L^{1-\sigma})^{\frac{1}{1-\sigma}}$$

The land supply is fixed at $\bar{L}$. Given population N, the income per capita is

$$x \equiv \frac{X}{N} = A\left[1 + \left(\frac{\bar{L}}{N}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Assume that the growth rate of the population increases linearly with x:

$$\frac{\dot{N}}{N} = ax - b$$

where a and b are positive parameters. The subsistence level of per-capita output is defined as $\overline{x} = b/a$, the level that leads to zero population growth. Note then that if $\epsilon > 1$ and $A > \overline{x}$ then as population grows, per-capita output approaches A, which is strictly above the subsistence level.

[7] Jones (2005) attempted to justify the Cobb-Douglas production function by the Pareto distribution of two variables that characterize the generation of ideas. However Growiec (2008) shows that Jone's result relies on the assumption that the two variables are independent to each other. Relaxing the assumption will make the production function no longer Cobb-Douglas.

[8] Wu (2013) explores the second possibility in detail.

Hence we immediately obtain a non-Malthusian conclusion. If we consider the case where there is a constant rate of technological change so that productivity A grows at a constant rate $g > 0$:

$$\frac{\dot{A}}{A} = g$$

then we find that income per-capita will stagnate if and only if $\epsilon = 1$, while it will rise and then decline if $\epsilon < 1$ and will grow steadily if $\epsilon > 1$.[9]

The possibility of growth under $\epsilon > 1$ contradicts the conventional Malthusian results. The combined growth of technology and labor input raise output more than is proportional to population growth. As a result, income per-capita keeps rising despite the Malthusian headwind.

We simulate the model using the following parameterization. The income sensitivity of population growth a is set to be consistent with English demographic data between 1500 and 1800. We normalize $b = a$ to make population stagnate when $x = 1$. The land endowment $\bar{L}$ is the total land area of the world; the initial level of population is 1 million, equal to the number of souls at the dawn of the agricultural era. The growth rate of productivity, 0.4% per decade, approximates the pre-industrial growth rate of population. The initial level of productivity A_0 is set contingently to fix the initial income per-capita at the subsistence level: $x_0 = 1$. The simulation runs for $1,200$ periods, with each period representing a decade. Since the beginning of agriculture (10,000 BC), it had taken about $12,000$ years to reach the Industrial Revolution.

Figure 3.1 shows the simulation results. If $\epsilon < 1$, income per-capita first climbs until it peaks at around the two-thousandth decade. From then on, each generation will live a worse (poorer) life than the last. However, if $\epsilon > 1$, income per-capita will grow at a pace that accelerates over time. The absolute constancy of income per-capita is achieved only when $\epsilon = 1$. In this sense, the classical Malthusian theory is a very special case, implicitly assuming a unit elasticity of substitution.

The case where $\epsilon < 1$ is problematic for four reasons. First, this scenario begins with a period of prosperity followed by a lasting decline. Neither the initial prosperity nor the subsequent decline has sufficient archeological support. Second, population would grow extremely fast in the prosperous early stage. As a result, population would easily exceed 10 billion after 12,000 years. Third, as figure 3.2 shows, if ϵ is smaller than one, the growth of population would decelerate rather than accelerate in the last few millennia, which contradicts historical facts. Fourth, in England, between 1200 and 1300, and between 1600 and 1850, the two periods for which we have data with no abrupt jumps, the expansion of agricultural population was accompanied by an increase in the share of agricultural wage relative to the rent of land—a definite sign that $\epsilon > 1$.

The size of the elasticity is therefore an empirical question, but the empirical estimates in the agricultural economics literature do not apply in the current context. Empirical estimates are adequate for use in short-run production functions, but when it comes to millennia of technological progress, what is relevant is the long-run production function, which envelopes the short-run functions. Even if all short-run functions take a Leontief form, the long-run envelope still allows

[9]This is shown in the web appendix.

flexible substitution between the factors of production. In particular, as population N grows with productivity A, people will develop more labor-intensive technologies to accommodate the increase in labor supply. As a result, the long-run elasticity of substitution is much larger than the short-run elasticities. While the short-run elasticity describes a given technology, the long-run elasticity measures how responsive agricultural innovation is to changes in the availability of factors.

Therefore, it is difficult to rule out the possibility that the long-run elasticity of substitution between land and labor is larger than one. When this is the case, the model predicts slow but accelerating growth both in the world population and in income per-capita. The pattern is consistent with Maddison's data, which shows that population growth accelerated over time and that Western European living standard were far above subsistence in the centuries prior to 1800. Never-the-less, the model has one prediction which is implausible: it predicts that as population grows to infinity, per-capita output remains bounded away from zero. While this leads us to conclude that the production function cannot always be CES with elasticity greater than one, we do not know whether population has ever been so large as to truly render this assumption void. Regardless, the point that Malthusian conclusions do not follow from Malthusian assumptions has been made, and we turn now to what we think is empirically the most significant shortcoming of Malthusian models—the one-sector approach.

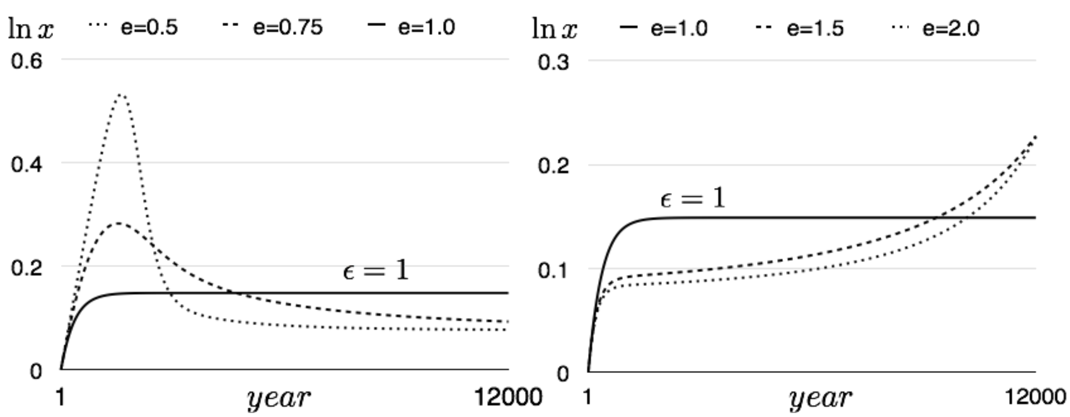

Figure 3.1: The simulated growth patterns of income per capita under different elasticities of substitution

4. Bread and Circuses: Malthus in a Two Sector World

4.1. How Economists Have Misinterpreted Historical Data

Having shown that Malthusian findings arise from an unlikely knife-edge unit elasticity of substitution between factors of production, we move to a more realistic model in which human beings must eat to survive, but also enjoy other things. Before delving into this model, we take a closer look at common claims that available historical data show clear evidence of non-existent pre-1760 growth.

Previous research taking pre-1760 stagnation for granted is problematic for several reasons. First, the data may be wrong. Nearly all of the papers mentioned in Table 1 rely to some extent on

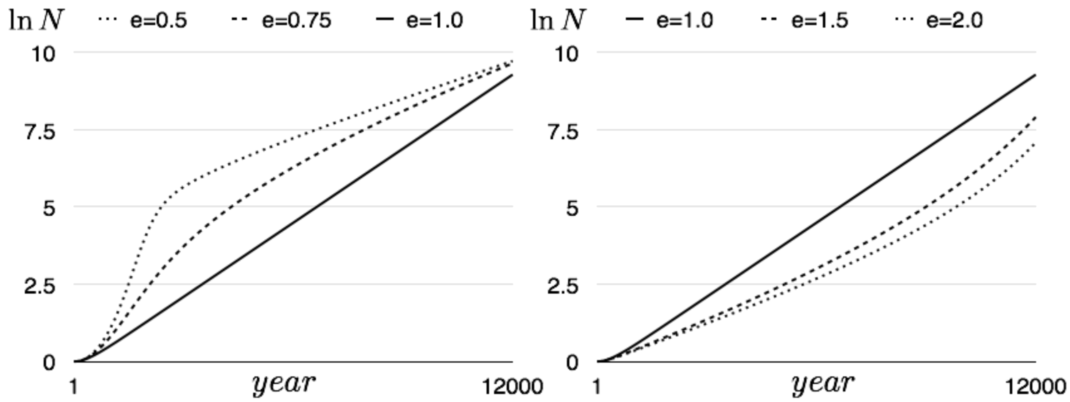

Figure 3.2: The simulated growth patterns of population under different elasticities of substitution

data from Maddison (1982) to motivate their models explaining historical growth patterns. These data are currently undergoing revisions as detailed in van Zanden and Bolt (2013). Further, and as pointed out in a footnote in Galor and Weil (2000), the Maddison (1982) data may have been imputed with a pre-Industrial Revolution Malthusian framework in mind. In other words, data presupposing Malthusian stagnation is used to motivate models explaining Malthusian stagnation.[10] Third, other research using historical data (and detailed in Table 2) has shown evidence of early growth (Finley, 1965; Pritchett, 1997; van Zanden and van Leeuwen, 2012).

Never-the-less, even if we grant that data from Maddison (1982) are correct, it remains problematic to use them to infer that pre-1760 economic growth was flat. The reason is quite simple. These data mostly record historical food production, and yet they are used to draw conclusions about income and consumption in general. Inferring growth based on per-capita agricultural production is misguided and misleading. Though human beings do indeed require food to survive, our well-being is also a function of many things that we consume that are not food. To underscore this point, a cursory glance at growth in U.S. food production in the 20th century shows that it did not move a lot, at least until about 1980 (Figure 4.1). If we (or posterity) were to infer U.S. growth in the last 100 years using the same methodology as scholars currently studying pre-1760 growth, it would erroneously be inferred that no growth took place! As we know, this is not the case, as can be seen in Figure 4.2, which plots per-capita GDP and calories from 1929-2000 in the U.S., normalized to $1.00 and 1, respectively, in 1929. What we see is a five-fold growth in GDP per person, which marks a striking departure from nearly flat growth in calories per person over the same time period.[11]

Closely related to the previous point is that GDP includes many goods that do nothing to encourage survival, but do constitute economic growth. Economists do not see individuals as maxi-

[10]Maddison (1995), it should be noted, refutes that data imputation in Maddison (1982) presupposes Malthusian stagnation.

[11]Underscoring the absurdity of using per-capita food supply as the sole measure of societal wealth and well-being is that in the U.S. and other developed (and even developing) countries, obesity has been associated with poverty rather than affluence. See, for example, Miech et al. (2006).

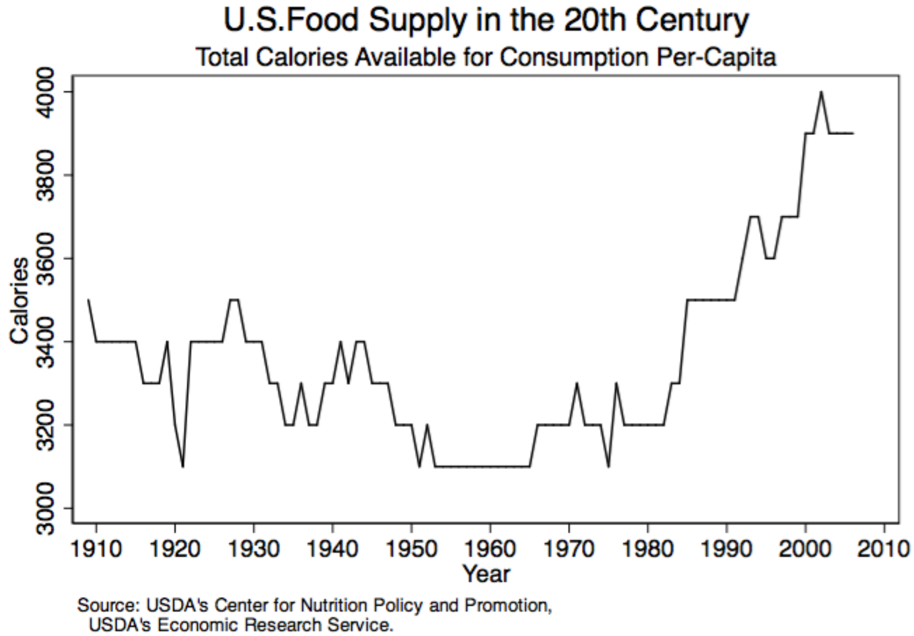

Figure 4.1: Calories per-capita in the U.S. from 1909-2000.

mizing survival. Rather, agents trade off longevity with other consumables that improve the quality of life.[12] To highlight this point, we consider growth in the dollar value of musical instruments per person, again from 1929 to 2000, in comparison to food (Figure 4.1, again normalized to $1.00 and 1 calorie per person, respectively, in 1929). As before, calorie growth is flat. However, the dollar value of the stock of musical instruments per person is anything but flat, a fact that likely did nothing to improve survival, but certainly led to an improvements to the quality of life for many people. Similarly, we argue that many improvements to the quality of life occurred prior to 1760, which did not necessarily increase survival, but did make agents better off.

Underlying some of the issues outlined above seems to be a conceptual misunderstanding that amounts to a confusion between mean and median consumption. On average, a society can become incredibly wealthy even though subsets of its population remain subject to a binding Malthusian constraint. GDP per capita can be very high even though the typical, median individual eats near subsistence. Whether or not this is the case depends on the distribution of wealth. However, to say that no growth occurred prior to 1760 and to base that claim on the fact that many people were hungry, is to misunderstand what average wealth does and does not imply. Great leaps in average wealth have occurred throughout history—and continue to occur—in societies where many, if not most, individuals are separated from hunger by little more than a bad storm, a flood, a drought, a medical emergency or cuts to government-sponsored handouts like food stamps.

[12]In health economics, the value of a medical innovation is sometimes misleadingly computed using its effect on survival. Recent work has shown that this value should take into account that dynamically optimizing agents are willing to forfeit years of life when doing so can increase comfort or consumption.

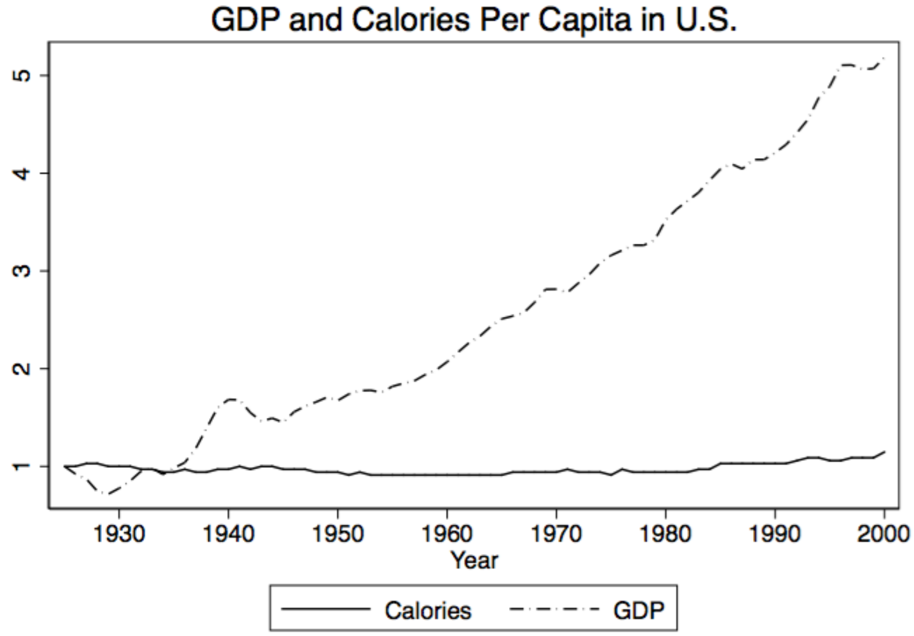

Figure 4.2: Calories per-Capita and GDP per-capita in the U.S. from 1929-2000. For ease of comparison, calories per-capita are normalized to 1 in 1929 and GDP per-capita is normalized to $1.00 in 1929. Source (calories): USDA's Center for Nutrition Policy and Promotion, USDA's Economic Research Service. Source (GDP): Bureau of Economic Analysis.

Among authors who seem to accept pre-1760 growth as flat, there is some hint of acknowledgement of the issues endemic to traditional "Malthusian" models that we outlined above. We focus upon one interesting example: Baumol (1990). We do so because, although like other authors he seems to generally accept flat pre-1760 growth as given, in other passages he seems to express some of the doubts regarding the common practice of relying on food production to infer growth patterns. This leads to a noticeable tension in the paper.

The model in Baumol (1990) focuses on the allocation of entrepreneurial resources to explain growth differences, the aim being to explain great leaps in economic growth. However, the historical evidence he offers points to opportunities in eras and places—that are neither Great Britain nor post-Industrial Revolution—where great wealth could be gained. An example is the High Middle Ages, when a smaller and shorter industrial revolution occurred. After listing a number of improvements, including, for example, better woven woolen goods which surely raised utility, Baumol (1990) states, "In a period in which agriculture probably occupied some 90 percent of the population, the expansion of industry in the twelfth and thirteenth centuries could not by itself have created a major upheaval in living standards." However, in an accompanying footnote, he writes, "But then, much the same was true of the first half century of 'our' industrial revolution," by which he refers to the one beginning in the 18th century.

Nonetheless, Baumol (1990) goes on to say: "[I]t has been deduced from what little we know of European gross domestic product per capita at the beginning of the eighteenth century that its

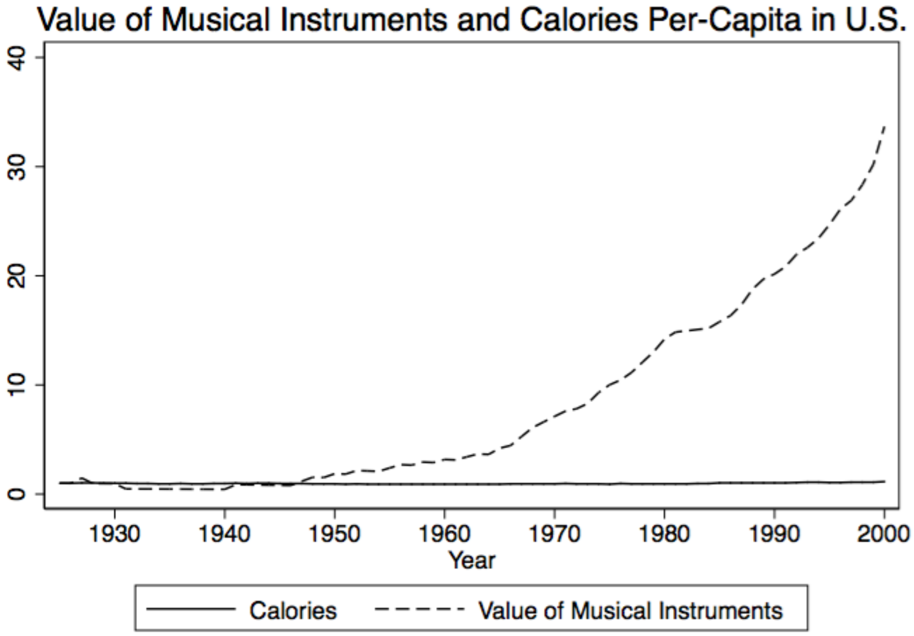

Figure 4.3: Calories per-capita and value of musical instruments per-capita in the U.S. from 1929-2000. For ease of comparison, calories per-capita are normalized to 1 in 1929 and value of musical instruments per-capita is normalized to $1.00 in 1929. Source (calories): USDA's Center for Nutrition Policy and Promotion, USDA's Economic Research Service. Source (value of musical instruments): Bureau of Economic Analysis.

average growth in the preceding six or seven centuries must have been modest, since if the poverty of the later time had represented substantial growth from eleventh-century living standards, much of the earlier population would surely have been condemned to starvation." Here, as in other studies, food and living standards are again used interchangeably, which overlooks how people living today and who eat near subsistence levels, through centuries of technological advances, enjoy better living standards than their pre-1760 counterparts. Consistent with this line of thinking, we present a model that enforces a Malthusian dynamic on population growth, but allow agents to consume (and enjoy) something other than food.

4.2. The Model

We examine a simple two-sector Malthusian model. There are two goods, which we call bread X and circuses Y. These are produced using two inputs, land L and people N. The endowment of land is fixed at L, while at a particular moment of time the endowment of people (labor) is denoted by N. The number of people, however, will be endogenous as it is determined by Malthusian population dynamics. To get a clear picture of what is driving our results we make the assumption that we know leads to stagnation in the one-sector case—that the production function is Cobb-Douglas.

The technologies for bread and circus production are of the Cobb-Douglas form.

$$X = BL_B^{1-\gamma_B} N_B^{\gamma_B}$$

$$Y = CL_C^{1-\gamma_C} N_C^{\gamma_C}$$

with resource constraints,

$$L_B + L_C \leq L$$

$$N_B + N_C \leq N.$$

The fact that the output elasticity in the bread sector with respect to land must be strictly greater than that in the circus sector is captured by the assumption of $\gamma_B \leq \gamma_C$. The goods are assumed to be equally distributed across the population. Per-capita consumption for the two goods is denoted by,

$$x = X/N, y = Y/N.$$

There is a representative consumer who has a Cobb-Douglas utility function,

$$U(x, y) = x^{1-\beta} y^{\beta}.$$

In other words, man does not live by bread alone.[13]

As we have noted, economists do not see individuals as maximizing survival–indeed the Malthusian model does not suppose that population grows above the level of subsistence and that everybody instantly dies the moment food consumption falls below subsistence. Rather it is assumed that with lower food consumption birth-rates decline and death-rates increase. Hence, we assume that the trade-off between bread and circuses for an individual is the usual type of trade-off studied by economists. Specifically, for any given population level N we model the economy as a competitive market for bread and circuses. Equivalently, according to the first welfare theorem, we assume that labor is allocated between the two sectors to maximize the utility of the representative consumer.

Population itself is assumed to move according to the usual Malthusian dynamic with respect to food consumption. We continue to use $\bar{x}$ to denote the subsistence level of bread—circuses being pleasant but unnecessary for survival. Recalling that x is the per-capita output of bread, if follows that population increases if $x > \bar{x}$ and declines if $x < \bar{x}$. Hence we will be interested in the Malthusian equilibrium population level where $N = \frac{X}{\bar{x}}$.

4.3. The Malthusian Equilibrium and Comparative Statics

The Malthusian equilibrium is given by choosing the allocation of land and labor to maximize utility for a given population level N. This results in a level of per-capita bread consumption x. Adding the Malthusian constraint of bread consumption being at the subsistence level, $x = \bar{x}$ allows us then to identify the equilibrium levels of population and circus consumption.

[13] In capturing preferences by a Cobb-Douglas utility function, we are assuming that the two goods are sufficiently substitutable. Nonetheless, we are implicitly assuming the necessity of food for procreation by imposing the Malthusian dynamic.

Proposition 1. *There is a unique Malthusian equilibrium. The equilibrium level of per-capita consumption of circuses is the unique solution to*

$$y = C\left(\frac{\overline{x}}{B}\right)^{\frac{1-\gamma_C}{1-\gamma_B}} f_y(\gamma_B, \gamma_C, \beta)$$

The equilibrium population density is given by,

$$\frac{N}{L} = \left(\frac{B}{\overline{x}}\right)^{\frac{1}{1-\gamma_B}} f_d(\gamma_B, \gamma_C, \beta)$$

The implications of improvements in bread and circus sector productivity on per capita consumption and population density follow immediately.[14]

Proposition 2. *(i) An increase in circus sector productivity increases per capita income while having no impact on population.*
(ii) An increase in bread productivity increases population while decreasing per capita income.

Increases in productivity of the two sectors may often go hand in hand. It is then important to ask which of the forces having an impact on per-capita income would dominate. In order to address this question, suppose that technological changes tend to be proportional. In particular $B = K$ and $C = \lambda K$. A proportional technological change is then characterized by an increase in K. This yields the following result.

Proposition 3. *A proportional increase in productivity results in higher population density* and *higher per-capita income.*

4.4. Discussion of the Two Sector Model

A simple and yet powerful innovation of our model is that it allows for the study of technological changes in the bread and circus sectors separately. Importantly, bread and circuses are different not just in terms of production technology but also in the manner in which they enter the two distinct considerations of consumer preference and population dynamics. The key contrast with earlier studies is that we stay true to the Malthusian premise of population dynamics being determined solely by the amount of bread consumption. The comparative static results above highlight the role that advances in circus sector productivity play in delivering higher per-capita income to a population otherwise squarely held down by Malthusian population dynamics. That increases in bread sector productivity lead to greater population density while bringing down average income is more in line with earlier Malthusian results. Evidently, earlier studies considering a single sector would be left with only this latter result.

[14] In the Propositions, $f_y(\gamma_B, \gamma_C, \beta) = \left(\frac{\gamma_C}{1-\gamma_C}\right)^{\gamma_C - 1} \left(\frac{\gamma_B}{1-\gamma_B}\right)^{1-\gamma_B} \left(\frac{1}{\left(\frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\right)^{\frac{\gamma_C-1}{\gamma_B-1}}}\right)$ and $f_d(\gamma_B, \gamma_C, \beta) = \frac{1}{\left(\frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\right)^{\frac{1}{\gamma_B-1}}\left[\left(\frac{\gamma_B}{1-\gamma_B}\right)\left(\frac{1}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}-1\right)\right]}$.

Here, it is important to highlight that our equilibrium conditions are derived under the assumptions that resources are employed efficiently between the bread and circuses sectors and that both markets clear. These assumptions are entirely standard: so standard that one might overlook their importance. Our findings imply that an efficient use of factors of production might sometimes entail a lower population in favor of more circuses per person. This should not be surprising as it does not imply that agents prefer starvation over not going to the circus. As noted above, in Malthusian models, "subsistence" does not mean starvation. Nor does it mean that individuals are about to die. It simply means that livings standards are sufficiently low that the reproduction rate falls below the replacement rate. A key contribution of our approach is to clearly identify the trade-off between comfort and survival of the species. A simple way to do so has been to allow one good to enhance the quality of life without affecting population growth.

The few studies that have considered a two sector framework have implicitly ruled out the possibility that one of the goods only affects the quality of life without having any influence on population dynamics. As mentioned earlier, Hansen and Prescott (2002) assume that food and non-food items have essentially the same impact on demographic considerations. In Yang and Zhu (2013) it is total consumption, agricultural and non-agricultural, that drives population growth. Further, defining consumer preferences by requiring food consumption to constitute a specific amount of total income with the rest being spent on non-food items essentially makes non-food production the true driver of population growth. Finally, Voigtländer and Voth (2013) assume a form of lexicographic preference in which non-food items are considered only after a subsistence level of food is acquired by the consumer. Population dynamics are driven both by food and non-food consumption, with increases in the latter raising mortality rates. Again, something other than food drives population dynamics and there is no good in the utility function that solely improves the quality of life. The model studied in Voigtländer and Voth (2013) does echo Mathus' own concern regarding working conditions in the manufacturing sector, i.e., that growth in average non-food consumption could make the poor worse off. However, it is difficult to reconcile higher non-food consumption (including better housing, transportation, sanitation, garments, labor-saving devices, and so on) with higher mortality rates. If anything, higher consumption of these goods should lower mortality, though our results go through if we simply assume that non-food consumption has no impact on mortality.

5. Conclusion

Malthusian theory, in its many variants, has a persistent hold upon the imagination of researchers. As we document in this paper, this despite the fact that, both empirically and theoretically, the theory has little support. We especially emphasize what we see as the key deficiency in current historical analyses based on Malthusian theory and the derivative conclusion that the Industrial Revolution is somehow different: the focus on the one-sector model. As we show theoretically, when there are two consumption goods: bread and circuses, even if bread consumption is held down to subsistence by Malthusian forces, this does not imply that circus consumption has any particular level. Rather, increases in circus sector productivity raise per-capita income by increasing the consumption of circuses while leaving bread consumption at the subsistence level. A moment of reflection will show that this is in fact the main source of increased consumption over time: we provide evidence showing that during the Industrial Revolution, bread consumption per-capita did not change much, and that in modern times it is consumption of other things that has skyrocketed. We are not so much better off today because we eat so much better—but because we enjoy a vastly improved quality of life, ranging from entertainment, communication and transportation—and even toilet facilities. It is sometimes remarked that a medieval king would envy the life-style of even a relatively poorly-off citizen of the modern world. Not, however, because we eat so much better.

A second moment of reflection shows that what is true in modern times has been true throughout history. While unequally distributed over the population, there is no doubt that on the basis of per-capita GDP, Romans were quite a lot better off than, for example, stone-age tribes. Again: not because of the amount of food that they ate—but because of the (literal) circuses that they enjoyed—not to speak of stone houses, glass windows, elegant jewelry and on and on.

The major contribution of this paper has been to explore the implications of a model where a Malthusian constraint is imposed upon the population, but where there is also a good to enjoy besides food, where this good has zero impact on population dynamics. Having shown that this simple addition to previous models is enough to permit sustained, long-term economic growth, we now briefly discuss possible extensions. It is possible that non-food consumption is not solely for entertainment purposes. For example, the non-food consumption good might also be permitted to contribute to the individual's survival. One example would replace circuses with warmer homes. Another would allow circuses to be a form of conspicuous consumption that attracts potential mates. In other words, by highlighting the value of including a strictly non-food sector, we have provided a basic framework for future research that can explore the implications of putting more structure on it.

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Appendix: Proofs of Propositions

Proof. **Proposition 3.1**
The first order conditions from the maximization exercise gives the following two equations

$$(1-\beta)(1-\gamma_B)(L-L_B) = \beta(1-\gamma_C)L_B$$

$$(1-\beta)\gamma_B(N-N_B) = \beta\gamma_C N_B$$

This gives us

$$L_B = \frac{(1-\beta)(1-\gamma_B)}{(1-\beta)(1-\gamma_B)+\beta(1-\gamma_C)}L$$

$$N_B = \frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}N$$

We also get

$$\frac{N_B}{L_B} = \left(\frac{\gamma_B}{1-\gamma_B}\frac{1-[(1-\beta)(\gamma_B)+\beta(\gamma_C)]}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\right)\frac{N}{L}$$

Per-capita consumption of bread is

$$x = \frac{AL_B^{1-\gamma_B}N_B^{\gamma_B}}{N} = B\left(\frac{N_B}{L_B}\right)^{\gamma_B-1}\frac{N_B}{N}$$

Since in equilibrium per-capita consumption of bread is at subsistence level, we get

$$\overline{x} = B\left(\frac{N}{L}\right)^{\gamma_B-1}\frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\left[\left(\frac{\gamma_B}{1-\gamma_B}\right)\left(\frac{1}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}-1\right)\right]^{\gamma_B-1} \tag{5.1}$$

Per-capita consumption of circuses is

$$y = C\left(\frac{N}{L}\right)^{\gamma_C-1}\frac{\beta(\gamma_C)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\left[\left(\frac{\gamma_C}{1-\gamma_C}\right)\left(\frac{1}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}-1\right)\right]^{\gamma_C-1} \tag{5.2}$$

Equation 5.1 gives us the equation determining equilibrium population density

$$\frac{N}{L} = \frac{\overline{x}^{\frac{1}{\gamma_B-1}}B^{\frac{1}{1-\gamma_B}}}{\left(\frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\right)^{\frac{1}{\gamma_B-1}}\left[\left(\frac{\gamma_B}{1-\gamma_B}\right)\left(\frac{1}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}-1\right)\right]} \tag{5.3}$$

This in turn gives us the equation for per-capita consumption of circuses

$$y = C\left(\frac{\gamma_C}{1-\gamma_C}\right)^{\gamma_C - 1}\left(\frac{\gamma_B}{1-\gamma_B}\right)^{1-\gamma_B}\left(\frac{\overline{x}^{\frac{\gamma_C-1}{\gamma_B-1}}}{B^{\frac{1-\gamma_C}{1-\gamma_B}}\left(\frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\right)^{\frac{\gamma_C-1}{\gamma_B-1}}}\right)\frac{\beta(\gamma_C)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)} \tag{5.4}$$

Proposition 3.3
Setting $B = K$ and $C = \lambda K$, Equation 5.4 can then be written as

$$y = \lambda K^{1-\frac{1-\gamma_C}{1-\gamma_B}}\left(\frac{\gamma_C}{1-\gamma_C}\right)^{\gamma_C - 1}\left(\frac{\gamma_B}{1-\gamma_B}\right)^{1-\gamma_B}\left(\frac{\overline{x}^{\frac{\gamma_C-1}{\gamma_B-1}}}{\left(\frac{(1-\beta)(\gamma_B)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}\right)^{\frac{\gamma_C-1}{\gamma_B-1}}}\right)\frac{\beta(\gamma_C)}{(1-\beta)(\gamma_B)+\beta(\gamma_C)}$$

Since γ_C is greater than γ_C it follows that

$$\frac{1-\gamma_C}{1-\gamma_B} < 1$$

This in turn yields the required result. □

Web Appendix: One Sector CES Dynamics

Suppose the productivity A grows at a constant rate $g > 0$:

$$\frac{\dot{A}}{A} = g$$

Proposition 4. *The income per capita will stagnate if $\epsilon = 1$, will climb up and then decline if $\epsilon < 1$, and will be growing if $\epsilon > 1$.*

Proof. The income per capita is

$$x = A\left[1 + \left(\frac{\bar{L}}{N}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Taking logarithm of both sides of the above equation and differentiating it, we get

$$\frac{\dot{x}}{x} = g - \frac{\left(\frac{\bar{L}}{N}\right)^{1-\sigma}}{1+\left(\frac{\bar{L}}{N}\right)^{1-\sigma}}(ax - b)$$

The above equation and the population dynamics $\frac{\dot{N}}{N} = ax - b$ form a system of differential equations. The null-clines of the system are

$$\frac{\dot{x}}{x} = 0: \qquad x = \frac{b}{a} + \frac{g}{a}\left[\left(\frac{N}{\bar{L}}\right)^{1-\sigma} + 1\right]$$

$$\frac{\dot{N}}{N} = 0: \qquad x = \frac{b}{a}$$

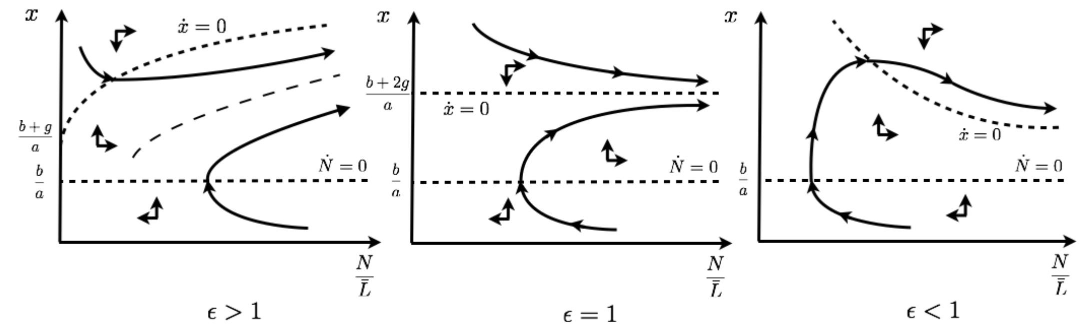

Figure 5.1: The phase diagrams under different elasticities of substitution

Figure 5.1 shows the phase diagrams under different elasticities of substitution. When the elasticity is smaller than or equal to one, income per-capita will remain suppressed in the long run as Malthusian theory predicts, but when the elasticity exceeds one, income per-capita will keep growing. □