A SIMPLE DURABLE GOODS MODEL*

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The durability of a good has two implications. First, it can be stored in inventories by producers. Second, if it provides a stream of services to consumers, consumers may wish to defer purchases to take advantage of price fluctuations. The most significant conclusion is that the stockpiling of demand that results when consumers defer purchases explains why the variance of output exceeds the variance of sales even if demand shocks are serially independent.

I. INTRODUCTION

Since inventories appear empirically to play an important role in the business cycle, considerable effort has been made to study the theoretical aspects of inventory fluctuations. Most models that have been studied are of the production smoothing type. In these models there is increasing marginal cost of production, and inventories are held to smooth the irregular pattern of demand in order to lower overall production costs. Unfortunately, available empirical evidence (Section V.B) has consistently shown that production varies more than sales, casting doubt on production smoothing as a model of inventories.

The conflict between production smoothing and production varying more than sales has been partially resolved by Blinder [1981, 1982], who showed that production may vary more than sales, provided that shocks to demand are serially correlated for reasons exogenous to the model. One theme of this paper is that the behavior of demanders can lead to endogenous serial correlation in sales, even though shocks to demand are serially independent. This also can explain why production varies more than sales.

This paper differs from previous papers on inventories by reducing the emphasis on the production and storage of durables and increasing emphasis on the fact that durables yield a stream of services to consumers. Thus, in previous models demand is taken to be independent of past and anticipated future prices. With a durable good this assumption makes little sense: typically not only is a durable storable for the firm (frozen orange juice), but also it provides a relatively long stream of services to the

*I am grateful to Andrew Abel, Drew Fudenberg, Bengt Holmstrom, the economic theory workshop at the University of Western Ontario, the Econometric Society session on Stochastic Models of the firm, and two anonymous referees.
consumer (cars). Thus, the consumer must decide when to buy the durable, and anticipated future prices matter. The timing of consumer purchases appears to be an important part of the business cycle and is thus deserving of study in its own right. In this paper I study the simultaneous decisions of firms to hold inventories and consumers to defer purchases in a simple rational expectations model.

Except for Reagan and Weitzman [1983] (and to some extent Blinder), the inventory literature has followed Zabel [1972] and focused on the inventory decisions of a single monopolistic producer. This is because at the firm level inventories in a competitive market are not well-behaved. However, there is no problem at the industry level, and thus I elect to study what is probably the more interesting case of perfect competition.

To what extent can the stylized facts of inventory fluctuations be explained by a very simple competitive model? To explore this question, I assume a constant marginal cost of production, zero storage cost (except for forgone interest), and identical consumers. Except in the earlier Arrow-Karlin-Scarf [1958] literature (which focused on indivisibilities on production), these are assumptions frequently employed in the literature on inventories.

The conclusions of the analysis are mixed. With serially independent shocks sales do exhibit a positive serial correlation, but follow an MA(1) process that does not exhibit the long lag structure characteristic of macro data. In this respect the model represents a step back from Blinder and Fischer [1981], who showed how a richer model of production can lead to a long lag structure through the behavior of inventories.

The model is more successful in explaining why production varies more than sales. Production varies solely in order to correct past mistakes; in determining sales, the irregular path of production is actually smoothed by consumers deferring purchases. Thus, focus on the supply side has led primarily to models of production smoothing. Focus on the demand side leads instead to a theory of sales smoothing.

Finally, I study the issue of "price stickiness" or the "insensitivity" of prices to demand shocks. Blinder [1981, 1982] and especially Reagan [1982] and Reagan and Weitzman [1983] have also considered this issue. Unlike Reagan and Weitzman, I find that the sensitivity of prices to demand shocks does not depend on the size of the shock to any great extent. Indeed, while they found that prices should be most sensitive to the largest demand shocks, I find that when consumers can defer purchases, prices
should be most sensitive to demand shocks of an intermediate size—the possibility of deferring purchases puts an upper bound on how much consumers are willing to pay now, and this limits the extent to which demand shocks, no matter how large, can raise current prices. As a result, if the production lag is very short, prices are almost completely insensitive to demand shocks of any size.

II. THE MODEL

At time $t$ cars sell for $p_t$. There are $C_t$ cars available for sale (notional supply) and $D_t$ consumers wishing to buy a car (notional demand). Excess demand is $X_t = D_t - C_t$. Sales of cars are $S_t$. Borrowing and lending can take place in a market with discount factor $0 < \beta < 1$. Cars last forever, and there is no resale market.

Cars are produced by a single risk-neutral competitive producer who views both price and market inventories as outside his control. Output of cars at $t$ is $Y_t$, the cost of producing a car is $c$ and is incurred at time $t - 1$ when the production decision must be made. Thus, cars are produced at constant marginal cost with a one-period lag. Inventories available for sale at $t$ are $I_t$. Notional supply is just

$$C_t = Y_t + I_t,$$

while inventories are determined by

$$I_t = C_{t-1} - S_{t-1}.$$  

The firm begins in period zero with no cars. Demand begins one period later.

Each car buyer demands exactly one car, and after purchase keeps it forever. In period $t$, $\varepsilon_t$ new car buyers arrive at the market. These are assumed to be i.i.d. nonnegative random variables with a continuous c.d.f. $F(\varepsilon)$. The stock of potential car buyers is

$$D_t = D_{t-1} - S_{t-1} + \varepsilon_t.$$  

Consumers are identical and risk neutral and receive a money equivalent utility of $u - p$ in the period they buy a car for price $p$; they receive zero in all other periods.

A car produced in period $t - 1$ costs $c$ to produce and, if purchased when available in period $t$, yields a money equivalent utility of $u$. Discounting this back to $t - 1$ shows that if the value of cars is to exceed their production cost, then
I shall always assume this to be the case.

A rational expectations competitive equilibrium of this market is characterized by the following:

A. Rational Expectations—probability distributions perceived by agents are the same as those generated by the model.

B. Zero Profits—the firm has an expected present value of zero.

C. Optimal Production—the firm believes that it can sell all it wants at the prevailing market price. Given this belief, it cannot profit by altering its production plan.

D. Voluntary Exchange—if the firm sells a car, it prefers this to selling later; if a buyer buys a car, he prefers this both to waiting and to not buying a car at all.

E. Feasibility\(0 \leq S_t \leq \min(D_t,C_t)\).

Let \(H_t\) be all information available to agents at the close of period \(t\)—all current and past output, sales and especially notional supply and demand (which are presumed to be observable). In accordance with (A) all random variables conditioned on \(H_t\) have the distribution generated endogenously by the model.

III. THE EQUILIBRIUM THEOREM

The market described above has a unique rational expectations equilibrium. The equilibrium production plan is

\[
\begin{align*}
Y_1 &= \gamma \equiv F^{-1}(1 - \lambda), \quad \text{where } \lambda \equiv c/\beta u \\
Y_t &= \epsilon_{t-1}, \quad t > 1.
\end{align*}
\]

Sales are determined by

\[
S_t = \min(D_t,C_t).
\]

and prices by

\[
\begin{align*}
\epsilon_t > \gamma, \quad p_t &= \bar{p} = c + u(1 - \beta); \\
\epsilon_t < \gamma, \quad p_t &= \bar{p} \equiv c.
\end{align*}
\]

Note that \(pr(\epsilon_t = \gamma) = 0\) so that (7) almost surely determines prices.

As a preliminary to proving this theorem, I first develop some of its qualitative properties. The probability that the high price \(\bar{p}\) occurs is \(1 - F(\gamma) = \lambda\) by the definition of \(\gamma\). Expected next period price is
which is the cost of producing a car this period in next-period dollars. It follows from (8) that $E(p_{t+1}|H_{t-k}) = c/\beta$, $k \geq 1$.

The low price $\bar{p} = c$ is less than the production cost $c/\beta$. However, the firm is indifferent between selling at $p$ and selling next period for the expected price of $c/\beta$. The high price $\bar{p} = c + u(1 - \beta)$ is, by virtue of (4), above production cost and leaves the consumer indifferent between buying now for a present value of $u - \bar{p}$ or next period for the expected present value of $\beta[u - (c/\beta)]$.

In period 1 production is $\gamma$, and demand is $\varepsilon_1$. Thus, excess demand is $X_1 = \varepsilon_1 - \gamma$. In subsequent periods by (3) and (6), demand is $D_t = \varepsilon_t + \max(X_{t-1},0)$, while by (1), (2), and (6), notional supply is $C_t = \varepsilon_{t-1} - \min(X_{t-1},0)$. By inductive hypothesis $X_{t-1} - \varepsilon_{t-1} = \gamma$, so equilibrium excess demand is

$$X_t = \varepsilon_t - \gamma.$$

Thus, by (7) when $p_t = \bar{p}$, there is excess demand, the firm sells all its cars, and some consumers wait to buy a car. Also, by (7), when $p_t = p$, all consumers buy a car, and the firm holds inventories.

I now prove that the system of equations above does indeed define an equilibrium. I consider conditions (B), (C), and (D). Condition (E) follows direction from (6). Later in the section I prove that this is the only equilibrium.

Define $V_t$ to be the expected present value of the selling price of a car stocked at time $t$ conditional on $H_{t-1}$.

**Lemma 1.**

$$V_t = c/\beta.$$

**Proof of Lemma 1.** Let $f_t$ be the probability that the car is sold now when $p$ occurs. Then

$$V_t = \lambda \bar{p} + (1 - \lambda)f_t \bar{p} + (1 - \lambda)(1 - f_t)\beta V_{t+1}.$$

Algebraic manipulation of (10), making use of (8), shows that if $Z_t = \beta V_t - p$, then

$$Z_{t+1} = [\beta(1 - \lambda)(1 - f_t)]^{-1} Z_t.$$

However, $Z_t$ is bounded, since $0 \leq V_t \leq \bar{p}$, and the only solution of (11) that remains bounded is $Z_t \equiv 0$. Since $Z_t = \beta V_t - p$,

$$V_t = p/\beta = c/\beta.$$

Q.E.D.
The lemma implies that the expected selling price of a car equals its cost of production. Thus, the zero profit condition (B) holds, and any production plan is profit maximizing, given the perception that the firm cannot control prices and can sell all it wants when \( \bar{p} \) occurs. Thus, (C) holds. Finally, (D) holds for the firm. If \( p_t = \bar{p} \), it sells all its cars and strictly prefers to sell them now. If \( p_t = \bar{p} \), it is indifferent between selling now and waiting and is willing to hold the required inventories. A similar argument shows that exchange is voluntary for the consumer.

This proves that the equations above do indeed give rise to an equilibrium. I now show that this is the only equilibrium.

Observe that in equilibrium,

\[
E(p_{t+1}|H_{t-k}) = c/\beta, \quad k \geq 0
\]

(provided that there is a positive probability of sale in period \( t + 1 \)). For if \( E(p_{t+1}|H_t) > c/\beta \), the firm will wish to produce infinitely many cars at \( t \). Thus, \( E(p_{t+1}|H_{t-k}) \leq c/\beta \). But if strict inequality holds with positive probability of a sale, the present value of the firm is negative: it never expects a profit on a sale, but sometimes expects a loss. Thus, (12) must hold.

Since the firm cannot expect to earn more than a present value of \( c \) from selling a car later, it follows that all cars held by the firm are sold if the price now is \( p > \bar{p} = c \). Next we establish that when \( p \leq \bar{p} \), in equilibrium all consumers buy cars. Let \( U_t \) be the utility of (any) consumer in the market at time \( t \). Let \( \bar{U} = \sup_{H_t} E(U_{t+1}|H_t) \). Thus, \( \bar{U} \) is the most utility a consumer can get under any circumstances. We show that buying at \( \bar{p} \) (or less) now is always better than waiting until next period and getting \( \bar{U} \):

**Lemma 2.**

\[
\beta \bar{U} < u - p.
\]

**Proof of Lemma 2.** We examine \( E(U_{t+1}|H_t) \). If \( H_t \) is such that \( C_{t+1} \) (current inventories plus production) is zero, then

\[
U_{t+1} = \beta E(U_{t+2}|H_{t+1}) \leq \beta \bar{U}
\]

implying that \( U_{t+1} < \bar{U} \). Thus, \( \bar{U} \) is the supremum over \( H_t \) with \( C_{t+1} > 0 \). I claim that this implies

\[
U_{t+1} \leq (u - p_{t+1}) + \max(0, \beta \bar{U} - u + p).
\]

There are two cases. If \( p_{t+1} > \bar{p} \), the firm sells all \( C_{t+1} > 0 \) cars, and since consumers buy them voluntarily, \( U_{t+1} = u - p_{t+1} \). If \( p_{t+1} \leq \bar{p} \), we can write
\[ U_{t+1} \leq \max(u - p_{t+1}, \beta \bar{U}) \\
= (u - p_{t+1}) + \max(0, \beta \bar{U} - u + p_{t+1}). \\
\leq (u - p_{t+1}) + \max(0, \beta \bar{U} - u + \bar{p}). \]

Thus, since \( E[p_{t+1}|H_t] = c/\beta, \bar{U} \leq (u - c/\beta) + \max(0, \beta \bar{U} - u + \bar{p}). \) If \( \beta \bar{U} \geq u - \bar{p} \) (the converse of what we wish to show), then since \( \bar{p} = c, \)

\[ \bar{U} \leq u - c/\beta + \beta \bar{U} - u + c = c - c/\beta + \beta \bar{U}, \]

and \( (1 - \beta) \bar{U} \leq c - c/\beta < 0, \) which contradicts \( \beta \bar{U} \geq u - c > u - c/\beta > 0. \) Thus, \( \beta \bar{U} < u - \bar{p}. \)

Q.E.D.

Lemma 2 implies that all consumers buy cars if \( p < p, \) and we already know that all cars held by the firm are sold if \( p > \bar{p}. \) We conclude that

\[ S_t = \min(D_t, C_t). \]

If \( X_t > 0 \) and there is excess demand, the consumer must be indifferent between buying now and waiting. By virtue of (12) this implies that \( p_t = \bar{p}. \) If \( X_t < 0, \) the firm must be indifferent between selling now and waiting—by (12) this implies that \( p_t = p. \) Also if \( \lambda \) is the probability of \( \bar{p} \) at time \( t, \) conditional on \( t - 1 \) information, by (12) is

\[ \lambda \bar{p} + (1 - \lambda) \bar{p} = c/\beta, \]

which implies that \( \lambda = c/\beta u. \)

It remains only to show that the optimal production plan is unique. Production at \( t - 1 \) must be chosen so that \( pr(X_t > 0) = \lambda. \) A bit of algebraic manipulation then yields the production plan in (9).

This completes proof of the equilibrium theorem.

One aspect of the equilibrium deserves note. Firms do not charge inventory carrying costs: the expected price equals production cost. However, the average price paid by a consumer exceeds expected price. This is because when price is high, there are (typically) more consumers than when price is low. Since the firm has zero present value, the excess of average price paid by a consumer over expected price (equals production cost) must exactly compensate the firm for its cost of carrying inventories.
IV. PARETO EFFICIENCY

The first welfare theorem assures us that a finite horizon economy with finitely many states and complete contingent claims is Pareto efficient. The durable good economy described above has an infinite horizon, infinitely many states each period, and since consumers cannot order cars in advance, does not have complete contingent claims. However, consumers who defer purchases are risk neutral and pay the same expected price they would have to pay to order a car in advance. That is, the market is identical to one in which consumers who do not get cars order one for next period and pay the production cost of $c$. Thus, the market is equivalent to one with complete contingent claims. This makes it plausible that the market is Pareto efficient: it was shown by dynamic programming arguments in an earlier version of this paper [Levine, 1982] that the equilibrium production plan maximizes the excess of the present value of consumer utility over the present value of producing cars. In fact, from the equilibrium production plan (5), we see that the social optimum is to produce enough cars to fill previous unsated demand and $\gamma$ more for new customers. The reason for this is that with constant marginal cost, past errors can be wiped clean by producing enough cars for any currently unsatisfied demand.

If consumers are not risk neutral, then a forward market can provide insurance against price fluctuations next period, since consumers could order cars to hold as a hedge. Unfortunately, without complete contingent claims, not only is the equilibrium inefficient relative to the full set of markets, but it may actually be inefficient relative to the existing set of markets. A good discussion of the efficiency issues with incomplete contingent claims is in Hart [1975]. One advantage of the risk-neutrality assumption is that it enables us to avoid the empirical issue of whether or not there are adequate forward markets for durables—it makes no difference to the equilibrium. Furthermore, it would appear that (for reasons explored in Section V.C) durables prices do not fluctuate very much. Thus, the actual risk faced by consumers is small, and risk neutrality can be regarded as a good approximation to reality.

V. QUALITATIVE DYNAMICS

What qualitative characteristics does the unique equilibrium have, and how well does it explain the stylized facts of actual
inventory behavior? I explore this, maintaining the assumption that demand shocks are independent.

A. Serial Correlation in Quantities

Of primary interest are the three observables: production $Y_t$, inventories $I_t$, and sales $S_t$. Recall that excess demand $X_t = \varepsilon_t - \gamma$. It is convenient to let $X_t^+ = \max(\varepsilon_t - \gamma, 0)$ and $X_t^- = \min(\varepsilon_t - \gamma, 0)$. Thus, $X_t = X_t^+ + X_t^-$. Then production $Y_t = \gamma + X_{t-1}$, which is serially independent, and inventories $I_t = -X_{t-1}$, which is also serially independent. Sales, on the other hand, are $S_t = \gamma + X_{t-1}^+ + X_{t}^-$, which follows an MA(1) process, and since both $S_t$ and $S_{t-1}$ are increasing functions of $X_{t-1}$, $\text{cov}(S_t, S_{t-1}) > 0$.

The most striking fact is the relative absence of serial correlation. Actual time series are typically better described by AR(1) than MA(1) processes and exhibit dependence on innovations from the relatively distant past. However, with constant marginal cost the slate is wiped clean each period, and innovations from previous periods become irrelevant. One possible reason long lags appear in economic series is that the innovations themselves—the $X_t$ (or equivalently $\varepsilon_t$)—are serially dependent. Another possibility is that there are diminishing returns to scale in production. However, since industry studies seem to show that actual cost curves exhibit roughly constant marginal cost between a minimum efficient scale and a capacity constraint, the constant marginal cost of production assumption should not be dropped lightly.

B. Variance of Output Versus Sales

Most inventory models exhibit production smoothing: with increasing marginal cost of production, inventories are used to smooth output, while sales are permitted to fluctuate. Unfortunately, empirical evidence indicates that production fluctuates more than sales. At an aggregate level this is a well-known stylized fact reported, for example, in Feldstein and Auerbach [1976, Table 1]: GNP varies less if investment in total business inventories is subtracted. A study of the automobile industry shows that this is true at a disaggregated level as well: Blanchard [1983] reports that the variance of output is roughly 1.3 times that of sales.

How does the variance of output compare with that of sales in this model with constant marginal cost? Since
\[ Y_t = \gamma + X_{t-1} \]
\[ = \gamma + X_{t-1}^+ + X_{t-1}^- \]
\[ S_t = \gamma + X_{t-1}^+ + X_t^- \]

and \( X_t \) and \( X_{t-1} \) are independent, we have

\[
\text{var}(Y_t) - \text{var}(S_t) = 2 \text{cov}(X_{t-1}^-, X_{t-1}^-)
\]
\[ = 2[EX_{t-1}^+X_{t-1}^- - EX_{t-1}^+EX_{t-1}^-] \]
\[ = -2EX_{t-1}^+EX_{t-1}^- > 0. \]

Thus, consumers deferring purchases and constant returns to scale imply that \textit{production} will vary more than sales! Intuitively, the reason production varies over time is to correct past mistakes. The resulting variations in production are then smoothed by consumers deferring purchases between periods, yielding a flatter and more regular sales profile. This is illustrated in Figure I, where a particular realization of the sequence \( \epsilon_t \) is shown. Output, of course, just lags \( \epsilon_t \) by one period. However, the shifting of

\[ t \]

\[ t \]

\[ t \]

\[ \text{FIGURE I} \]

Output Versus Sales
demand between periods yields a sales profile that is completely flat. Blinder [1981] showed that exogenous positive serial correlation in demand gave this result; the analysis here shows that endogenous serial correlation in sales explains the same fact.

C. Price Variation

Like output and inventories, price changes are serially independent. Of more interest is the extent to which they actually vary over time and the elasticity of price with respect to demand shocks. Reagan and Weitzman [1983] argue that since at any moment of time the supply curve is J-shaped—flat until a stockout occurs, then completely inelastic. When demand is low, small demand shocks will not affect price; while demand is high, small demand shocks will have a large impact on price. With consumer waiting the picture changes radically. When demand is low, price is \( p \) and insensitive to small increases in \( \varepsilon \); when demand is high, price is \( \bar{p} \) and is still insensitive to small demand shocks. Only in the intermediate range where demand \( \varepsilon \) is close to \( \gamma \) will price respond much to small demand shocks: in this case it will jump between \( p \) and \( \bar{p} \). As Figure II shows, the reason for this is that not only is the supply curve shaped like a \( J \), but the demand curve is shaped like a "7." Thus, the behavior of consumers truncates prices above as well as below.

How much does price actually jump between \( p \) and \( p \)? A calculation shows that

\[
\frac{\bar{p}}{p} - 1 = \frac{1 - \frac{\beta}{\lambda}}{\beta}\frac{1}{\lambda},
\]

where \( \lambda = c/\beta u = 1 - F(\gamma) \) is the probability of a stockout. Thus, with \( \lambda \) held fixed, the price variation ranges from zero to infinity and decreases with \( \beta \). Thus, if the periods are very short so \( \beta \) is close to one, prices do not vary very much—regardless of how large inventories or unsatisfied demand is (these depend on how large \( \varepsilon \) is and not on \( \lambda \)). As I remark below, this means that there may be large inventories and many unsatisfied buyers simultaneously and in every period, yet even as these fluctuate wildly, prices never seem to change very much! This may partially explain why prices appear to be "sticky."

D. Stockouts and Aggregation

One implication of the model is that stockouts should occur frequently. This rarely happens at the level of an industry. The
straightforward explanation of this is that many small independent markets (car dealerships) are served by a common constant return producer. In this case aggregate inventories are \( \Sigma_{i=1}^{N} - X^-_i(i) \), where \( i \) denotes the \( i \)th submarket, and unsatisfied demand is \( \Sigma_{i=1}^{N} X^+_i(i) \). Notice that at an aggregate level stockouts will rarely occur and that unfilled orders and inventories will exist simultaneously (but actually in different submarkets). Of course, at a less aggregate level, a car dealership say, we should (and do) observe frequent stockouts.

VI. CONCLUSION

The analysis shows that a simple representative consumer model with constant marginal cost of production, no storage cost, and serially independent demand shocks explains serial correlation in sales; why output varies more than sales; and why prices appear relatively rigid. Serial correlation in demand shocks can be introduced into the model with little additional cost: in this case the target production \( \gamma \) will simply become a random variable.
γ, based on the distribution of arriving consumers conditioned on current information. This would explain (although by an exogenous mechanism) the long lags empirically observed in the correlation of sales.

Several types of consumers may be added to the model with little additional cost as well. The major implication appears to be that there will be more than two possible market prices. Each additional type of consumer will have a price that makes that group indifferent between buying now and next period. The least patient consumers (those who value the product most highly or have the highest discount rates) will buy currently available cars. The available supply of cars will determine which group of consumers is the marginal group that will have its demand only partially satisfied, and the market clearing price will be the one that makes this group indifferent to waiting. More patient types will, of course, strictly prefer to wait.

A constant marginal cost of storage is also easy to analyze. In this case, the relevant consideration for holding inventories is next-period expected price net of inventory holding costs. Thus, the price that makes the firm indifferent will be higher by the storage cost. Otherwise, the model is unchanged.

The change that would substantially complicate the model would be to allow increasing marginal cost. This eliminates the "sweeping away" of past errors property that makes the model so easy to analyze. The equilibrium no longer will be stationary, but prices will depend on the level of inventories or leftover consumers, since this affects the cost of producing for newly arriving consumers. As a result, the decision problem of consumers is enormously complicated. An obvious effect is to introduce longer lags into the model. Another effect is production smoothing, which would tend to reduce the variation in output relative to sales. A model of this type would probably prove tractable only insofar as it could be reduced to a social optimization problem.

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REFERENCES


