Self Control, Risk Aversion, and the Allais Paradox

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Introduction

Explain quantitatively Allais paradox as a consequence of a self-control problem

A common explanation with effect of cognitive load on decision making

Explains also Rabin paradox

Use self-control framework of Fudenberg and Levine [2006]

Shiv and Fedorikhin [1999]

memorize either two- or a seven-digit number

walk to table with choice of two desserts: chocolate cake or fruit salad

pick a ticket for one dessert

report number and dessert choice in a different room

seven-digit number: cake 63% of time

two-digit number: cake 41% of time

(statistically as well as economically significant)

our interpretation: cognitive resources used for self-control are substitutes for cognitive resources used for memorizing numbers plus increasing marginal cost of cognitive resource usage

An Implication

replace desserts with lotteries giving a probability of a dessert self-control problem reduce, so fewer should give in to temptation of chocolate cake

violates independence axiom

will argue that Allais paradox has a similar nature

Self-Control with a Cash Constraint

Fudenberg and Levine [2006] "self control game" between single long run patient self and sequence of short-run impulsive selves equivalent to "reduced form maximization" by single long-run agent

Reduced form: maximize expected present value of per-period utility *u* net of self control costs *C*:

$$U = \sum_{t=1}^{\infty} \delta^{t-1} \big(u(a_t, y_y) - C(a_t, y_t) \big), \qquad (2.1)$$

a_t action chosen in period *t*

y_t state variable such as wealth

“opportunity-based cost of self control”: cost C depends only on realized short-run utility and highest possible value of short-run utility in current state

latter *temptation utility*

note that preferences here are time-consistent

infinite-lived consumer savings decision

periods $t = 1, 2, \ldots$ divided into two sub-periods

bank subperiod and *nightclub* subperiod

state $w \in \Re_+$ wealth at beginning of bank sub-period

"bank" subperiod, no consumption, wealth w_t divided between savings s_t (remains in bank) and cash x_t carried to nightclub (also durable spending)

consumption not possible in bank, so short-run self indifferent between all possible choices, and long-run self incurs no cost of self control

in nightclub consumption $0 \le c_t \le x_t$ determined, with $x_t - c_t$ returned to bank at end of period

$w_{t+1} = R(s_t + x_t - c_t)$ no borrowing possible, and no source of income other than return on investment.

Extension of Fudenberg and Levine [2006]

Choice of nightclubs indexed by quality of nightclub $c^* \in (0,\infty)$

"target" level of consumption expenditure

low value of $c\,*$ cheap beer bar

high value of $c\,*$ expensive wine bar

base preference of short-run self $u(c,c^*)$

$u(c,c) = \log c$, ($\log(0) = -\infty$)

$u(c,c^*) \le u(c,c)$ so best to choose nightclub of same index as amount you want to spend

convenient functional form (with $\rho > 1$)

$$u(c,c^*) = \log c\,* - \frac{(c\,/\,c^*)^{1-\rho} - 1}{\rho - 1}.$$

$g(d+\overline{u}-u)$ cost of self-control when maximum (temptation) utility attainable for short-run self is $\overline{u}$, actual realized utility u, and cognitive load due to or activities d

in calibrations use quadratic: $g(u)=\gamma_0 u+(1/2)bu^2$.

reduced form preferences for long-run self are (w/o durable)

$$U_{RF}=\sum_{t=1}^{\infty}\delta^{t-1}\left[u(c_t,c_t^*)-g(u(x_t,c_t^*)-u(c_t,c_t^*))\right]. \qquad (2.2)$$

no cost of self-control in bank so choose $c_t^*=c_t=x_t=(1-\delta)w_t$

same as solution without self-control

utility as function of wealth:

$$U_1(w_1)=\frac{\log(w_1)}{1-\delta}+K.$$

Risky Drinking: Nightclubs and Lotteries

Suppose at door to nightclub you are greeted by Maurice Allais who insists that you choose between two lotteries, A and B' with returns $\tilde{z}_1^A, \tilde{z}_1^B$ (losses not to exceed pocket cash)

Assume choice completely unanticipated

Assume that no further lotteries at nightclubs are expected in the future

highest possible short-run utility comes from consuming entire outcome of lottery, temptation utility calculated as
$\max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\}$

where $\tilde{z}_1^j$ realization of lottery $j = A, B$

$\tilde{c}_1^j(z_1^j)$ consumption chosen contingent on realization of lottery *j,* self-control cost

$$\overline{g}(x_1, \tilde{c}_1^j, c_1^*) = g\Big(\max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\} - Eu(\tilde{c}_1^j, c_1^*)\Big)$$

random unanticipated income $\tilde{z}_1^j$ at nightclub

z_1 realized income, short-run self constrained to consume $c_1 \leq x_1 + z_1$.

Period 2 wealth given by

$$w_2 = R(s_1 + x_1 + z_1 - c_1) = R(w_1 + z_1 - c_1)\,.$$

utility of long-run self starting in period 2 given by solution of problem without self control

$$U_2(w_2) = \frac{\log(w_2)}{1-\delta} + K$$

$\tilde{c}_1$ optimal response to unanticipated income $\tilde{z}_1$

overall objective of long-run self to maximize

$$Eu(\tilde{c}_1^j, c_1^*) - \overline{g}(x_1, \tilde{c}_1^j, c_1^*) + \frac{\delta}{(1-\delta)} E \log(w_1 + \tilde{z}_1^j - \tilde{c}_1^j) + K$$

$$\overline{u}(x_1, c_1^*) = \max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\}$$

marginal cost of self-control:

$$\gamma = g'\Big(\overline{u}(x_1, c_1^*) - Eu(\tilde{c}_1^j, c_1^*)\Big) = g'\Big(\overline{u}(x_1, c_1^*) - \sum\nolimits_{z_1^j} \Pr(z_1^j) u(z_1^j, c_1^*)\Big),$$

can show objective function globally concave w.r.t. first period consumption

maximimum characterized by first order condition

$$\begin{aligned}(c_1^j)^\rho &= \left(c_1^*\right)^{\rho-1} \frac{(1-\delta)(1+\gamma)}{\delta}\left(w_1 + z_1^j - c_1^j\right) \\ &= \bar{K}\left(w_1 + z_1^j - c_1^j\right)\end{aligned}$$

first order condition is optimum provided constraint $c_1^j \leq x_1 + z_1^j$ satisfied; othewise spend all available cash $c_1^j = x_1 + z_1^j$

find cutoff for which constraint is satisfied

$$\hat{z} = \left(c_1^*\right)^{\frac{\rho-1}{\rho}} \left[\frac{1-\delta}{\delta}(1+\gamma)\left[w_1 - x_1\right]\right]^{1/\rho} - x_1. \quad (3.3)$$

Note that for arbitrary x_1, c_1^* we may have $\hat{z}$ negative.

$z_1^j < \hat{z}$ optimal to spend all cash

$z_1^j > \hat{z}$ optimum given by FOC

$\hat{\gamma}^j(\gamma) =$

$$g'(\max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\} - Eu(\min\{\hat{c}_1^j(\gamma)(\tilde{z}_1^j), x_1 + \tilde{z}_1^j\}, c_1^*))$$

.

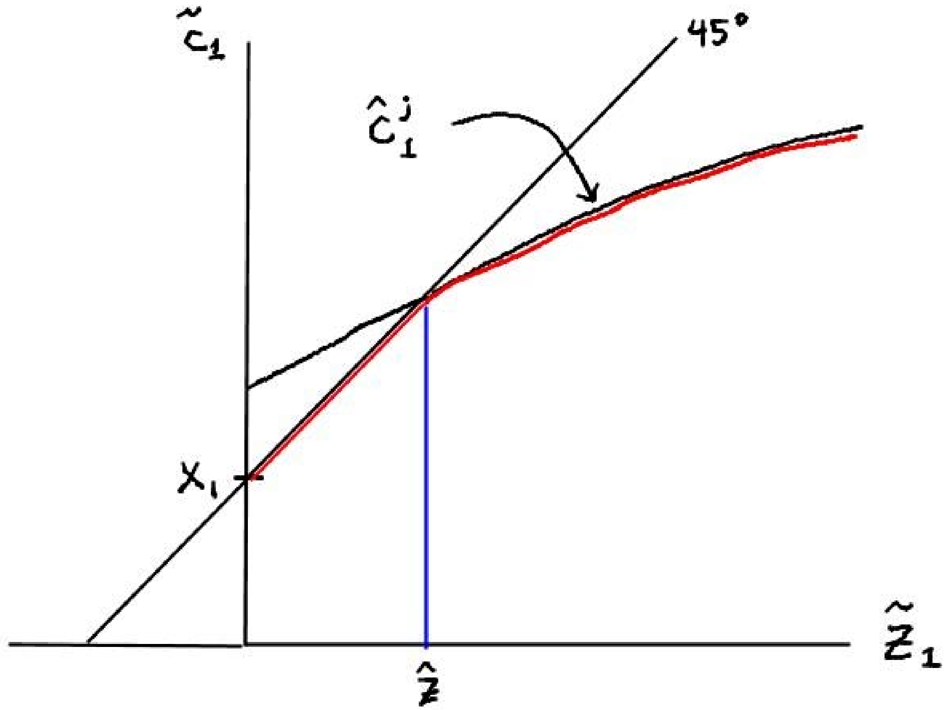
We show in Appendix that we can characterize optimum by

Theorem 1: For given (x_1, c_1^*) and each $j \in \{A, B\}$ there is a unique solution to

$$\gamma^j = \hat{\gamma}^j(\gamma^j)$$

and the solution together with $\tilde{c}_1^j = \min(\hat{c}_1^j(\gamma^j)(z_1^J), x_1 + z_1^j\}$ and choice of j that maximizes the objective function is necessary and sufficient for an optimal solution.

"consumption function" $\tilde{c}_1^j = \min(\hat{c}_1^j(\gamma^j)(z_1^j), x_1 + z_1^j\}$



Making Evening's Plans: Pocket Cash and Choice of Club

Simple case: you didn't anticipate Maurice Allais, no self-control problem at bank, so choose $c_1^* = x_1$ and plan to spend all pocket cash in nightclub of choice. Problem purely logarithmic, so solution to choose $x_1 = (1 - \delta) w_1$

Basic Calibration

Department of Commerce Bureau of Economic Analysis, real per capital disposable personal income in December 2005 was $27,640. will use three levels of income $14,000, $28,000, and $56,000.

do not use currently exceptionally low savings rates, but higher historical rate of 8% (see FSRB [2002])

gives us consumption from income; then wealth is consumption divided by subjective interest rate

pocket cash

expenditures not subject to temptation: housing, durables, and medical expense

adjust basic model of utility by assuming it is separable (and logarithmic) between “durable” consumption c^D that not subject to temptation, with weight on “tempting” or “nightclub” consumption equal to “temptation factor” τ

NIPA Q4 2005

personal consumption expenditure $8,927.80.

$1,019.60 durables, $1,326.60 housing, and $1,534.00 medical care

gives temptation factor $\tau = 0.57$.

subjective interest rate real market rate, less growth rate of per capita consumption

Shiller [1989]

average growth rate of per capita consumption has been 1.8%

average real rate of returns on bonds 1.9%

real rate of return on equity 7.5%

don’t try to solve equity premium puzzle here

plausible range 0.1% to 5.7% for subjective interest rate

use three values: 1%, 3%, and 5%

time horizon of short-run self

most plausible period based on evidence from the psychology literature seems to be about a day

level of pocket cash – about $84 for a person with $56K of income seems implausibly low relative to, for example, daily limit on teller machines

(mental accounting?)

consider two different horizons: an daily horizon and a weekly horizon

Percent interest			14K Income			28K Income			56K Income		
r	Dy	Wk	Wlth	Dy	Wk	Wlth	Dy	Wk	Wlth	Dy	Wk
1	.003	.020	1.3M			2.6M			5.2M		
3	.008	.058	.43M	20	141	.86M	40	282	1.7M	80	563
5	.014	.098	.30M			.61M			1.2M		

reasonable range of self control costs

how does marginal propensity to consume “tempting” goods change with unanticipated income?

Older literature on permanent income hypothesis

study using 1972-3 CES data Abdel-Ghany et al [1983]

examine marginal propensity to consume semi- and non-durables out of windfalls

windfalls = “inheritances and occasional large gifts of money from persons outside family...and net receipts from settlement of fire and accident policies”

windfalls less than 10% of total income MPC is 0.94

windfalls more than 10% of total income MPC of 0.02

reason for 10% unclear so take it as a general indication in our modle consumption cutoff between high MPC of 1.0 and low MPC of order $\tau(1-\delta)$ given by

$$\begin{aligned}\hat{c} &= (x_1)^{\frac{\rho-1}{\rho}}\left[\frac{\tau(1-\delta)}{\delta}(1+\gamma)[w_2]\right]^{1/\rho} \\ &\approx x_1(1+\gamma)^{1/\rho}\end{aligned}$$

Note that γ here is $g'(0)$: since all gains are spent below cutoff – so there is no cost of self-control

$$\mu = (x_1/y)(1+\gamma_0)^{1/\rho}$$

cutoff relative to income, will report this rather than marginal cost of self-control

Rabin Paradox

A $(.5 : -100, .5 : 105)$

B to get nothing for sure

Many people choose B

However, this implies also rejecting lose \$4,000 win \$635,670

For large gambles, we have logarithmic preferences, so that isn't a problem

What about rejecting the small gamble

Our model predicts all the income should be spent, so individual is risk averse with wealth equal to pocket cash and risk aversion coefficient ρ

A logarithmic consumer with pocket cash of \$2100 would reject this gamble, so not much to see here

Rabin gamble $(.5 : -100, .5 : 105)$ chosen to make a point

Actual laboratory risk aversion much greater

Holt and Laury [2002]

subjects given a list of ten choices between an A and a B lottery.

Option A		Option B		Fraction Choosing A			
\$2.00	\$1.60	\$3.85	\$0.10	1X	20X	50X	90X
0.1	0.9	0.1	0.9	1.0	1.0	1.0	1.0
0.2	0.8	0.2	0.8	1.0	1.0	1.0	1.0
0.3	0.7	0.3	0.7	.95	.95	1.0	1.0
0.4	0.6	0.4	0.6	.85	.90	1.0	1.0
0.5	0.5	0.5	0.5	.70	.85	1.0	.90
0.6	0.4	0.6	0.4	.45	.65	.85	.85
0.7	0.3	0.7	0.3	.20	.40	.60	.65
0.8	0.2	0.8	0.2	.05	.20	.25	.45
0.9	0.1	0.9	0.1	.02	.05	.15	.40
1.0	0.0	1.0	0.0	.00	.00	.00	.00

Yellow 50%, blue 85%

Paid one row picked at random, then can turn in payment for a higher value lottery

stakes plus pocket cash well below our estimate of $\hat{c}$

so fit a CES with respect to our pocket cash estimates of $21, $42, $84, $155, $310 and $620, in each case estimating value of ρ that would leave a consumer indifferent to given gamble

		Pocket Cash x_1					
		$20	$40	$80	$141	$282	$563
ρ 50th		1.06	1.3	1.8	2.4	3.8	6.5
ρ 85th		2.1	2.8	4.3	6.3	12	22

Allais Paradox

Kahneman and Tversky [1979] version of Allais Paradox

A_1 $(.01 : 0, .66 : 2400, .33 : 2500)$

B_1 2400 for certain

$A_2 = (.33 : 0, .34 : 2400, .33 : 2500)$

$B_2 = (.32 : 0, .68 : 2400)$

paradox: choose B_1 and A_2

base case

annual interest rate $r = 3\%$

annual income is \$28,000

wealth is \$860,000

short-run self's horizon a single day

pocket cash and chosen nightclub are $x_1 = c_1^* = 40$.

linear cost of self-control

$b = 0 \ \gamma^A = \gamma^B = \gamma_0$

solve for numerically unique value γ^* ($\mu* = 1.36$)

such that indifference between A and B (same in scenario 1 and scenario 2 because of linearity)

quadratic cost of self-control

from decision problem

$$\gamma^A = \gamma_0 + b\Big(\overline{u}_1 - Eu(\tilde{c}_1^A(\gamma^A))\Big)$$

$$\gamma^B = \gamma_0 + b\Big(\overline{u}_1 - Eu(\tilde{c}_1^B(\gamma^B))\Big)$$

so given γ^A, γ^B find corresponding values of γ_0, b

solution must satisfy $\infty > \gamma_0, b \geq 0$

if satisfied, say that γ^A, γ^B are *feasible marginal costs of self control*

in the data, the constraint on γ^A, γ^B is very tight

when long-run self is indifferent $Eu(\tilde{c}_1^B(\gamma^*)) > Eu(\tilde{c}_1^A(\gamma^*))$ so short-run self prefers the sure outcome B

fact that drives the Allais paradox

$b \geq 0$ implies $\gamma^B < \gamma^A$

difference in first period utility between A and B small

so: we reduce γ^B corresponding b blows up to infinity “fast”

means that numerically any feasible value of γ^B is close to γ^A

for given μ^A corresponding μ^B that are feasible

$\mu^A(\%)$	$\mu^B(\%), b$				
1.22	1.21, 9.73	1.21, 25.8	1.20, 57.7	1.19, 152	1.18, 23K
1.29	1.28, 9.51,	1.28, 24.3	1.27, 50.9	1.26, 112	1.26, 425
1.36	1.36, 9.33	1.35, 23.24	1.34, 46.3	1.34, 92.4	1.33, 231
1.44	1.43, 9.18	1.42, 22.36	1.42, 42.9	1.40, 79.8	1.40, 593
1.51	1.50, 9.06	1.50, 21.6	1.49, 40.4	1.48, 71.4	1.46, 16K

Green = A optimal; White = B optimal

To a good approximaton

$\mu^A > \mu*$ the optimal choice will be B

$\mu^A < \mu*$ the optimal choice will be A

T^{j_k} temptation for decision problem k when the choice is j

i.e. $\overline{u}(x_1, x_1) - Eu(\tilde{c}_1^j, x_1)$

r	income	w	$x_1 = c_1^*$	ρ		$\mu * (\%)$	T^{B_1}	T^{A_2}	$\overline{\mu} - \mu *$(%)
3%	28K	.86M	40	1.3	Day	1.37	.72	.51	0.41

key to Allais paradox: $T^{B_1} > T^{A_2}$

when the less tempting alternative is chosen in problem 1, and the more tempting alternative in problem 2

temptation is still greater in problem 1

so any small curvature does the trick

$\overline{\mu}$ bounds curvature above

36 different combinations of parameter values

summarize by regressing $\mu *$ on them ($R^2 = 0.63$)

	coefficient	standard error
constant	8.84	1.8
r	-0.0029	.11
$\log(income)$	-0.74	0.18
ρ	-0.04	0.02
week dummy	0.94	0.22

- results are not sensitive to the interest rate
- coefficient of relative risk aversion little role: increase ρ by 10 decreases $\mu *$ by only 0.4%
- week dummy raises $\mu *$ roughly 1%
- income: 10% increase in income increases the cutoff by 2.6% in absolute dollars
- Hence if we fit the date with higher incomes, then higher marginal

original Allais paradox

A_1 $(.01 : 0, .89 : 1{,}000{,}000, .1 : 5{,}000{,}000)$

B_1 1,000,000 for certain, paradoxical choice B_1.

$A_2 = (.90 : 0, .10 : 5{,}000{,}000)$ paradoxical choice being A_1

$B_2 = (.89 : 0, .11 : 1{,}000{,}000)$

logarithmic long-run preferences B_1 will never be chosen

assumption of logarithmic preferences with respect to prizes vastly in excess of wealth implausible

modify the utility function so that $u(5{,}000{,}000) = \log Y \approx 1{,}149{,}500$ then optimal choices are B_1 and A_2 for similar self-control parameters to those explaining the lower stakes paradox

r	incm	w	$x_1 = c_1^*$	ρ		$\mu*(\%)$	T^{B_1}	T^{A_2}	$\overline{\mu} - \mu*$(%)
3%	28K	.8M	40	1.3	Dy	1.58	1.06	.13	5.93

our explanation of the paradox requires near indifference in both scenarios

required “indifference” may be easier to achieve for thought experiments than for actual ones

Cognitive Load

experiment by Benjamin, Brown and Shapiro [2006] shows the impact of cognitive load on risk preferences

Chilean high school juniors

choices about uncertain outcomes both under normal circumstances and under the cognitive load of having to remember a seven digit number while responding

key fact: students responded differently to choices involving increased risk when the level of cognitive load was changed

real not hypothetical reward; safe option was 250 pesos

paid in cash at end of session

1 $US= 625 pesos; average weekly allowance including lunch money around 10,000 pesos

Fraction Choosing Risky Option

“X”	No load	Cognitive Load
200	1/15	1/22
350	4/15	8/22
500	6/14	9/22
650	9/13	5/21
800	10/13	8/21

We assume the non-monotonicity is artifactual: assume that when no load, and prize is increased to 650, some subjects switch to the risky alternative but not when they are under cognitive load

"Safe" option is 50-50 randomization

"X"	No load	Cognitive Load
200	2/13	3/22
350	0/15	2/22
500	4/14	7/22
650	11/15	15/22
800	13/15	19/22

Parameters needed to explain Chilean data

r	incm	w	$x_1 = c_1^*$	ρ		$\mu * (\%)$ 1	$\mu * (\%)$ 2
5%	1.6K	29K	2.29	1.06	Dy	3.543	3.550

Key fact: $\mu *$ in second scenario higher than in first: the risky “safe” option *lowers* the marginal cost of self-control

Note that the self-control parameters are consistent with the Allais calibration