Collusion Constrained Equilibrium

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An Example

game with three players each has two actions C or D

bimatrix payoffs

player 3 plays C players 1 and 2 are in a symmetric Prisoner's Dilemma game in which player 3 prefers that 1 and 2 both play C

	C	D
C	$6,6,5$	$0,8,0$
D	$8,0,0$	$2,2,0$

player 3 plays D players 1 and 2 are in a coordination game in which player 3 prefers that either 1 or 2 plays D

	C	D
C	$10,10,0$	$0,8,5$
D	$8,0,5$	$2,2,5$

Groups

main question: what happens when players are exogenously partitioned into collusive groups?

in the example an additional primitive: the first two players form a collusive group

- they have a private randomizing device
- they have agreed upon group objectives
- agreements they reach are non-binding (only incentive compatible plans may be agreed to)

Analysis of Player 3

α^i denotes the probability with which player i plays C

ρ^1 denotes the correlated strategy of the group

for illustrative purposes suppose that the randomize only between C, C and D, D so that ρ^1 is the probability of C, C

best response of 3

$\rho^1 < 1/2$ then $\alpha^3 = 0$ (player 3 plays D)

$\rho^1 = 1/2$ player 3 indifferent

$\rho^1 > 1/2$ then $\alpha^3 = 1$ (player 3 plays C)

Analysis of the Group: Correlation

$\alpha^3 \leq 1/2$ coordination game

any value of ρ^1 is an equilibrium for the group

$\alpha^3 > 1/2$ pd

only $\rho^1 = 0$ is an equilibrium for the group

there are no binding agreements: the group must play an equilibrium

correlated equilibria:

$\rho^1 \leq 1/2$ and $\alpha^3 = 0$ (player 3 plays D)

$\rho^1 = 1/2$ and $\alpha^3 = 1/2$

The Paradox of 3 as King

focus on the correlated equilibrium D, D, D (also static Nash)

payoffs $2, 2, 5$ very good for player 3

BUT:

players 1 and 2 can collude and C, C is incentive compatible given that 3 is expected to play D: they would get 10 instead of 2

so no “collusion constrained” equilibrium with $\alpha^3 = 0$

what about $\rho^1 = 1/2$ and $\alpha^3 = 1/2$

	C	D
C	$8, 8$	$0, 8$
D	$8, 0$	$2, 2$

the group should agree on C, C, but then $\alpha^3 = 1$

No Strict Collusion Constrained Equilibrium

the example is robust

driven by the fact that the Nash correspondence fails to be LHC

think like a behavioral economist

we have a discontinuous change at $\alpha^3 = 1/2$ when the group loses C, C as an equilibrium

but how can the group be dead certain of what α^3 is?

suppose they agree on joint beliefs, but it is a random function of the "true" value of α^3

so if $\alpha^3 = 1/2$ about half the time they think it is a bit less than ½ and about half the time they think it is a bit more than ½

so in effect they randomize between C, C and D, D

$\rho^1 = 1/2$ and $\alpha^3 = 1/2$ is the unique collusion constrained equilibrium

Basic Notions

strict collusion constrained equilibrium

each group maximizes its objective among incentive compatible correlated plans given the play of other groups

group reservation utility: the worst of the best equilibria for nearby beliefs (when $\alpha^3 = 1/2$ it is 2 in the example)

collusion constrained equilibrium

each group randomizes among incentive compatible correlated plans that give at least the group reservation utility given the play of other groups

allows *shadow mixing* randomization onto plans that are not optimal but are no worse than the worst of the best equilibria for nearby beliefs

collusion constrained equilibria **exist** and are correlated equilibria of the underlying game; strict collusion constrained equilibria are collusion constrained equilibria

Three Conceptual Experiments

- Random Belief Equilibrium

beliefs of each group are random. strict collusion constrained equilibrium exists and as the randomness vanishes the limit is a collusion constrained equilibrium

- Costly Enforcement Equilibrium}

groups can overcome incentive constraints with a costly enforcement technology (as in the peer monitoring model of Levine/Modica and the voting application of Levine/Mattozzi). strict collusion constrained equilibrium exists and as the costs become large the limit is a collusion constrained equilibrium

- Leader/Evaluator Equilibrium

leaders with valence give orders and are punished for orders that are not incentive compatible – Bayesian perfect equilibrium exists and as the valence vanishes the limit is a collusion constrained equilibrium

Lower Hemi-Continuity

collusion constrained equilibrium is “big enough” but is it “too big?”

in examples we see that the three different conceptual experiments can lead to different collusion constrained equilibria in the limit

consider:

belief perturbation, costly enforcement and a small perturbation of the group objective functions

in the limit we get collusion constrained equilibrium AND **every** collusion constrained equilibrium arises as such a limit

so you can't really use a smaller set