Production Chains

David K. Levine

September 6, 2010
Introduction

- Wealth of nations driven by degree of specialization
- Over 6,000,000 parts in a Boeing 747
- Specialization implies many stages of production
- Long chains of production are potentially fragile
- Motivated by “Keynesian multiplier”, Leijonhufvud’s credit chains
- Chains have been studied in macro, but a single wide chain
- Chains have been studied in IO, but with respect to integration of the firm
Chains and Fragility

- Are capitalist economies more prone to “crisis”?  
- If so is this a bad thing?  
- More specialization implies more output and more welfare – but also more unemployment and more volatility  
- The importance of the correlation of shocks across chains: small shocks may have big consequences  
- Analysis in the context of insurance market imperfections
The Technology

- infinite sequence intermediate goods \( j = 1, 2, \ldots \) and one final good 0
- endowment: one unit of labor
- anyone can use \( x \) units of labor to produce \( \beta x \) units of any type of intermediate good or final good.
- anybody can be specialist of any type
- type \( j \) produces \( \lambda_j x \geq x \) units of \( j - 1 \) from \( x \) units of \( j \)
- utility only for consumption \( c \) of the final good: strictly increasing, strictly concave function and smooth utility function \( u(c) \)
Production Chains

- $k$-production chain has one generalist and specialists $j = 1, \ldots, k - 1$
- labor of the chain used exclusively within chain
- output of chain divided among members
- per capita output of chain $f(k) = \prod_{j=1}^{k-1} \lambda_j \beta, k \geq 1; f(1) = \beta$
- after chain formed probability $0 \leq p \leq 1$ of individual “failure”
- chain with one or more failures produces 0
- sick, accident, machine breaks, bankruptcy, and so forth
Correlation of Failures

- three chains producing cars: Jaguars, BMWs, and Toyotas
- each chain three links: tires, pistons, and unspecialized production
- three failures
- three patterns of failure: good, intermediate and bad
<table>
<thead>
<tr>
<th></th>
<th>Jaguar</th>
<th>BMW</th>
<th>Toyota</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>good</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tires</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pistons</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unspecialized</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>intermediate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tires</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pistons</td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Unspecialized</td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><strong>bad</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tires</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Pistons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unspecialized</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Consequences of Failure

- good case: only Jaguar chain fails – best possible outcome, at least one chain must fail
- bad case all tire producers fail: it is impossible to produce any cars
- intermediate case: can the chains be reorganized?
- market organization, the availability of information, the quality of business connections, and the degree of substitutability between specialists
Expected Output of Chains

- **reliability** of chain \( R(k) \) is probability chain succeeds
- \( y = f(k) \) per capita output
- shocks highly correlated \( R(k) \) falls slowly
- expected output \( R(k)f(k) \)
- easy to construct models so \( k \rightarrow \infty \) increases expected output
- example: \( f(k) \rightarrow \infty \) much more quickly than \( R(k) \rightarrow 0 \)
- \( R(k) \) and \( f(k) \) not bounded away from infinity
- prediction: rarely produce, but occasionally produce giant amounts
- avoid this case
Simple Model of Chain Correlation

- one: positive probability of being in a chain of all successes
- bounds $R$ away from zero, which we wish to avoid
- two: positive probability of being in a chain of all failures
- probability $r \leq p$ that individual is in chain of all failures
- probability $1 - r$ in chain with independent $(1 - p)/(1 - r)$ chance of success
- overall failure probability $r + (1 - r)(1 - (1 - p)/(1 - r)) = p$
- reliability of the chain $R(k) = (1 - r)((1 - p)/(1 - r))^k$. 

The Production Process

$\lambda_j$ constant?

probability of success and expected output exponential, so either shortest or longest possible chain

specify $f(k)$ with specialization coefficients backed out

$$\lambda_j = \frac{f(j + 1)}{f(j)}$$

Invert reliability function $R(k) = (1 - r)((1 - p)/(1 - r))^k$

$$k = \frac{\log(1 - r) - \log R}{\log(1 - r) - \log(1 - p)}$$

expected output as a function of reliability rate:

$$Y = F(R) = R_y \left( \frac{\log(1 - r) - \log R}{\log(1 - r) - \log(1 - p)} \right)$$
Assumption on Production Function

**Assumption 1:** \( F(R) \) is strictly concave.

\[
f(k) = (b + k)^\alpha, \quad \alpha > 0, \quad b \geq 0, \quad \beta = (b + 1)^\alpha.
\]

Increasing function, \( \alpha \) measures return to specialization

**Lemma 2:** \( F(R) \) is strictly concave if and only if either \( \alpha \leq 1 \) or

\[
b \geq \frac{1 - \alpha}{\log(1 - r) - \log(1 - p)}
\]

“returns to specialization are not too greatly increasing for small chains”
Strict Concavity

Lemma 2: \(F(R)\) is strictly concave if and only if

\[
\frac{f_{kk}(k)}{f_k(k)} \leq \log(1 - r) - \log(1 - p).
\]

sufficient that \(f(k)\) be concave

don’t want \(f(k)\) to be too concave

Definition 3: We say that \(f(k)\) is moderately concavity if

\[
\frac{f_{kk}(k)}{f_k(k)} \geq \frac{f_k(k)}{f(k)} - k^{-1}
\]

satisfied by \(f(k) = (b + k)^\alpha\), for \(b \geq 0, \alpha > 0\)
An Example

expected output maximization when \( f(k) = k^\alpha \)

expected output of chain \( F(k) = R k^\alpha \)

\[
\hat{k} = \frac{\alpha}{\log(1 - r) - \log(1 - p)}.
\]

\[
\hat{R} = (1 - r)e^{-\alpha}
\]
production function indexed \( y = f(k, \alpha) \)
sole friction: an insurance market imperfection
only fraction \( \gamma \) of output can be used to make insurance payments
\( \gamma = 1 \) frictionless full insurance world
\( \gamma = 0 \) no insurance is possible
Stages of the Economy

- Complete contingent insurance markets
- Determination of chain length.
- Realization of aggregate shock \((\alpha, p, r, \gamma) \in S\), \(S\) a finite set.
- Realization of individual shocks
- Output produced and insurance claims paid

Public information economy

Notion of equilibrium constrained efficiency

May be decentralized as competitive equilibrium
No Aggregate Shock

Proposition 5: The optimal lottery is degenerate and places weight one on a single value of $k$.

full insurance $R \geq 1 - \gamma$, maximize expected output

partial insurance $R < 1 - \gamma$: welfare function

$$W(R, Y) = Ru\left((1 - \gamma)Y / R\right) + (1 - R)u\left(\gamma Y / (1 - R)\right)$$

Proposition 6: The welfare function is concave in $R, Y$ and strictly increasing in $Y$. Indifference curves are for $R \leq 1 - \gamma$ smooth and downward sloping, and for $R \geq 1 - \gamma$ horizontal. Indifference curves are differentiable including at $R = 1 - \gamma$. 
Comparative Statics

coefficient of relative risk aversion $\rho(c)$

**Definition 7:** We say that risk aversion is moderate if

$$(\rho(c)/c)[1 - \rho(c)] + \rho'(c) \geq 0$$

CRRA aversion case true if and only if $\rho \leq 1$

**Proposition 8:** With moderate risk aversion, if there is partial insurance then increasing $\gamma$ lowers reliability $R$, raises unemployment $U = 1 - R$ and increases specialization and chain length $k$.

Note: unemployment goes up!!
Further Comparative Statics

Definition 9: We say that $\alpha$ increases returns to specialization if $y_\alpha(k) > 0$ and

$$\frac{d(y_k(k, \alpha)/y(k, \alpha))}{d\alpha} > 0$$

This says output increases but that the marginal product of specialization is increased more.

Proposition 10: Suppose that preferences are a moderate CRRA. (i) If $\alpha$ increases returns to specialization then higher $\alpha$ leads to higher expected output $Y$ more specialization $k$ and more unemployment $U$. (ii) Under moderate concavity a lower failure probability $p$ or a higher correlation of shocks $r$ leads to higher expected output $Y$ more specialization $k$ and no less unemployment $U$; higher correlation of shocks leads to strictly more unemployment.

Everything goes as expected, except unemployment
A Low Probability Negative Shock

baseline shock \((\alpha_0, 1 - p_0, r_0, \gamma_0)\) with probability \(1 - \pi\)

negative shock \((\alpha, 1 - p, r, \gamma) \leq (\alpha_0, 1 - p_0, r_0, \gamma_0)\) with probability \(\pi\)

probability of the negative shock sufficiently small that optimal chain length \(\hat{k}\) is approximately what it would be when \(\pi = 0\)
Production Function Shocks

reduction in benefits of specialization $\alpha$

- lowers welfare
- lowers aggregate output
- no effect on unemployment
- financial constraint binding if and only if it was binding in base state
- if returns to specialization are increases, the effect of $\alpha$ on aggregate output are greater for greater $k$
Failure Rate Shocks

increase in the failure rate $p$

- lowers welfare
- lowers aggregate output
- either raises unemployment or leaves it unchanged
- no effect on TFP since the output per employed worker does not change
- financial constraint may bind following shock

$$R_p = k \left( \frac{1}{1 - r} \right)^{k-1} (1 - p)^{k-1}.$$ 

“reverse Keynesian multiplier” higher $r$ and longer $k$ mean more sensitivity to shocks
Measured *TFP versus actual TFP*

we assume everyone working for an unproductive chain is counted as “unemployed”

those who play cards at home because they have been laid off, and those who play cards at work because the production line has been shut down by a shortage of parts are equally unproductive from an economy wide perspective
suppose that unemployment $U$ translates as fraction of workers who are measured as unemployed $\eta U$ and fraction who are measured as employed but unproductive $(1 - \eta)U$

measured TFP output divided by hours worked

$$\frac{(1 - U)f(k)}{(1 - \eta U)} < f(k)$$

so measured TFP falls, although actual TFP does not

warning against paying too much attention to measures such as unemployment or TFP
Correlation Shocks

decrease in the correlation \( r \)
- lowers welfare
- lowers aggregate output
- raises unemployment
- no effect on TFP although may reduce measured TFP
- financial constraint may bind following shock

same kind of sensitivity as \( p \)
What do we Measure?

- when we think of the size of a shock, we think of $p$
- yet two shocks with the same $p$ can have a very different impact depending on the value of $r$
- example: crude oil price shock versus linseed oil price shock where the change in the value of the input used is the same
- crude oil price shock probably has very low $r$
Dynamic Shocks

the bad state is “sticky” so probability $\pi$ of going back again

period one, base shock: output, TFP high, unemployment low

period two, bad shock: output falls, TFP depends on measurement, unemployment high

period three: choose lower $\kappa$, output falls more, TFP definitely falls, and unemployment also falls to lower than the base level
reduction in $\gamma$ lowers welfare, nothing else
long term reduction in $\gamma$ reduces specialization so reduce output and TFP, but decreases unemployment
what determines financial sector efficiency?
  - savings and borrowing
  - unemployment insurance
  - bankruptcy
we cleverly made it harder to go bankrupt right before the crisis, lowering $\gamma$
one-time-transfer from debtors to creditors
Quasi-Self-Confirming Equilibrium

\[ \gamma \] not binding in base state, hard to know what it is following shock
financial sector has lots of incentive to exaggerate (which it did)
shock leads to a crisis: promised \[ \gamma \] can’t be delivered, who loses?
at which point Lloyd Blankfein gets on the phone to Hank Paulson and
says “It shouldn’t be us – grab 800 billion from the Treasury for us”
but who bears the burden isn’t really the point: a natural response is to
“tighten up” regulation – meaning that the actual \[ \gamma \] is reduced, leading
in the long run to less output, and so forth.
**Short Chains**

Efficient economies with large values of $\gamma, \alpha, r, 1 - p$ have long chains and lots of specialization. They are more vulnerable to shocks, and will have higher unemployment.

This can be avoided: have shorter chains. Less welfare.

You may manage to make chains so short that the employed in the good state of the “socialist” economy are worse off than the unemployed in the bad state of the “capitalist” economy.

Better to be employed in Cuba, or unemployed in the United States? The old Soviet Bloc is a natural experiment: did they have shorter chains and less volatility?
International Trade Considerations

- chains that overlap between countries: when the United States coughs, Mexico catches cold
- FDI may allow longer chains, since foreign firms may have access to better $r$
- this can be contagious in the long-run explaining the disproportionate effect of FDI
Conclusion

the capitalist economy is more prone to crisis – and this is a good thing