Evolving to the Impatience Trap: The Example of the Farmer-Sheriff Game

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Why are we horribly impulsive?

“cost” of getting a copy of a new book or the last model of a computer decreases substantially with time. But few people choose to wait.

However evolution favors the very long run.

Wealth accumulation problem, evolution favors the patient so strongly that it favors the patient over the smart (Blume and Easley [1992] and Bottazzi and Dindo [2011])

Possible explanation we are impatient because we may not live to see tomorrow. (Chowdhry [forthcoming])

However this does not in itself explain why we should evolve impatience: even a very patient individual will behave impatiently in the face of uncertain life.

We explore an alternative explanation of the evolution of impatience.
Short-sightedness is dysfunctional in an individual investment problem.

The same is not true in a game

- Preferences can act as a form of commitment device
  - Reputation for laziness is desirable to avoid requests for referee reports or letters of recommendation
  - In a repeated game an impatient player can not be threatened with future punishment, and so is harder to exploit

The idea of impatience as commitment requires two elements:

- credibility
- publicity

Evolutionary forces, building impatience into preferences makes impatient behavior credible

Player’s play is observed after the end of his life. Thus, there are no incentives to mimic other types. The children could gain from parents impatience
We study a simple game designed to illustrate:

1. how impatience can emerge as an evolutionary outcome
2. how different social roles may result in different degrees of patience.

This paper is designed to advance the literature on the evolution of preferences.

- Evolution of altruism has been studied, for example, in Bowles [2001].
- Evolution of impatience (as opposed to patience) has not been much studied.
We study a simple question of why intertemporal preferences with a low geometric discount factor might emerge in an evolutionary setting.

We also look at the inefficiency of equilibrium.
Before only talked about private gains from impatience. However, there could be social gains from impatience.

- Literature on conflict.
  - People can a) produce or b) appropriate others’ production (that is, through conflict).
  - Resources spent in conflict are a social waste.
  - It is best for society that people do not engage in appropriation.
  - As a second best, it is best to have non producers impatient, so that they do not invest in technologies that are detrimental to social welfare.
The Model

- Continuum of players; two populations
- Farmers fraction \( \phi \) and Sheriffs fraction \( 1 - \phi \)
- Each round randomly matched; Probability of meeting Farmer-Sheriff \( 2\phi(1 - \phi) \).
- Rest of the population is unmatched.
Unmatched Farmer (the investment game):

- Period 1: invest $k_I \in [0, 1]$, consume $1 - k_I$
- Period 2: receive and consume output $y_I = A k_I^\alpha$, where $\alpha A \leq 1$ and $0 < \alpha < 1, A > 0$
- Period 3: nothing
The Model III

- **Unmatched Sheriff:**
  - Period 1: consume endowment of 1
  - Period 2: nothing
  - Period 3: nothing
Farmer-Sheriff game:

- Period 1a: Sheriff invests $k_S \in [0, 1]$, consumes $1 - k_S$ and states a demand $d_S \geq 0$.
- Period 1b: Farmer invests $k_F \in [0, 1]$, consume $1 - k_F$ and agrees to pay the Sheriff $d_F \geq 0$.
- Period 2: Farmer produces output $y_F = A k_F^\alpha + G$, consumes $y_F - d_F$ the Sheriff consumes $d_F$. $G > 0$ gain to trade.
- Period 3: if $d_F \geq d_S$ nothing, otherwise Sheriff produces punishment cost to Farmer $AB k_S^\alpha$, $B > 0$. 
Preferences

- Players preferences are characterized by a discount factor $\delta_F, \delta_S$
  - Investment Game objective function
    \[ 1 - k_I + \delta_F y_I \]
  - Unmatched Sheriff get 1
  - Farmer-Sheriff game
    - Farmer Objective function
      \[ 1 - k_F + \delta_F (y_F - d_F) - (\delta_F)^2 1_{d_F < d_S} AB k_s^\alpha \]
    - Sheriff Objective function
      \[ 1 - k_S + \delta_S d_F \]
Before each match Farmer and Sheriff know their discount factor and have independent common knowledge beliefs about the others $\mu_F(\delta_S), \mu(\delta_F)$ (only matters for the Farmer-Sheriff game).

Punishment takes place with a delay. Thus, Farmers can gain from being impatient.
Equilibrium

Investment Game

\[
\max_{k_I} 1 - k_I + \delta_F A \ k_I^\alpha 
\]

solution is

\[
k_I = (\alpha A)^{1/(1-\alpha)} \ \delta_F^{1/(1-\alpha)}
\]
Equilibrium II

- Farmer-Sheriff Game
  - Farmer’s problem
    \[
    \max_{k_F} 1 - k_F + \delta_F (A \ k_F^\alpha - d_F + G) \quad \text{if} \quad d_F \geq d_S \\
    \max_{k_F} 1 - k_F + \delta_F (A \ k_F^\alpha - d_F + G) - (\delta_F)^2 AB \ k_s^\alpha \quad \text{if} \quad d_F < d_S
    \]
    solution is
    \[
    k_F = k_I = (\alpha A)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)}
    \]
  - Sheriff’s problem (ask maximum that farmer is willing to pay)
    \[
    \max_{k_S} 1 - k_S + \delta_S \delta_F AB \ k_s^\alpha
    \]
    solution is
    \[
    k_S = (\alpha AB)^{1/(1-\alpha)} \delta_F \delta_S^{1/(1-\alpha)}
    \]
    and
    \[
    d_S = d_F = \alpha^{\alpha/(1-\alpha)} (AB)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)} \delta_S^{\alpha/(1-\alpha)}
    \]
    increasing in believed discount factor of the Farmer
Evolutionary Process

- Evolution of preferences measured by $\delta$ and the number of Farmers and Sheriffs. Overall fitness does not depend on preferences but on total undiscounted expected utility.
- Two types of each group $\delta_S, \delta_F \in \{\delta, 1\}$
- Each group gives birth to offspring identical in preferences and type: offspring are commonly observed. Beliefs in a round are fixed $\Rightarrow$ incentive to maximize with respect to true preferences. Preferences are inferred from behavior.
- $\phi$ the fraction of Farmers; let $\psi_F$ fraction of impatient Farmers. $\psi_S$ and the fraction of impatient Sheriffs.
- $V_F(\delta_F), V_S(\delta_S)$ evolutionary fitness of Farmers and Sheriffs as a function of their preferences.
Model of evolution is the standard replicator dynamics based on evolutionary fitness.

\( \phi_j \) is the population fraction of group \( j \), \( V_j \) its fitness and \( \bar{V} \) average fitness of the population then

\[
\dot{\phi}_j = \phi_j (V_j - \bar{V})
\]

Sheriffs evolve strictly towards greater patience: \( \dot{\psi}_j < 0 \)

Thus in the long-run has only three types: patient Sheriffs, and both patient and impatient Farmers. We now focus in this case.

The replicator dynamics is summarized by

\[
\dot{\psi} = \psi (1 - \psi) \left[ V_F (\delta) - V_F (1) \right]
\]

\[
\dot{\phi} = \phi (1 - \phi) \left\{ [V_F (\delta) - V_S] - (1 - \psi) [V_F (\delta) - V_F (1)] \right\}\]
Theorem

Given non vacuous assumptions. For any $0 < \delta < 1$ there exists an open set of $G'$'s such that there is a unique interior steady state and it is dynamically stable. At the steady state

$$\phi = \phi^* = 1 - \frac{1 - \alpha - \delta^\alpha/(1-\alpha)(1 - \alpha\delta)}{B^{1/(1-\alpha)}\left(1 - \delta^{1/(1-\alpha)}\right)}$$

- Since $D_\delta \phi^* > 0$. If the impatient Farmers are less impatient there will be more of them at the steady state.
- Key observation here is that at a stable interior steady state in the long-run there is a positive fraction of farmers who are impatient: evolution leads to impatience.
With only two possible discount factors the level of impatience is specified exogenously.

Similar results are found with $\delta \in [0, 1]$ where the levels emerge endogenously.
Many Types

- individuals with every discount factor in the interval $\delta \in [0, 1]$
- use a simple approximation that enables us to determine a steady state value of $\delta$
- Sheriffs with $\delta = 1$ always have higher fitness than those with lower discount factors
- assume a single group of patient Sheriffs
a density function over discount factors $\psi_\delta$ and near an interior steady state

density function approaches spike as every type of Farmer evolves towards optimal discount factor

replicator dynamic given by

$$\dot{\psi}_\delta = \psi_\delta(V_F(\delta) - \overline{V}_F),$$

where $\overline{V}_F$ is mean fitness of farmers
approximate mean fitness $\overline{V_F}$ by the fitness $V_F$ evaluated at the mean discount factor $δ_F$:

$$
\dot{ψ}_δ = ψ_δ(V_F(δ) - \overline{V_F})
$$

$$
≈ ψ_δ(V_F + DV_F[δ - δ_F] - V_F)
$$

$$
= ψ_δDV_F[δ - δ_F]
$$
after short interval of time $\tau$ the system will evolve according to

$$\psi_\delta(t + \tau) \approx \psi_\delta(t) + \dot{\psi}_\delta(t)\tau$$

$$\approx \psi_\delta(t) + \psi_\delta(t)DV_F[\delta - \delta_F]\tau$$

compute the mean discount factor by integrating:

$$\delta_F(t + \tau) = \int \delta \psi_\delta(t + \tau) d\delta$$

$$\approx \int \delta [\psi_\delta(t) + \psi_\delta(t)DV_F[\delta - \delta_F]\tau] d\delta$$

$$= \int \delta \psi_\delta(t) d\delta + \int \delta \psi_\delta(t)DV_F[\delta - \delta_F]\tau d\delta$$

$$= \delta_F(t) + DV_F\tau \int \delta \psi_\delta(t)[\delta - \delta_F] d\delta$$

$$= \delta_F(t) + \sigma^2(t)DV_F\tau$$
approximate dynamic equation for the mean discount factor of the Farmers

\[ \dot{\delta}_F \approx \sigma^2(t)DV_F. \]

variance \( \sigma^2 \) is time varying does not matter for stability analysis, so hold it fixed, and study dynamic equation

\[ \dot{\delta}_F = \sigma^2 DV_F \]

the continuous time best response dynamic!!

mean moves in the direction of increasing fitness
dynamics of $\phi$ are replicator dynamics, now based on the mean discount factor, so

$$\dot{\phi} = \phi(1 - \phi)(V_F - V_S).$$

**Theorem**

Assume $G > (\alpha AB)^{1/(1-\alpha)}$. Then there is a unique interior steady state and it is dynamically stable.
Theorem

(1) The steady state value of $\phi$ is larger than $1/2$, and larger the larger is $G$.

The comparative statics with respect to $G$ and $B$ are the following:

(2) $D_G \delta_F > 0$, $D_G \phi > 0$, $D_B \delta_F < 0$, and for sufficiently large $G$, $D_B \phi < 0$. 
Efficiency

- Measure of welfare is average fitness for the whole population.
- We show that inefficient impatience trap arises in which the wrong population becomes impatient.
- Social planner chooses distribution over discount factors for Farmers $f_F(\delta_F)$ and Sheriffs, $f_S(\delta_S)$ and what fraction of the population is assigned the role of a Farmer $\phi$ in order to maximize fitness. Each individual chooses investment optimal level. Planner is constrained to choose discount factors, we refer to this as the second best.
Theorem

The second best distribution is given by

$$
\phi = \min \left\{ 1, \frac{1}{2} + \frac{1}{2} \frac{A^{1/(1-\alpha)}}{G} \left( \frac{\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}}{G} \right) \right\}
$$

whereas $f_F$ and $f_S$ assign point mass at $\delta_F = 1$ and $\delta_S = 0$, respectively.

- Sheriff’s investment are a social waste, if they are impatient would not invest. Farmer’s invest optimally if they are patient.
- Optimal fraction of Farmers, less than 1 because there is a social gain of $G$ whenever a Farmer and a Sheriff meet.
A related question has to do with the optimal mix of Farmers and Sheriffs when their discount factors are at their equilibrium values.

\[ \phi = \min \left\{ 1, \frac{1}{2} + \frac{1}{2} \frac{y_F - G - k_F}{G - k_S} \right\} \]

It is less than 1 for \( G \) large enough, and tends to 1/2 as \( G \) grows.
The steady state value of $\phi > 1/2$ and increasing in $G$

The fact that steady state $\phi > 1/2 \Rightarrow$ if $G$ is large enough, there are inefficiently many Farmers, and too few Sheriffs.

- The intuition is that this arises because the Sheriff’s have to pay to collect their share.

This is the impatience trap. Interpreting the model as one where Sheriffs are Buyers and Farmers are Sellers, viewing $k_S$ as the short run cost of enforcing reliability and $G$ as the long run gain of partnership and trust. Inefficiency worsens the larger is $G$. 
A necessary condition for impatient’s survival is type observability.

- Patient player maximize utility=fitness.
- Impatient can only have a higher fitness if the other players action depends on rivals’ patience
- Thus, opponents’ type must be observable.
Impatient Farmers survive against efficiency.

Consider this symmetric game, Payoff matrix for Row Player (present payoffs, future payoffs)

\[
\begin{array}{cccc}
  & a_1 & a_2 & a_3 & a_4 \\
 a_1 & (0,8) & (2,9) & (0,0) & (0,5) \\
 a_2 & (0,0) & (0,0) & (0,2) & (0,0) \\
 a_3 & (2,0) & (10,0) & (0,0) & (0,0) \\
 a_4 & (0,0) & (0,0) & (1,0) & (4,0) \\
\end{array}
\]

The Best Reply correspondence is

\[
\begin{array}{cccc}
  & a_1 & a_2 & a_3 & a_4 \\
 B_P & a_1 & a_1 & a_2 & a_1 \\
 B_I & a_3 & a_3 & a_4 & a_4 \\
\end{array}
\]
Robustness III

- Unique pure-strategy equilibrium,
  - patient chooses
    - $a_1$ against a patient
    - $a_2$ against an impatient
  - impatient chooses
    - $a_3$ against a patient
    - $a_4$ against an impatient

- The resulting fitness is given

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- This is a prisoner's dilemma.
  - The optimal profile is for both players to be patient,
  - The only equilibrium outcome is for both players to be impatient.
  - The Farmer-Sheriff game does not follow from its particular sequencing structure, nor the particularities of the action spaces.
Conclusions

- Impatience survives evolutionary forces when it keeps down punishment by the opponents. In contrast to the single-person investment (Blume and Easley, 1992).
- Interpreting the model as one of buyer and seller, long run gains of partnership, not fully exploited in equilibrium due to the presence of too many impatient sellers.
Extensions

- Alternative sequences of the game.

  In the model there is a “non-standard” result, in a bargaining situation being impatient might be better. Usually we get the opposite result (for example, Rubinstein’s model). Punishments are applied in the future and as such, a more patient Farmer is more influential by threats, weakening his bargaining position to the advantage of Sheriffs that get paid more.

  - Results are qualitatively robust if Sheriff makes his investment decision before knowing his opponent’s and if the Sheriff make his investment decision before knowing if he would be matched or not.
  - If the Sheriff makes both, his investment decision and his demand before knowing his opponent’s results do not hold. Also results disappear if the punishment were to take place in period 2 rather than in period 3.

- Regarding welfare, the results are independent of the sequence of the game. The efficient distribution of the population is independent of the sequence.