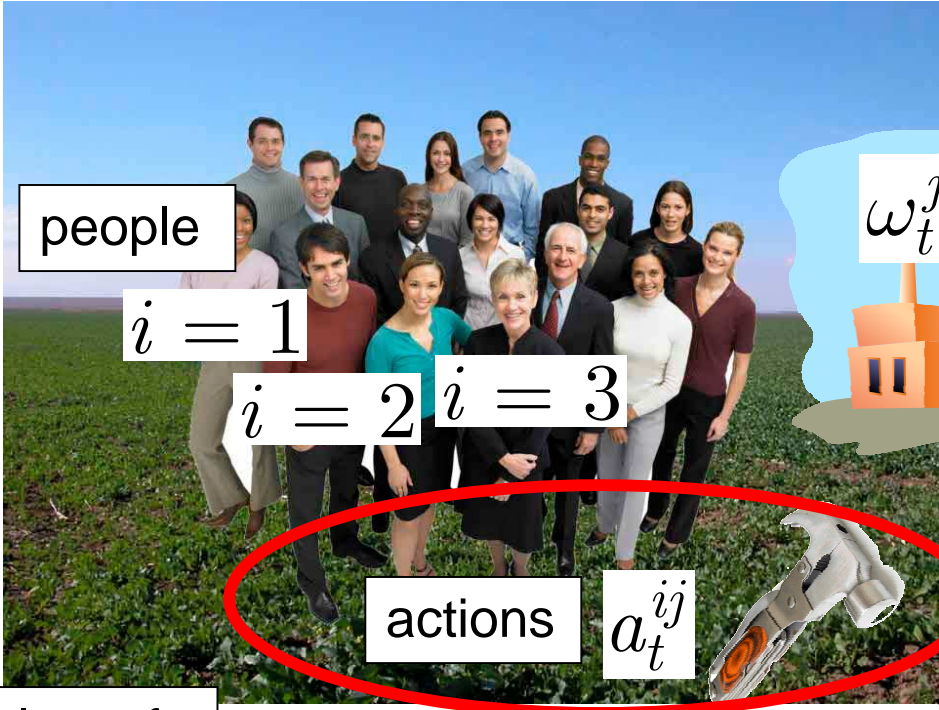


# **Anti-Malthus: Conflict and the Evolution of Societies**

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## ***Evolution of Societies***

- Does not evolution favor more efficient societies?
- Must have incentive compatibility: evolutionarily better everyone else contributes to the common good and you free ride
- So selection takes place within Nash equilibria
- Evolution + voluntary migration = efficiency within the set of equilibria
- Isn't the way the world works:
  - The United States didn't become rich because the Native Americans had such a great equilibrium and everyone wanted to move there
- More often than not ideas and social organization spread at the point of the sword



people

$i = 1$

$i = 2$

$i = 3$

actions

$a_t^{ij}$

$\omega_t^j$

stocks of capital

$j = 2$

plots of land

$j = 1$

global interaction



$j = 3$



$j = 4$

## Consequences (Stage Game)

- Utility  $u^i(a_t^j, \omega_t^j)$
- Capital/investment dynamics  $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$

Assumptions about capital dynamics on an individual plot

**Irreducibility:** any environment can be reached

**Steady state:** if everyone plays the same way repeatedly the environment settles to a steady state.

## ***Disruption***

At most one plot per period disrupted, probability of plot  $k$  being disrupted (forced, conquered) to play action  $a_t^j$  (at time  $t + 1$ ) given actions and capital stocks on all plots  $a_t, \omega_t$  is

$$\pi^k(a_t^j, a_t, \omega_t)[\varepsilon]$$

[conflict resolution function] depends on “noise”  $\varepsilon$  and everything on all plots

- this is how plots interact
- it is global (no geography)
- details to follow

## ***Definition: Steady State Nash Equilibrium***

a pair  $a_t^j, \omega_t^j$  that is as it sounds

(note pure strategies; will assume existence; interested in environments with many equilibria not few)

## Malthus Example

- capital stock is population  $\omega_t^j \in \{1, \dots, N\}$
- actions are target population  $a_t^{ij} \in \{1, \dots, N\}$
- utility  $u^i(a_t^j, \omega_t^j) = a_t^j$ : want lots of kids
- $\bar{a}_t^j$  average target (those who live are picked at random)
- population grows or declines depending on whether it is above or below the target
- $\omega_{t+1}^j = \omega_t^j + \begin{cases} -1 & \bar{a}_t^j < \omega_t^j - 1/2 \\ +1 & \bar{a}_t^j > \omega_t^j - 1/2 \\ 0 & \text{otherwise} \end{cases}$  stickiness to assure convergence to steady state
- unique steady state Nash equilibrium at  $N$
- will consider models with multiple equilibria later

## Behavior

- behavior based on finite histories  $s_t$  is the *state*
- if plot was disrupted, players play as required otherwise play  $B^i(s_{t-1})$
- *quiet state* for player  $i$ : capital stock and action profile constant *and* player  $i$  is playing a best response
- otherwise: *noisy state*
- in a quiet state the probability of all actions except the status quo are zero
- in a noisy state all actions have positive probability
- absent disruption (for example  $J = 1$ ) – Nash steady states are absorbing, all have positive probability of being reached (from non-absorbing state)



## ***Free Resources and Conflict Resolution***

What happens to the subsistence farmers when they get invaded by a society that has population control? Nothing good.

- Free resources  $f(a_t^j, \omega_t^j) > 0$  are those above and beyond what is needed for subsistence and incentives; they are what is available for influencing other societies and preventing social disruption, less discretionary income (nobles consume swords versus jewelry)
- What matters is free resources aggregated over a society  $F$
- Monaco versus China
- These things help determine the conflict resolution function  $\pi^k(a_t^j, a_t, \omega_t)$

## Societies

- attitudes towards expansion and willingness to belong to a larger society: a consequence of the actions taken by individuals on that plot of land; represented by  $\chi(a_t^j) \in \mathbb{Z}$
- three possible attitudes towards expansion and social organization: given by positive (expansionist), negative (non-expansionist) and the zero values
- expansionist: Christianity after the Roman period; Islam
- non-expansionist – leave neighbors alone: Judaism after the diaspora; Russian Old Believers
- do not wish to belong to a larger society or unable to agree:  $\chi(a_t^j) = 0$ : *isolated* plot; otherwise value of  $\chi(a_t^j)$  indexes the particular society to which the plot is willing to belong – society formation by mutual agreement
- assume: at least one steady state Nash is expansionary

## ***Aggregation of Free Resources***

- it is free resources of the entire society that matters
- aggregate free resources increasing function of average free resources per plot and fraction of plots belonging to society
- $\bar{f}(x, a_t, \omega_t)$  average free resources per plot in society  $x \neq 0$
- $J(x)$  number of plots
- aggregation function:  $F(x, a_t, \omega_t) = \Phi(\bar{f}(x, a_t, \omega_t), J(x) / J)$
- $\Phi(\bar{f}, \phi)$  smooth and  $\lim_{\phi \rightarrow 0} \Phi(\bar{f}, \phi) = 0$

## ***Appreciable versus Negligible Probabilities***

Will consider a limit as a noise parameter  $\varepsilon \rightarrow 0$

- Probabilities that go to zero are *negligible*
- Probabilities that do not go to zero are *appreciable*

Definition of resistance:

More resistance (to change) = smaller probability (of change)

$Q[\varepsilon]$  a function of the noise parameter  $\varepsilon$

$Q$  is regular if

the resistance  $r[Q] \equiv \lim_{\varepsilon \rightarrow 0} \log Q(\varepsilon) / \log \varepsilon$  exists

and  $r[Q] = 0$  implies appreciable probability  $\lim_{\varepsilon \rightarrow 0} Q(\varepsilon) > 0$

if  $r[Q] > 0$  then negligible probability

## ***Disruption***

- probability of society  $x$  being disrupted,  $\Pi(x, a_t, \omega_t)[\varepsilon]$  probability that one of its plots is disrupted to an alternative action
- interested in the resistance of  $\Pi(x, a_t, \omega_t)[\varepsilon]$  - resistance to disruption
  - sum of  $\pi^k(a_t^j, a_t, \omega_t)[\varepsilon]$  over all  $a_t^j \neq a_t^k$  and all plots  $k$  belonging to that society
  - assumed to be regular
  - resistance bounded above and normalized so that  $r[\Pi(x, a_t, \omega_t)] \leq 1$

## Assumptions About Conflict

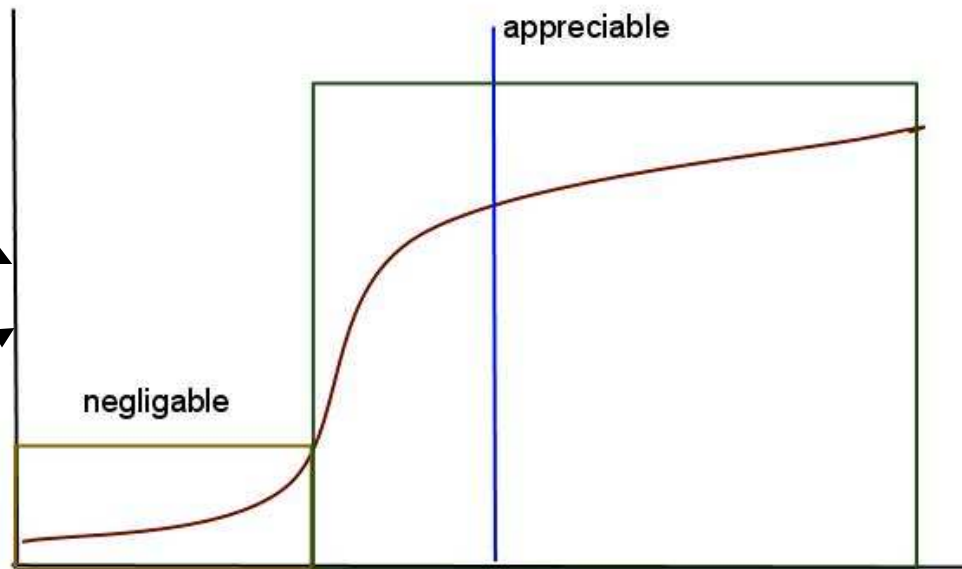
- a society with more free resources has at least the same resistance as the one with fewer free resources
- an expansionary society with at least as many free resources as a rival has an appreciable chance of disrupting it.
- Given free resources, divided opponents are no stronger than a monolithic opponent
- Expansionary:  $E(x) = 1, 0$  as  $x > 0, \leq 0$
- Binary case: see figure

$r[\Pi(x, a_t, \omega_t)] = q(F'/F, E')$ , non-increasing left-continuous in first argument: weakly decreasing, left continuous,  $q(0, E) = q(\phi, 0) = 1$ , for some  $\phi > 0$   $q(\phi, 1) > 0$

$\Pi(a_t^k, a_t, \omega_t)$  probability of disruption

No opposition:  
Spontaneous  
disruption

Non-  
expansionary  
opponent



1

$\phi$

opposing free resources  
friendly free resources

## ***Main Result***

*Stochastically stable states* are where the system spends most of its time

Don't converge there and stay there

*Monolithic steady state*: a single expansionary society each plot in a Nash steady state

**Theorem:** characterization of stochastically stable states

*Maximum free resource among monolithic steady states are stochastically stable*

*As  $J \rightarrow \infty$  the least free resources in any stochastically stable state approach this as a limit*



## *Intuition*

- Consider monolithic: it takes one coincidence to go anywhere after which will almost certainly wind up back where you started before a second coincidence happens
- So: need some minimum number of coincidences before an appreciable chance of being disrupted
- More free resources = more coincidences required
- Think in terms of layers of protecting a nuclear reactor: redundancy - a second independent layer of protection double the cost, but provides an order of magnitude more protection (1/100 versus 1/10,000)
- What happens if you need more than equal free resources before chance of disrupting becomes appreciable? can have two expansionary societies living side by side, neither having much chance of disrupting the other

## ***Social Norm Games***

Discuss the fact that you can have equilibria at well above subsistence, real question: which equilibrium?

- Repeated games, self-referential games
- Here a simple two-stage process
- Add a second stage in which each player has an opportunity to *shun* an(y) opponent
- If everyone shuns you utility is less than any other outcome of the game

Transparently a folk theorem class of games

## ***Malthus Revisited***

$Y(z)$  output as function of population

suppose social norm game, what maximizes free resources?

Free resources:  $AY(z) - Bz$  where  $A$  is technology parameter

More than minimum population, less than subsistence

Technological change?  $A$  gets bigger

- Suppose that there is a labor capacity constraint on each plot
- Once constraint reached can increase free resources only by increasing per capita income
- Anti-Malthus

## ***What are Free Resources: Bureacracy***

Individuals produce output  $y$  with continuous positive density on  $[0, \infty)$

Risk neutrality

Subsistence is  $B$  which must be met *on average* in the population

(some people could reproduce more slowly, others more rapidly)

$Ey > B$  or else not much can happen

output unobservable so no free resources

## Commissars

Can monitor each other and  $\kappa$  other individuals

$\phi$  fraction of population who are commissars

commissars have to get the same expected utility as anyone else

monitored individuals may produce less  $y_S$  weakly stochastically dominated by  $y$

maximize free resources:

if  $Ey_S \geq Ey/2$  and  $\kappa > 1$  positive fraction of commissars

➤ What is missing? Why don't commissars collude to steal the output?

## *Summing Up*

- free resources are those that prevent disruption and allow expansion
- maximization of free resources provides a positive theory of institutions including the state and population
- the long-run may be a long-time, but institutions that are deficient on free resources are not likely to last long