Conflict and the Evolution of Societies

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Evolution of Societies

- Does not evolution favor more efficient societies?
- Must have incentive compatibility: evolutionarily better everyone else contributes to the common good and you free ride
- So selection takes place within Nash equilibria
- Evolution + voluntary migration = efficiency within the set of equilibria
- Isn’t the way the world works:
  The United States didn’t become rich because the Native Americans had such a great equilibrium and everyone wanted to move there
- More often than not ideas and social organization spread at the point of the sword
plots of land

people

actions

global interaction

$j = 1$

$i = 1$

$i = 2$

$i = 3$

$j = 2$

$j = 3$

$j = 4$
The Economy

- Utility $u^i(a^j_t)$; Nash Equilibrium means of this stage game on a plot

**Appreciable versus Negligible Probabilities**

Will consider a limit as a noise parameter $\varepsilon \to 0$

- Probabilities that go to zero are *negligible*
- Probabilities that do not go to zero are *appreciable*

**Definition of resistance:**

More resistance (to change) = smaller probability (of change)

$Q[\varepsilon]$ is regular if resistance $r[Q] \equiv \lim_{\varepsilon \to 0} \log Q(\varepsilon) / \log \varepsilon$ exists

and $r[Q] = 0$ implies $\lim_{\varepsilon \to 0} Q(\varepsilon) > 0$; if $r[Q] > 0$ then negligible probability
Disruption

At most one plot per period disrupted, probability of plot $k$ being disrupted (forced, conquered) to play action $a_t^j$ (at time $t+1$) given actions on all plots $a_t$ is

$$\pi_k^k(a_t^j, a_t)[\varepsilon]$$

[conflict resolution function] depends on “noise” $\varepsilon$ and everything on all plots

- this is how plots interact
- it is global (no geography)
- details to follow
**Behavior**

- behavior based on finite histories \( s_t \) is the *state*
- if plot was disrupted, players play as required otherwise play \( B_i(s_{t-1}) \)
- *quiet state* for player \( i \): action profile constant and player \( i \) is playing a best response
- otherwise: *noisy state*
- in a quiet state the probability of all actions except the status quo are zero
- in a noisy state all actions have positive probability
- absent disruption (for example \( J = 1 \)) – Nash states are absorbing, all have positive probability of being reached
Free Resources and Conflict Resolution

What happens to the subsistence farmers when they get invaded by a society that has population control? Nothing good.

- Free resources \( f(a_t^j) > 0 \) are those above and beyond what is needed for subsistence and incentives; they are what is available for influencing other societies and preventing social disruption, less discretionary income (nobles consume swords versus jewelry)

- What matters is free resources aggregated over a society \( F \)

- Monaco versus China

- These things help determine the conflict resolution function \( \pi^k(a_t^j, a_t) \)
Societies

- attitudes towards expansion and willingness to belong to a larger society: a consequence of the actions taken by individuals on that plot of land; represented by $\chi(a^j_i) \in \mathbb{Z}$

- three possible attitudes towards expansion and social organization: given by positive (expansionist), negative (non-expansionist) and the zero values

- expansionist: Christianity after the Roman period; Islam

- non-expansionist – leave neighbors alone: Judaism after the diaspora; Russian Old Believers

- do not wish to belong to a larger society or unable to agree: $\chi(a^j_i) = 0$: isolated plot; otherwise value of $\chi(a^j_i)$ indexes the particular society to which the plot is willing to belong – society formation by mutual agreement

- assume: at least one Nash state is expansionary
Aggregation of Free Resources

- it is free resources of the entire society that matters
- for \( x \neq 0 \) we have \( F(x, a_t) = \sum \chi(a_t^j) = x f(a_t^j) \)
Disruption

- probability of society $x$ being disrupted, $\Pi(x, a_t)[\varepsilon]$ probability that one of its plots is disrupted to an alternative action
- interested in the resistance of $\Pi(x, a_t)[\varepsilon]$ - resistance to disruption
  - sum of $\pi^k(a^j_t, a_t)[\varepsilon]$ over all $a^j_t \neq a^k_t$ and all plots $k$ belonging to that society
  - assumed to be regular
  - resistance bounded above and normalized so that $r[\Pi(x, a_t)] \leq 1$
Assumptions About Conflict

- A society with more free resources has at least the same resistance as the one with fewer free resources.
- An expansionary society with at least as many free resources as a rival has an appreciable chance of disrupting it.
- Given free resources, divided opponents are no stronger than a monolithic opponent.
- Expansionary: $E(x) = 1, 0$ as $x > 0, \leq 0$.
- Binary case: see figure.

$$r[\Pi(x, a_t)] = q(F'/F, E'),$$ non-increasing left-continuous in first argument: weakly decreasing, left continuous, $q(0, E) = q(\phi, 0) = 1$, for some $\phi > 0$ $q(\phi, 1) > 0$. 
General Results on Stochastic Stability

Theorem [Young]: Unique ergodic distribution

- Monolithic (expansionary) steady states
- Mixed steady states (only one expansionary)
- Non-expansionary steady states

Theorem [Young]: Unique limit of ergodic distribution as $\epsilon \to 0$ putting weight only on the above. These are called stochastically stable states.

\[ \Pi(a^k_t, a_t) \]

- No opposition: Spontaneous disruption
- Non-expansionary opponent

\[ \phi \]

Opposing free resources

Friendly free resources
**Strongest and Stochastically Stable Societies**

*Stochastically stable states* are where the system spends most of its time; don’t converge there and stay there

$x$ is a *strongest expansionary society* if $f(x) = \max_{x^1 > 0} f(x)$

by assumption there is one

For positive $x$ (in the image of $\chi$) we have the non-empty set $\chi^{-1}(x)$ of profiles that are compatible with that society; $f(x)$ least free resources of any of Nash profile in $\chi^{-1}(x)$

*Monolithic expansionary states* $S(x) \subseteq S$ are made up of combinations of Nash equilibrium profiles in $\chi^{-1}(x)$

*stochastically stable society*: all the corresponding monolithic states $S(x)$ are stochastically stable
Main Theorem

**Theorem:** characterization of stochastically stable states

*If $x$ is a strongest expansionary society then it is stochastically stable*

*For $J$ large enough every stochastically stable state $s_t \in S(x)$ for some strongest expansionary society $x$*

This is not trivial to prove although it uses standard types of arguments.

Note that all the states in a strongest expansionary society are stochastically stable, but the strength of the society is measured by the weakest state.
Consider monolithic: it takes one coincidence to go anywhere after which will almost certainly wind up back where you started before a second coincidence happens.

So: need some minimum number of coincidences before an appreciable chance of being disrupted.

More free resources = more coincidences required.

Think in terms of layers of protecting a nuclear reactor: redundancy - a second independent layer of protection double the cost, but provides an order of magnitude more protection (1/100 versus 1/10,000).

What happens if you need more than equal free resources before chance of disrupting becomes appreciable? can have two expansionary societies living side by side, neither having much chance of disrupting the other.
Social Norm Games

Discuss the fact that you can have equilibria at well above subsistence, real question: which equilibrium?

- Repeated games, self-referential games
- Here a simple two-stage process

Base game
First stage: choose an action in the base game and vote for a society
Public signal of what everyone did, possibly noisy
Second stage: choose which players to shun

Additively separable utility: stage game utility plus benefit of vote minus cost of being shunned $\Pi^i$. 
Folk Theorem

If perfect observability of first stage and $\Pi^i$ is big enough

- So free resource maximization over base game profiles
Malthusian Social Norm Game

\( Y(z) \) output as function of population

suppose social norm game, what maximizes free resources?

Free resources: \( A Y(z) - Bz \) where \( A \) is technology parameter

More than minimum population, less than subsistence

Technological change? \( A \) gets bigger

- Suppose that there is a labor capacity constraint on each plot
- Once constraint reached can increase free resources only by increasing per capita income
- Anti-Malthus
What are Free Resources: Bureacracy and the Mandarin Game

players choose an occupation $c_t^{ij} \in \{0,1\}$ where 1 corresponds is commissar and 0 is producer

producer produces $Y > 0$ with probability $\pi$, zero otherwise

successful producer chooses whether or not to reveal output $r_t^{ij} \in \{0,1\}$ where 1 is reveal

commissar audits $\kappa$ randomly chosen producers who have not revealed positive output and observes their production

enforcement specialists: can physically seize output

subsistence level zero, symmetric shunning penalty $\Pi$
Auditing Probability and Social Variables

\( \phi \) fraction of producers with positive unrevealed output \( K_t^j \) commissars

audit probability

\[
R_t^j = \frac{\kappa K_t^j}{(1 - \pi + \phi \pi)(N - K_t^j)}
\]

wage rate of the commissars \( w_t^j \)

amount \( W_t^j \) retained by a producer who admits to having output,

amount \( X_t^j \) retained by a producer whose positive output is discovered by a commissar.

In equilibrium: accept these or get shunned
Equilibrium

\( Y \leq \Pi \) shun anyone who refuses to admit output and take all from anyone who does not

everything is free resources without any commissars

shunning alone is enough

\( Y > \Pi \)

now a trade-off, commissars may complement shunning
Revelation Principle

Can get as many free resources by taking everything following an unsuccessful audit and having producers reveal truthfully, so only interesting case had $\phi = 0$ and $X^j_t = 0$

Any producer that does not declare output and is not audited is shunned
Free Resource Maximization

must have indifference between being a commissar and being a producer who refuses to be a commissar

\[ \hat{R} = \frac{\pi Y - 2\Pi}{\pi Y - \Pi} \] if numerator positive, zero otherwise

if enough commissars \( R_t^j > \hat{R} \) they can be paid zero, otherwise they get \((1 - R_t^j)(\pi Y - \Pi) - \Pi\)

[if there are many commissars then producers do not earn very much]
Cutoff

cutoff: \( \tilde{\kappa} = \frac{\kappa}{1 - \pi} \leq \frac{\Pi}{Y - \Pi} \)

[efficiency of commissars per number of audits needed versus strength of shunning]

below cutoff use no commissars

above cutoff 100% auditing

number of commissars not monotone in their effectiveness \( \tilde{\kappa} \)

low effectiveness, no commissars

hit threshold number of commissars jumps up

above thresholds number of commissars declines as you need fewer for 100% auditing
Summing Up

- free resources are those that prevent disruption and allow expansion
- maximization of free resources provides a positive theory of institutions including the state and population
- the long-run may be a long-time, but institutions that are deficient on free resources are not likely to last long