Anti-Malthus: Evolution, Population and the Maximization of Free Resources

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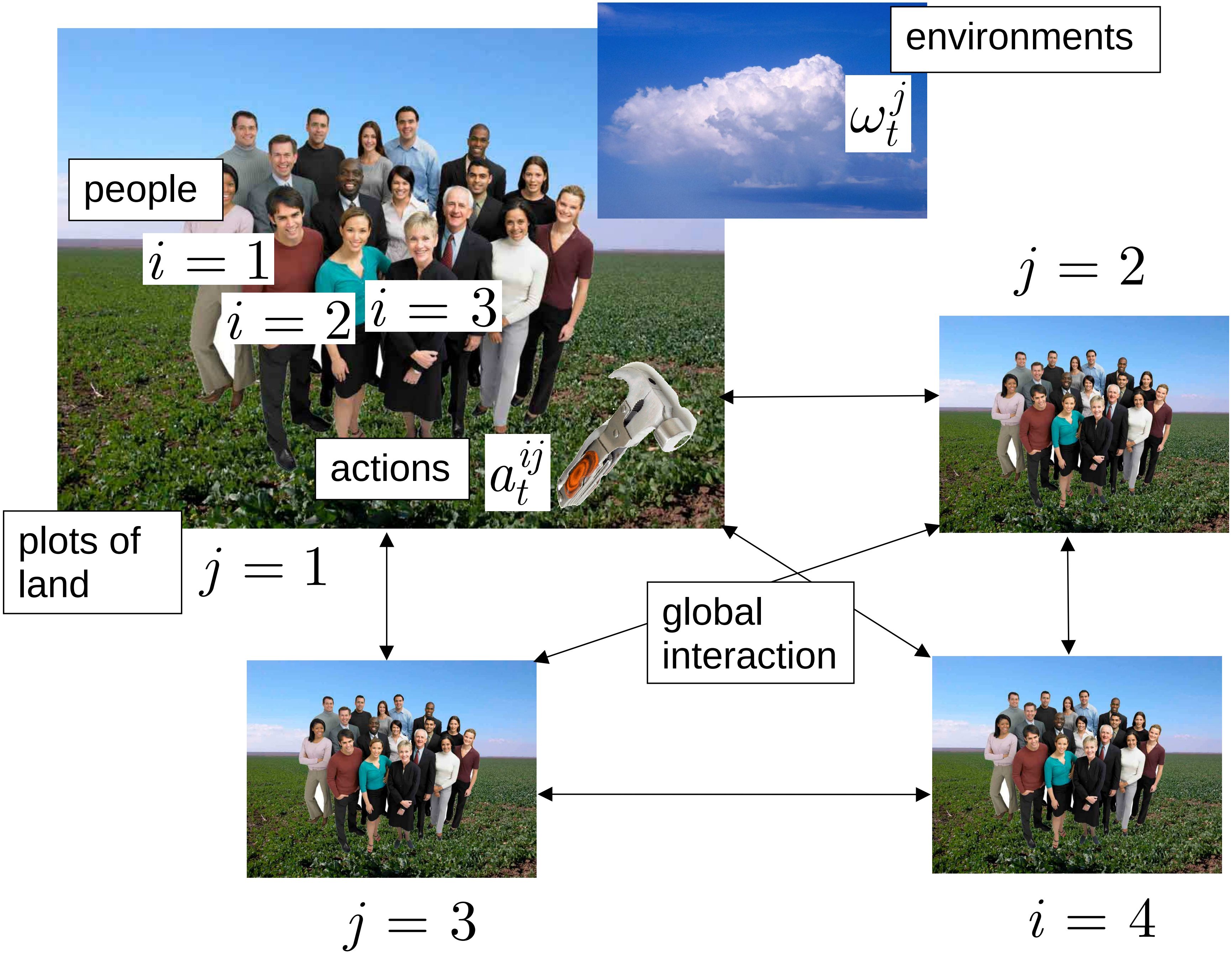
Ely

Evolution + voluntary migration = efficiency

Isn’t the way the world works


environments
ω_t^j
people
$i = 1$
$i = 2$
$i = 3$
$j = 2$
actions
a_t^{ij}
plots of land
$j = 1$
global interaction
$j = 3$
$i = 4$

Consequences (Stage Game)

- Utility $u^i(a_t^j, \omega_t^j)$
- Future environment $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$
- Free resources $f(a_t^j, \omega_t^j) > 0$ [discussed later]
- Expansionism $x(a_t^j) \in \{0,1\}$

Assumptions about an individual plot:

Irreducibility: any environment can be reached

Steady state: if everyone plays the same way repeatedly the environment settles to a steady state.

Disruption

At most one plot per period disrupted, probability of plot k being disrupted (forced, conquered) to play action a_t^j (at time $t+1$) given actions and environments on all plots a_t, ω_t is

$$\pi^k(a_t^j, a_t, \omega_t)$$

Definition: Steady State Nash Equilibrium

a pair a_t^j, ω_t^j that is as it sounds

Malthus Example

- Environment ω_t^j is current population $\in \{1, \dots N\}$
- Action stes A^i are desired target population $\in \{1, \dots N\}$
- Utility $u^i(a_t^j, \omega_t^j) = a_t^{ij}$ from target population
- ω_t^j dynamics $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$ well bahaved
  - Players chosen at random
  - Average target (average of averages) $\bar{a}_t^j = \sum_{i=1}^N a_t^{ij}/N$

$$\omega_{t+1}^j = \omega_t^j + \begin{cases} -1 \\ \;\;0 \\ \;\;1 \end{cases} \text{if} \begin{cases} \bar{a}_t^j < \omega_t^j - 1/2 \\ \omega_t^j - 1/2 \leq \bar{a}_t^j < \omega_t^j - 1/2 \\ \bar{a}_t^j > \omega_t^j + 1/2 \end{cases}$$

- Equilibrium: Unique SS NE with $a_t^{ij} = \omega_t^j = N$

Players' Behavior

- Players' behavior at t:
  - If in s_{t-1} plot j was disrupted, on j they do what they have to
  - Otherwise, player i in plot j plays distribution $B^i(h^j_{t-1})$ on A^i
- Quiet and noisy states, and assumption on play

Definition

A *quiet state* s_t for player i on plot j is a state where (a^j_t, ω^j_t) has been constant for L periods and where a^{ij}_t is best response. Noisy states for i are the other states.

Assumption

If s_{t-1} was a quiet state for player i then at t he plays best response for sure. Otherwise B^i is a full-support distribution on A^i.

Social Norm Games

Discuss the fact that you can equilibria at well above subsistence, real question: which equilibrium?

Social Norms and Finite Games

- Many social norms in infinitely repeated games but also in finite games
- Adopt two-stage approach with a *shunning* punishment giving utility of $\Pi \leq 0$
- Ensure that any profile is two-stage NE (in which defaulter is costlessly shunned)
- Focus on profiles which maximize free resources

Aggregation of Free Resources and Conflict Resolution

What happens to the subsistence farmers when they get invaded?

Free Resources

- We assume (a_t^j, ω_t^j) generates *free resources* $f(a_t^j, \omega_t^j) > 0$
- **Example, Malthus continued.**
  Maximum population size N and subsistence level B are defined by

$$Y(N)/N > B > Y(N+1)/(N+1)$$

  with Y production function (concave increasing).
  Population ω_t^j generates $f(a_t^j, \omega_t^j) = Y(\omega_t^j) - \omega_t^j B > 0$
- Free resources of society playing a_t^k

$$F(a_t^k, a_t, \omega_t) = \sum_{a_t^j = a_t^k} f(a_t^j, \omega_t^j)$$

  Pooling forces crucial for expansion

Expansion, Expansiveness and Free Resources

- Expansions/disruptions depend on Expansiveness and Free Resources
- Assume resistance to disruption lower when fewer free resources, zero (i.e. positive probability of disruption) if other is expansive

Assumption (Monotonicity)

Suppose $F(a_t^k, a_t, \omega_t) \leq F(a_t^j, a_t, \omega_t)$. *If* $x(a_t^k) = x(a_t^j) = 0$ *then* $r[\Pi(a_t^k, a_t, \omega_t)] \leq r[\Pi(a_t^j, a_t, \omega_t)]$; *if* $x(a_t^j) = 1$ *then* $r[\Pi(a_t^k, a_t, \omega_t)] = 0$.

- Next: when only two societies, resistance depends on ratio of free resources

Expansion, Expansiveness and Free Resources

Assumption (Binary Case)

If a_t has two societies then

$$r[\Pi(a_t^k, a_t, \omega_t)] = q(F(a_t^{-k}, a_t, \omega_t)/F(a_t^k, a_t, \omega_t), x(a_t^{-k}))$$

- *q non-increasing in the first argument*
- $q(0, x^j) = q(\phi, 0) = 1$
- $0 < \inf\{\phi | q(\phi, 1) = 0\} < 1$

- Comments
    - $q(0, x^j)$ resistance to mutants
    - $q(\phi, 0)$ resistance to insular groups
    - Exapnsive can disrupt you with positive probability for some $\phi < 1$

Expansion, Expansiveness and Free Resources

- Lastly, divided opponents can't do better than united:

Assumption (Divided Opponents)

If a_t is binary, $\tilde{a}_t$ has $F(a_t^k, a_t, \omega_t) = F(\tilde{a}_t^k, \tilde{a}_t, \omega_t)$ and $\sum_{k' \neq k} F(a_t^{k'}, a_t, \omega_t) \geq \sum_{k' \neq k} F(\tilde{a}_t^{k'}, \tilde{a}_t, \omega_t)$, then $r[\Pi(a_t^k, a_t, \omega_t)] \leq r[\Pi(\tilde{a}_t^k, \tilde{a}_t, \omega_t)]$.

- To sum up, 3 Assumptions:
  - Monotonicity, Ratio in Binary Case, Divided Opponents

Preliminary Results

Theorem [Young]: Unique ergodic

Assume expansive steady state

Monolithic (expansive) steady states

Mixed steady states

Non-expansive steady states

Proposition: When $\varepsilon = 0$ that is all

Main Result

- A *Nash State* is an s_t which is quiet for every player in every plot
- Characerizing ergodic sets $S[0, J]$

Proposition

The sets $S[0, J]$ are singleton Nash states, with either no expansive society, or a single expansive society with ratio of free resources less than $\bar{\phi}$ to all others (if any).

- What we show (abriged version) is

Theorem (Main Result)

For large enough J the stochastically stable states are exactly the Nash states with one expansive society playing the NE with maximum free resources (among all expansive steady states NE).

Technological Progress

- In Malthus example free resources where $f(a_t^j, \omega_t^j) = Y(\omega_t^j) - \omega_t^j B$ with population ω_t^j which depends on action path, with B subsistence income
- Take production

$$AY(z) \quad A \text{ technology level, } z \text{ population}$$

  so free resources are $AY(z) - zB$
- Which population maximizes free resources as A varies? What about income per capita?

Technological Progress

- Contrast Malthus case: for all A choose z such that $AY(z)/z = B$
  - Population increasing in A
  - Income per capita constant
- Our result

Proposition

The free resource maximizer z is increasing in A. Per capita output:

- *If $Y(z) = z^\alpha$ per capita output is independent of A.*
- *If $Y(z) = \log(a+z)$, $a > 0$ it is increasing for sufficiently large A; for large enough a it is decreasing in A then increasing.*

- log case of rapid decreasing return to population
  - In advanced economies income per capita grows with A
  - possibly hunter-gatherers better off than farmers

Bureaucratic State

- Gov provides public good free resources and pays the cost to extract them. Last section incentive payments
- Here monitoring of unobservable output, through *Commissars* ($\simeq$ tax collection for FR max, info rent for profit max)
- *Libertarian paradise* no commissars, no free resources (no gifts)

Bureaucratic State

- Monitoring: produce y if unmonitored, y_S if monitored
  y_S stochastically dominated by y. Assumed $Ey > B$
- Commissars, fraction ϕ of population
  - Produce no output
  - Monitor one another in circle plus κ other individuals (reducing their output)
  - Must be paid as much as the others

  But convert unobservable output into free resources
- Producers are fraction $1-\phi$ of population
  Monitored producers, wage w, are fraction $\kappa\phi/(1-\phi)$ of producers (fraction $\kappa\phi$ of population)
- Expected income of producer is

$$\bar{W} = \frac{\kappa\phi}{1-\phi} w + \left(1 - \frac{\kappa\phi}{1-\phi}\right) Ey$$

Bureaucratic State

- Per capita f come from monitored producers, fraction $\kappa\phi$
  Fraction ϕ of commissars must be paid $\bar{W}$. So expected f is

$$f = \kappa\phi(Ey_s - w) - \phi\bar{W}$$

- To max f subject to $\bar{W} \geq B$ and $\kappa\phi/(1-\phi) \leq 1$
- Alternative model: *Creepy Bureacracy*
  - Efficiency of commissars decreasing in ϕ
    "Heavy fraction calls more weight"

  κ decreasing function of ϕ

$$\kappa(\phi) = \kappa(1-\phi)$$

Bureaucratic State

- Result here is the following

Proposition

Assume $Ey_s > Ey/2$ *and* $\kappa > 1$ *and maximization of free resources.*

- *Fraction of commissars is positive*
- *Fraction of monitored producers is the same with or without creep*
- *Fraction of commissars is higher with creepy bureaucracy.*