Anti-Malthus: Evolution, Population and the Maximization of Free Resources

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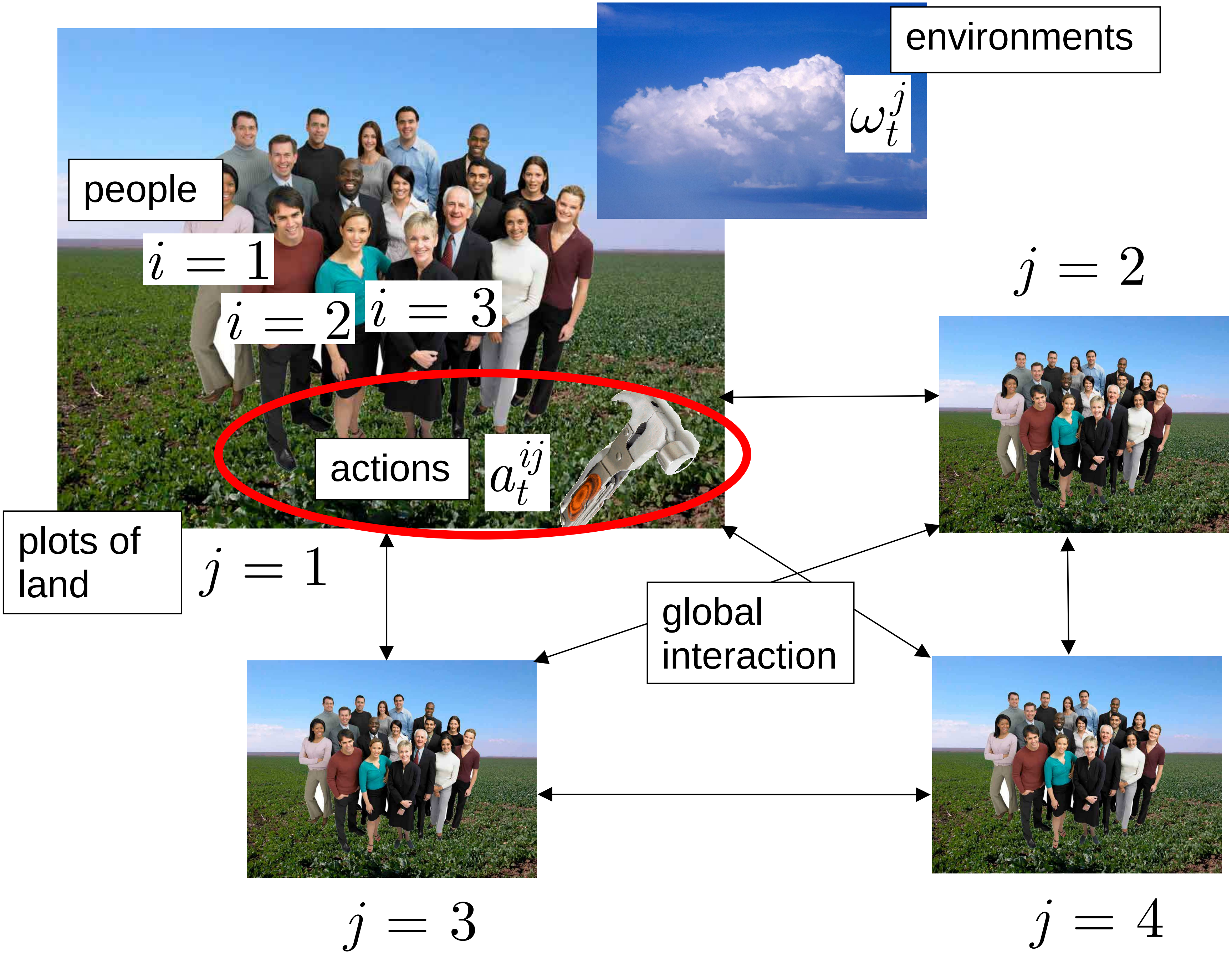
Evolution of Societies

- Does not evolution favor more efficient societies?
- Must have incentive compatibility: better everyone else contributes to the common good and you free ride
- Evolution + voluntary migration = efficiency within the set of equilibria
- Isn't the way the world works


environments
ω_t^j
people
$i = 1$
$i = 2$
$i = 3$
$j = 2$
actions
a_t^{ij}
plots of land
$j = 1$
global interaction
$j = 3$
$j = 4$

Consequences (Stage Game)

- Utility $u^i(a_t^j, \omega_t^j)$
- Future environment $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$
- Free resources $f(a_t^j, \omega_t^j) > 0$ [discussed later]
- Expansionism $x(a_t^j) \in \{0,1\}$

Assumptions about an individual plot:

Irreducibility: any environment can be reached

Steady state: if everyone plays the same way repeatedly the environment settles to a steady state.

Disruption

At most one plot per period disrupted, probability of plot k being disrupted (forced, conquered) to play action a_t^j (at time $t+1$) given actions and environments on all plots a_t, ω_t is

$$\pi^k(a_t^j, a_t, \omega_t)$$

[conflict resolution function]

disruption should depend on resources available to “defend” and “attack” and whether or not a society is intrinsically expansionary

- free resources
- expansionism

expansionist: Christianity after the Roman period; Islam

non-expansionist: Judaism after the diaspora; Russian Old Believers

Definition: Steady State Nash Equilibrium

a pair a_t^j, ω_t^j that is as it sounds

Malthus Example

- environment is population $\omega_t^j \in \{1,\ldots,N\}$
- actions are target population $a_t^{ij} \in \{1,\ldots,N\}$
- utility $u^i(a_t^j, \omega_t^j) = a_t^j$: want lots of kids
- $\overline{a}_t^j$ average target (those who live are picked at random)
- $$\omega_{t+1}^j = \omega_t^j + \begin{cases} -1 & \overline{a}_t^j < \omega_t^j - 1/2 \\ +1 & \overline{a}_t^j > \omega_t^j - 1/2 \\ 0 & \text{otherwise} \end{cases}$$
- unique steady state Nash equilibrium at N

Appreciable versus Negligable Probabilities

Will consider a limit as a noise parameter $\varepsilon \to 0$

- Probabilities that go to zero are *negligable*
- Probabilities that do not go to zero are *appreciable*

Definition of resistance:

$Q[\varepsilon]$ a function of the noise parameter ε

Q is regular if

the resistance $r[Q] \equiv \lim_{\varepsilon \to 0} \log Q(\varepsilon) / \log \varepsilon$ exists

and $r[Q] = 0$ implies appreciable probability $\lim_{\varepsilon \to 0} Q(\varepsilon) > 0$

if $r[Q] > 0$ then negligable probability

Behavior

- behavior based on finite histories s_t is the *state*
- if plot was disrupted, players play as required otherwise
- play $B^i(s_{t-1})$ a strictly positive probability distribution over A^i
- *quiet state* for player i: environment and action profile constant *and* player i is playing a best response
- otherwise: *noisy state*
- in a quiet state the probability of all actions except the status quo are negligable
- in a noisy state all actions have appreciable probability

Social Norm Games

Discuss the fact that you can equilibria at well above subsistence, real question: which equilibrium?

- Repeated games, self-referential games
- Here a simple two-stage process
- Add a second stage in which each player has an opportunity to *shun* an(y) opponent
- If everyone shuns you utility is less than any other outcome of the game
- Transparently a folk theorem class of games
- Also: *bloodlust games*: add a strategy that gives even less utility than being shunned, but generates more free resources than any steady state equilibrium

Aggregation of Free Resources and Conflict Resolution

What happens to the subsistence farmers when they get invaded? Nothing good.

- Free resources $f(a_t^j, \omega_t^j) > 0$ are those above and beyond what is needed for subsistence and incentives; they are what is available for influencing other societies and preventing social disruption
- Expansionism $x(a_t^j) \in \{0,1\}$ is willingness to deploy free resources in disrupting other societies
- A society are all plots playing a common action profile a_t^j
- What matters is free resources aggregated over a society F
- Monaco versus China
- These things help determine the conflict resolution function $\pi^k(a_t^j, a_t, \omega_t)$

Free Resources

- We assume (a_t^j, ω_t^j) generates *free resources* $f(a_t^j, \omega_t^j) > 0$
- **Example, Malthus continued.**
  Maximum population size N and subsistence level B are defined by

$$Y(N)/N > B > Y(N+1)/(N+1)$$

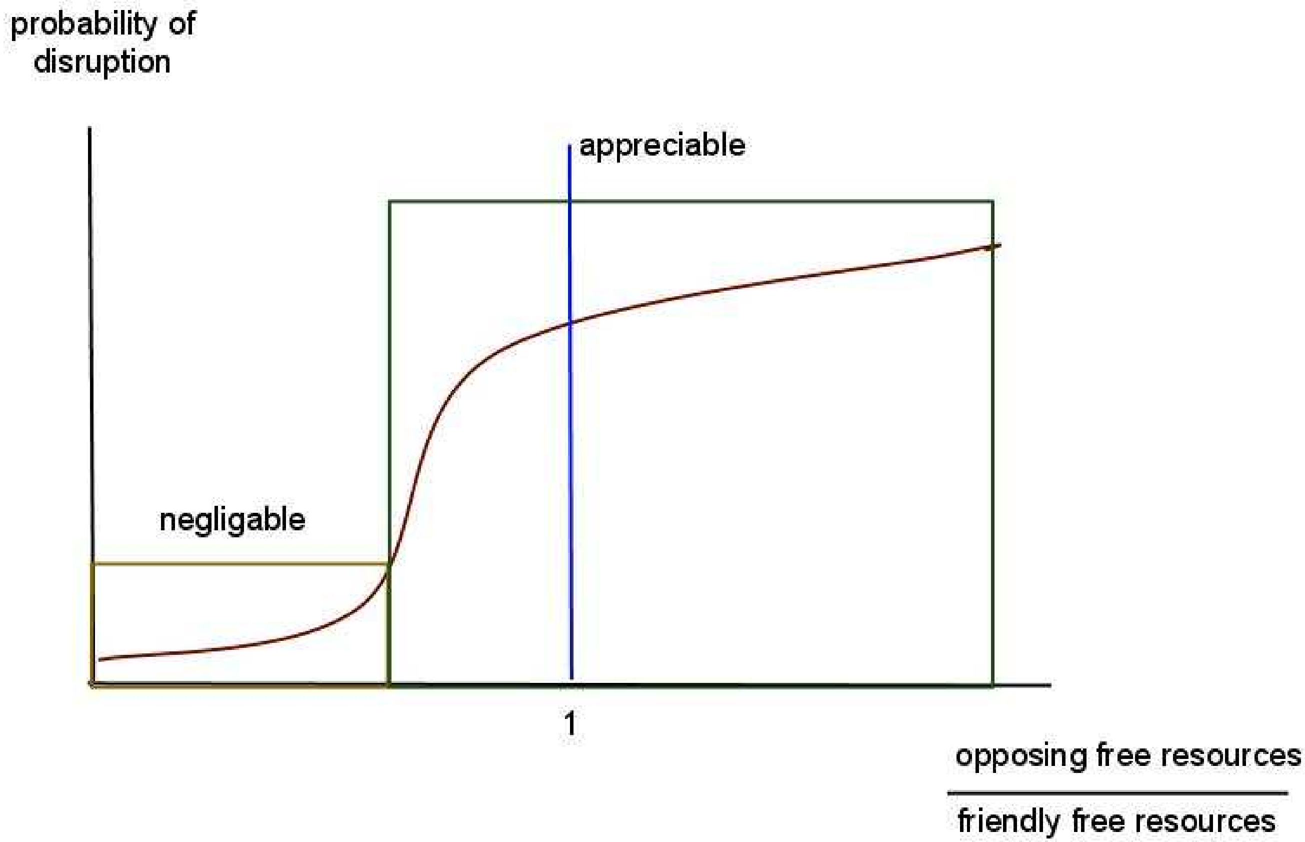
  with Y production function (concave increasing).
  Population ω_t^j generates $f(a_t^j, \omega_t^j) = Y(\omega_t^j) - \omega_t^j B > 0$
- Free resources of society playing a_t^k

$$F(a_t^k, a_t, \omega_t) = \sum_{a_t^j = a_t^k} f(a_t^j, \omega_t^j)$$

  Pooling forces crucial for expansion

Assumptions About Conflict

- Weak monotonicity: more free resources reduce chance of disruption, for expansionary societies they also increase the chance of disrupting others
- Given free resources, divided opponents are no stronger than a monolithic opponent
- Binary case


probability of
disruption
appreciable
negligable
1
opposing free resources
friendly free resources

Expansion, Expansiveness and Free Resources

Assumption (Binary Case)

If a_t has two societies then

$$r[\Pi(a_t^k, a_t, \omega_t)] = q(F(a_t^{-k}, a_t, \omega_t)/F(a_t^k, a_t, \omega_t), x(a_t^{-k}))$$

- *q non-increasing in the first argument*
- $q(0, x^j) = q(\phi, 0) = 1$
- $0 < \inf\{\phi | q(\phi, 1) = 0\} < 1$

- Comments
  - $q(0, x^j)$ resistance to mutants
  - $q(\phi, 0)$ resistance to insular groups
  - Exapnsive can disrupt you with positive probability for some $\phi < 1$

Preliminary Results

Theorem [Young]: Unique ergodic

Assume expansive steady state

Monolithic (expansive) steady states

Mixed steady states

Non-expansive steady states

Proposition: When $\varepsilon = 0$ that is all

Main Results

Theorem: general case are probability distributions over of $\varepsilon = 0$

assume existence of barbarian horde

Theorem: characterization of stochastically stable states

Maximum free resources if J the number of plots is big enough

Things that might not be necessary: existence of barbarian horde (probably not); assumption that cutoff between negligable and appreciable disruption is less than one; both are used in the existing proof

Intuition

- Consider monolithic: it takes one coincidence to go anywhere after which will almost certainly wind up back where you started before a second coincidence happens
- So: need some minimum number of coincidences before an appreciable chance of being disrupted
- More free resources = more coincidences required
- Think in terms of layers of protecting a nuclear reactor: redundancy - a second independent layer of protection double the cost, but provides an order of magnitude more protection (1/100 versus 1/10,000)
- What happens if you need more than equal free resources before chance of disrupting becomes appreciable? can have two expansionary societies living side by side, neither having much chance of disrupting the other

Technological Progress

- In Malthus example free resources where $f(a_t^j, \omega_t^j) = Y(\omega_t^j) - \omega_t^j B$ with population ω_t^j which depends on action path, with B subsistence income
- Take production

$$AY(z) \quad A \text{ technology level, } z \text{ population}$$

  so free resources are $AY(z) - zB$
- Which population maximizes free resources as A varies? What about income per capita?

Technological Progress

- Contrast Malthus case: for all A choose z such that $AY(z)/z = B$
  - Population increasing in A
  - Income per capita constant
- Our result

Proposition

The free resource maximizer z is increasing in A. Per capita output:

- *If $Y(z) = z^\alpha$ per capita output is independent of A.*
- *If $Y(z) = \log(a+z)$, $a > 0$ it is increasing for sufficiently large A; for large enough a it is decreasing in A then increasing.*

- log case of rapid decreasing return to population
  - In advanced economies income per capita grows with A
  - possibly hunter-gatherers better off than farmers

The principal-agent problem and the theory of the profit maximizing state

Bureaucratic State

- Gov provides public good free resources and pays the cost to extract them. Last section incentive payments
- Here monitoring of unobservable output, through *Commissars* ($\simeq$ tax collection for FR max, info rent for profit max)
- *Libertarian paradise* no commissars, no free resources (no gifts)

Bureaucratic State

- Monitoring: produce y if unmonitored, y_S if monitored
  y_S stochastically dominated by y. Assumed $Ey > B$
- Commissars, fraction ϕ of population
  - Produce no output
  - Monitor one another in circle plus κ other individuals (reducing their output)
  - Must be paid as much as the others

  But convert unobservable output into free resources
- Producers are fraction $1-\phi$ of population
  Monitored producers, wage w, are fraction $\kappa\phi/(1-\phi)$ of producers (fraction $\kappa\phi$ of population)
- Expected income of producer is

$$\bar{W} = \frac{\kappa\phi}{1-\phi} w + \left(1 - \frac{\kappa\phi}{1-\phi}\right) Ey$$

Bureaucratic State

- Per capita f come from monitored producers, fraction $\kappa\phi$
  Fraction ϕ of commissars must be paid $\bar{W}$. So expected f is

$$f = \kappa\phi(Ey_s - w) - \phi\bar{W}$$

- To max f subject to $\bar{W} \geq B$ and $\kappa\phi/(1-\phi) \leq 1$
- Alternative model: *Creepy Bureacracy*
  - Efficiency of commissars decreasing in ϕ
    "Heavy fraction calls more weight"

  κ decreasing function of ϕ

$$\kappa(\phi) = \kappa(1-\phi)$$

Bureaucratic State

- Result here is the following

Proposition

Assume $Ey_s > Ey/2$ *and* $\kappa > 1$ *and maximization of free resources.*

- *Fraction of commissars is positive*
- *Fraction of monitored producers is the same with or without creep*
- *Fraction of commissars is higher with creepy bureaucracy.*