Anti-Malthus: Evolution, Population and the Maximization of Free Resources

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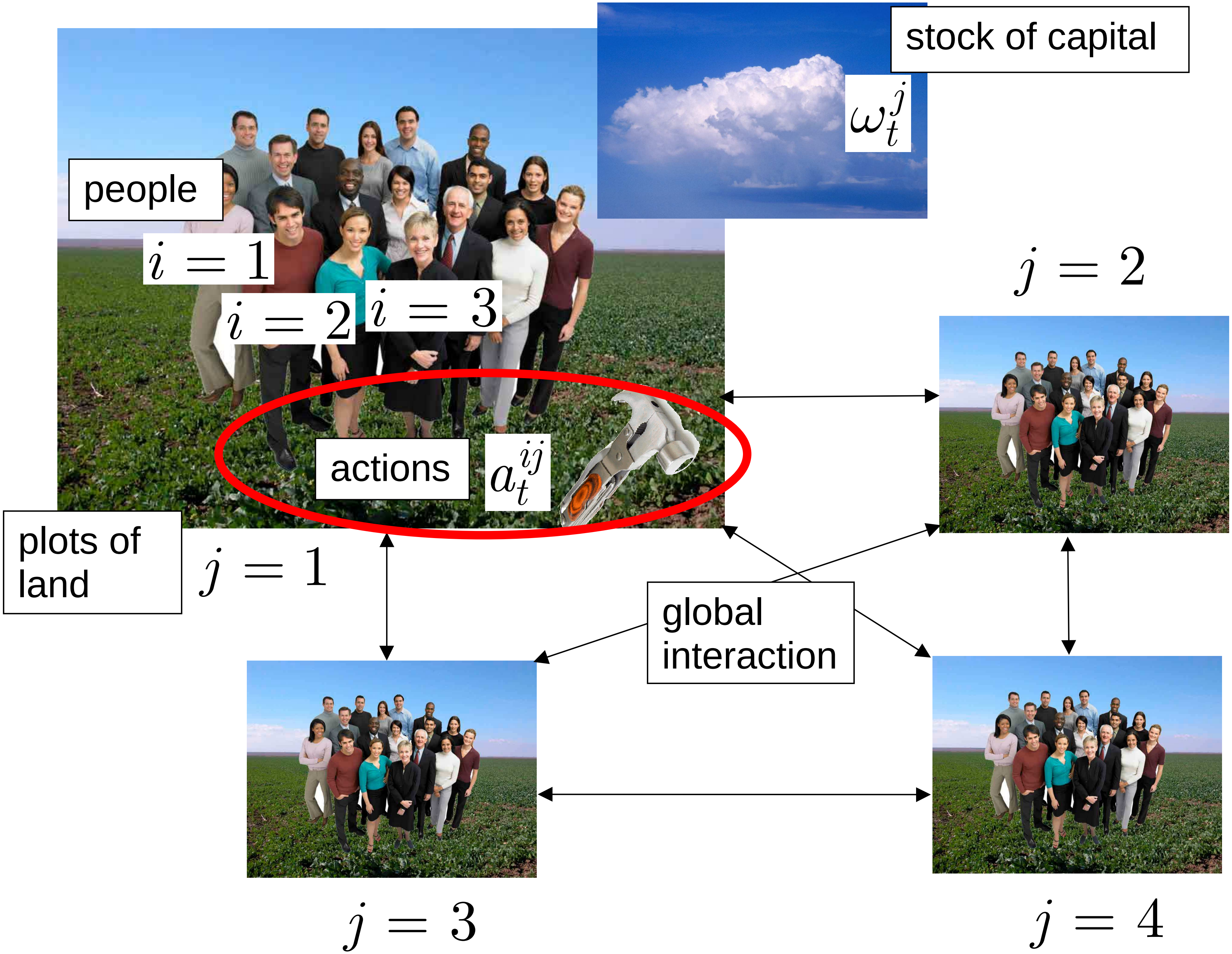
Evolution of Societies

- Does not evolution favor more efficient societies?
- Must have incentive compatibility: better everyone else contributes to the common good and you free ride
- Evolution + voluntary migration = efficiency within the set of equilibria
- Isn't the way the world works


stock of capital
ω_t^j
people
$i = 1$
$i = 2$
$i = 3$
actions
a_t^{ij}
plots of land
$j = 1$
$j = 2$
global interaction
$j = 3$
$j = 4$

Consequences (Stage Game)

- Utility $u^i(a_t^j, \omega_t^j)$
- Capital/investment dynamics $\omega_{t+1}^j = g(a_t^j, \omega_t^j)$
- Free resources $f(a_t^j, \omega_t^j) > 0$ [discussed later]
- Expansionism $x(a_t^j) \in \{0,1\}$ [discussed later]

Assumptions about capital dynamics on an individual plot:

Irreducibility: any environment can be reached

Steady state: if everyone plays the same way repeatedly the environment settles to a steady state.

Disruption

At most one plot per period disrupted, probability of plot k being disrupted (forced, conquered) to play action a_t^j (at time $t+1$) given actions and capital stocks on all plots a_t, ω_t is

$$\pi^k(a_t^j, a_t, \omega_t)$$

[conflict resolution function]

disruption should depend on resources available to "defend" and "attack" and whether or not a society is intrinsically expansionary

- free resources
- expansionism

expansionist: Christianity after the Roman period; Islam

non-expansionist: Judaism after the diaspora; Russian Old Believers

Definition: Steady State Nash Equilibrium

a pair a_t^j, ω_t^j that is as it sounds

(note pure strategies; will assume existence)

Malthus Example

- capital stock is population $\omega_t^j \in \{1,\ldots,N\}$
- actions are target population $a_t^{ij} \in \{1,\ldots,N\}$
- utility $u^i(a_t^j,\omega_t^j) = a_t^j$: want lots of kids
- $\overline{a}_t^j$ average target (those who live are picked at random)
- $\omega_{t+1}^j = \omega_t^j + \begin{cases} -1 & \overline{a}_t^j < \omega_t^j - 1/2 \\ +1 & \overline{a}_t^j > \omega_t^j - 1/2 \\ 0 & \text{otherwise} \end{cases}$
- unique steady state Nash equilibrium at N

Appreciable versus Negligable Probabilities

Will consider a limit as a noise parameter $\varepsilon \to 0$

- Probabilities that go to zero are *negligable*
- Probabilities that do not go to zero are *appreciable*

Definition of resistance:

$Q[\varepsilon]$ a function of the noise parameter ε

Q is regular if

the resistance $r[Q] \equiv \lim_{\varepsilon \to 0} \log Q(\varepsilon) / \log \varepsilon$ exists

and $r[Q] = 0$ implies appreciable probability $\lim_{\varepsilon \to 0} Q(\varepsilon) > 0$

if $r[Q] > 0$ then negligable probability

Behavior

- behavior based on finite histories s_t is the *state*
- if plot was disrupted, players play as required otherwise
- play $B^i(s_{t-1})$ a strictly positive probability distribution over A^i
- *quiet state* for player i: capital stock and action profile constant *and* player i is playing a best response
- otherwise: *noisy state*
- in a quiet state the probability of all actions except the status quo are negligable
- in a noisy state all actions have appreciable probability

Aggregation of Free Resources and Conflict Resolution

What happens to the subsistence farmers when they get invaded? Nothing good.

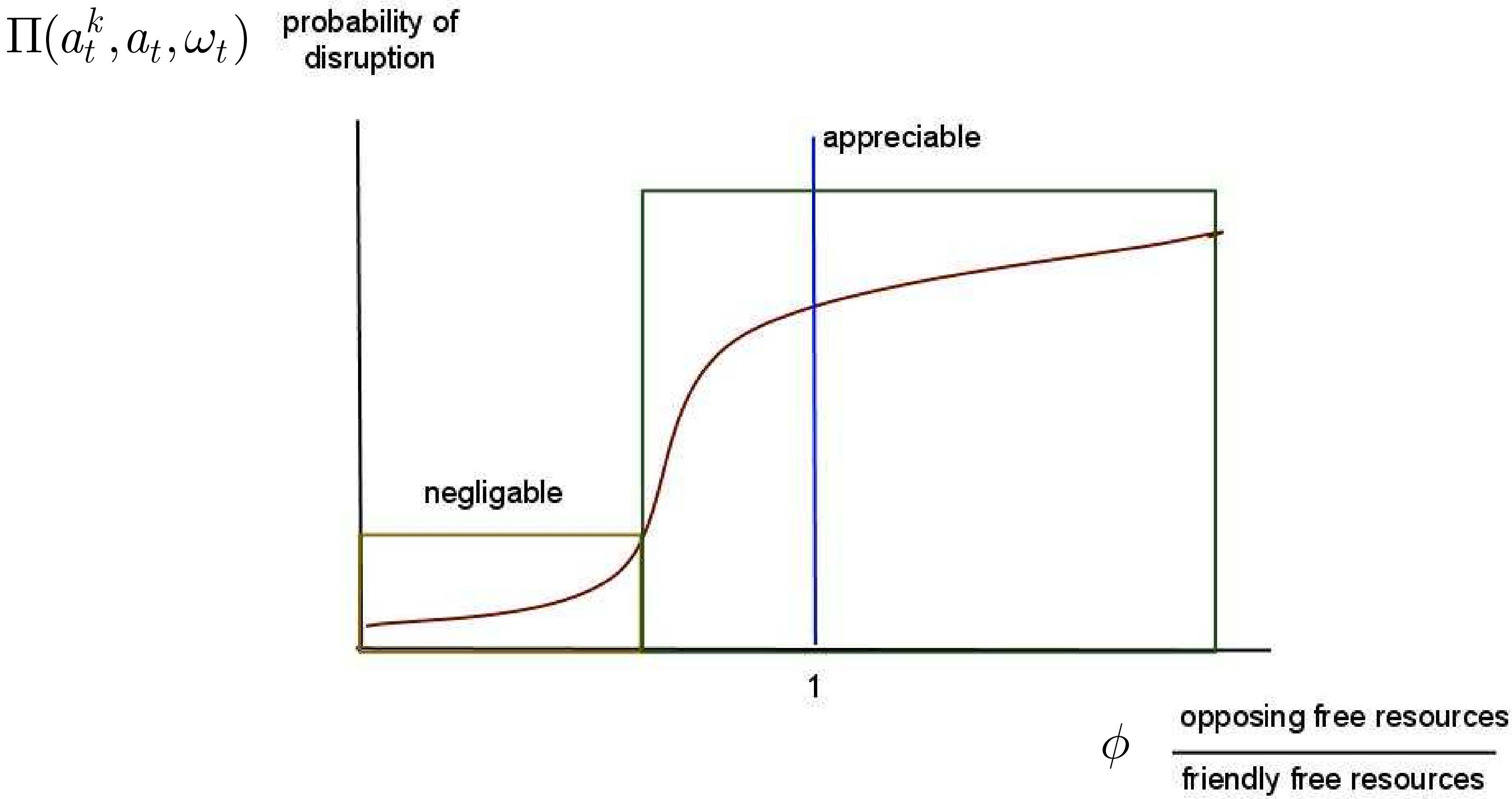
- Free resources $f(a_t^j, \omega_t^j) > 0$ are those above and beyond what is needed for subsistence and incentives; they are what is available for influencing other societies and preventing social disruption, less discretionary income
- Expansionism $x(a_t^j) \in \{0,1\}$ is willingness to deploy free resources in disrupting other societies
- A society are all plots playing a common action profile a_t^j
- What matters is free resources aggregated over a society F
- Monaco versus China
- These things help determine the conflict resolution function $\pi^k(a_t^j, a_t, \omega_t)$

Assumptions About Conflict

- Weak monotonicity: more free resources reduce chance of disruption, for expansionary societies they also increase the chance of disrupting others
- Given free resources, divided opponents are no stronger than a monolithic opponent
- Majority rule: if an expansionist opponent has as many free resources as you then you appreciable probability of disruption
- Binary case

Aggregation function: $F = \Phi(J, f)$ strictly increasing in both arguments where J are the number of plots in a society and f are the free resources per plot

$r[\Pi(a_t^k, a_t, \omega_t)] = q(F^{-k} / F^k, x^{-k})$, in first argument: weakly decreasing, left continuous, $q(0, x_t^j) = q(\phi, 0) = 1$, for some $\phi > 0$ $q(\phi, 1) > 0$


$\Pi(a_t^k, a_t, \omega_t)$
probability of disruption
appreciable
negligable
1
ϕ
opposing free resources
friendly free resources

General Results on Stochastic Stability

Theorem [Young]: Unique ergodic distribution

Assume expansive steady state exists

Types of steady states when $\varepsilon = 0$

Monolithic (expansionary) steady states

Mixed steady states (only one expansionary)

Non-expansive steady states

Theorem [Young] Unique limit of ergodic distribution as $\varepsilon \to 0$ putting weight only on the above

These are called *stochastically stable states*

Main Results

Theorem: characterization of stochastically stable states

Maximum free resources if J *the number of plots is big enough*

Might not need assumption that cutoff between negligable and appreciable disruption is less than one

Remark: also a monotonicity result in J

Intuition

- Consider monolithic: it takes one coincidence to go anywhere after which will almost certainly wind up back where you started before a second coincidence happens
- So: need some minimum number of coincidences before an appreciable chance of being disrupted
- More free resources = more coincidences required
- Think in terms of layers of protecting a nuclear reactor: redundancy - a second independent layer of protection double the cost, but provides an order of magnitude more protection (1/100 versus 1/10,000)
- What happens if you need more than equal free resources before chance of disrupting becomes appreciable? can have two expansionary societies living side by side, neither having much chance of disrupting the other

Social Norm Games

Discuss the fact that you can have equilibria at well above subsistence, real question: which equilibrium?

- Repeated games, self-referential games
- Here a simple two-stage process
- Add a second stage in which each player has an opportunity to *shun* an(y) opponent
- If everyone shuns you utility is less than any other outcome of the game

Transparently a folk theorem class of games

Malthus Revisited

$Y(z)$ output as function of population

suppose social norm game, what maximizes free resources?

Free resources: $AY(z) - Bz$ where A is techology parameter

More than minimum population, less than subsistence

$Y(z) = z^\alpha$ Malthusian result, per capita output independent of A

➢ why returns on a plot should decrease more rapidly

$Y(z) = \log(a + z)$ (note that $z \geq 1$)

per capita output increasing for large A

for large a it is also decreasing for small A

What are Free Resources: Bureacracy

Individuals produce output y with continuous positive density on $[0, \infty)$

Risk neutrality

Subsistence is B which must be met *on average* in the population

(some people could reproduce more slowly, others more rapidly)

$Ey > B$ or else not much can happen

output unobservable so no free resources

Commissars

Can monitor each other and κ other individuals

ϕ fraction of population who are commissars, w wage paid to those people who are monitored

commissars have to get the same expected utility as anyone else

monitored indivuals may produce less y_S weakly stochastically dominated by y

expected income of a producer

$$\bar{W} = \frac{\kappa\phi}{1-\phi}w + \left(1 - \frac{\kappa\phi}{1-\phi}\right)Ey$$

so per capita free resources are

$$f = \kappa\phi(Ey_s - w) - \phi\bar{W}$$

if $Ey_S \geq Ey/2$ and $\kappa > 1$ positive fraction of commissars

Summing Up

- free resources are those that prevent disruption and allow expansion
- maximization of free resources provides a positive theory of institutions including the state and population
- the long-run may be a long-time, but institutions that are deficient on free resources are not likely to last long