Voter Turnout with Peer Punishment

Noise, High Stakes Elections and Bimodality

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The Contest

large population of N voters divided into two parties of size $\eta_k N$ where $k = \{S, L\}$ *small* and *large* $\eta_S < \eta_L$

parties compete in an election – side that produces the greatest expected number of votes wins a prize worth V

parties by consensus or directed by leaders move first and simultaneously and non-cooperatively choose a social norm in the form of a participation rate for their members

individual party members move second and, given the social norm, optimally choose whether or not to vote in an election

Participation Costs

each identical party member privately draws a type y from a uniform distribution on $[0, 1]$

type determines a *net participation cost* of voting $c(y)$ continuously differentiable, strictly increasing and satisfies $c(\underline{y}) = 0$ for some $\underline{y} \in [0, 1]$

members for whom $y < \underline{y}$, those with a negative net cost of voting, are called *committed voters* and will always vote

social norm a threshold φ_k together with a rule prescribing voting if $y \leq \varphi_k$ it is also the expected participation rate of the party

N is large so assume φ_k also the actual participation rate of party k

social norm enforced through peer auditing and the possibility of imposing punishments on party members

Monitoring

each member of the party is audited by another party member

may think of party members as forming a circular network with each party member auditing the member to their left

auditor observes whether or not the auditee voted

if the auditee did not vote and the auditee violated the policy the auditor learns this for certain

if the auditee did not vote but did not violate the policy there is a probability $1-\theta \in [0,1]$that the auditor will learn this

θ represents the *monitoring inefficiency*: if $\theta = 1$ then the auditor learns nothing about y; if $\theta = 0$ the auditor perfectly observes whether y is above or below the threshold

Crime and Punishment

if auditee voted or is discovered not to have violated the policy, auditee is not punished.

if auditee did not vote and auditor cannot determine whether or not the auditee violated the policy auditee is punished with a loss of utility $P_k \geq 0$

Incentive Compatibility

social norm φ_k *incentive compatible* if and only if $P_k = c(\varphi_k)$

member with $y \leq \varphi_k$ willing to pay the participation cost $c(y)$ of voting rather than face certain punishment P_k

member with $y > \varphi_k$ prefers to pay the expected cost of punishment θP_k over the participation cost of voting $c(y)$

Cost of a Social Norm

punishment as it is paid by a member, is also a cost to the party

all costs per capita

total cost of choosing an incentive compatible social nor $\varphi_k > \underline{y}$ is denoted by $C(\varphi_k)$ with $C(\varphi_k) = 0$ for $\varphi_k \leq \underline{y}$

total cost $C(\varphi_k)$ has two parts

turnout cost $T(\varphi_k) = \int_{\underline{y}}^{\varphi_k} c(y)dy$ - the participation cost of voting to the member types who vote

monitoring cost $M(\varphi_k) = \int_{\varphi_k}^{1} \theta P_k dy$ - the cost of punishing party members who did not vote

Quadratic Case

for illustrative purposes suppose for $\varphi_k \geq \underline{y}$ participation cost is linear

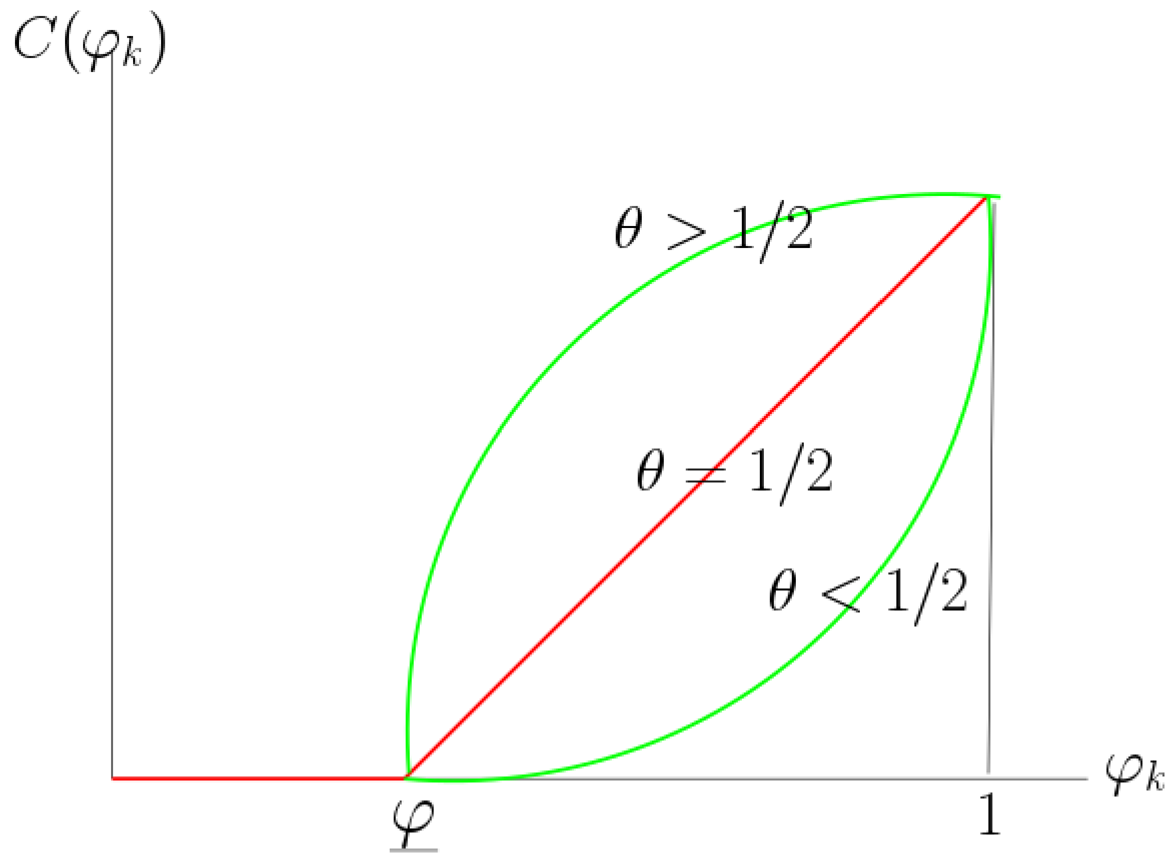
$$c(\varphi_k) = 2(\varphi_k - \underline{y})$$

turnout cost $T(\varphi_k) = (\varphi_k - \underline{y})^2$ is quadratic

total cost $C(\varphi_k) = (1-2\theta)(\varphi_k - \underline{y})^2 + 2\theta(1-\underline{y})(\varphi_k - \underline{y})$ also quadratic

strictly increasing

$C''(\varphi_k) = 2(1-2\theta)$ so $C(\varphi_k)$ is strictly concave if monitoring is sufficiently inefficient, that is $\theta > 1/2$, linear if $\theta = 1/2$ and strictly convex if $\theta < 1/2$


$C(\varphi_k)$
$\theta > 1/2$
$\theta = 1/2$
$\theta < 1/2$
$\underline{\varphi}$
1
φ_k

Bids and Equilibrium

both parties face the same per capita costs of turning out voters characterized by the same $\underline{y}$ and total cost $C(\varphi_k)$

outcome of the election is determined by the fraction of the electorate $b_k = \eta_k \varphi_k$ called the *bid* of party k

party that "submits the highest bid" wins: an all-pay auction - the highest bid wins, each party pays the cost for their bid

also endogenous tie-breaking rule

strategy for party k is a cumulative distribution function F_k over bids, that is, on $[\eta_k \underline{y}, \eta_k]$

An *equilibrium* consists of strategies for both parties together with a tie-breaking rule such that the strategy of a party is optimal given the strategy of the other party and the tie-breaking rule

Willingness to Bid

turning out φ_k and winning gets $V/\eta_k - C(\varphi_k)$ per capita

turnout out only committed voters and losing gets 0

if $C(1) < V/\eta_k$ party is willing to turn out all its voters to get the prize and willingness to bid is $\hat{b}_k = \eta_k$

if $C(1) \geq V/\eta_k$ then the party willingness to bid is determined by the indifference condition $C(\hat{b}_k/\eta_k) = V/\eta_k$

willingness to bid is endogenous in the sense that it depends on the parameters of the model: the cost function and the size of the party

not an equilibrium quantity in the sense that it is independent of any choices made by the other party, or indeed their characteristics

rule out the degenerate case where both parties are equally willing to bid

Party Advantage

a party *advantaged* if it has the higher willingness to bid and *disadvantaged* otherwise

disadvantaged party will be denoted by d so by definition $\hat{b}_d < \hat{b}_{-d}$

not always the case that the large party is advantaged

restrict attention to the interesting case in which the small party is willing to bid more than the committed voters of the large party that is $\hat{b}_S > \eta_L \underline{y}$ or equivalently $V/\eta_S < C(\eta_L \underline{y}/\eta_S)$

Tripartite Auction Theorem

There is a unique equilibrium. In this equilibrium the utility of the disadvantaged party is 0 and the utility of the advantaged party is $U_{-d} = V - \eta_{-d} C(\hat{b}_d / \eta_{-d})$.

- same result as second price auction
- advantage depends on the properties of the total cost function

Convexity and Concavity

if the small party is willing to bid $\hat{b}_S$ it incurs an *aggregate* total cost (that is total cost per size of the party) equal to

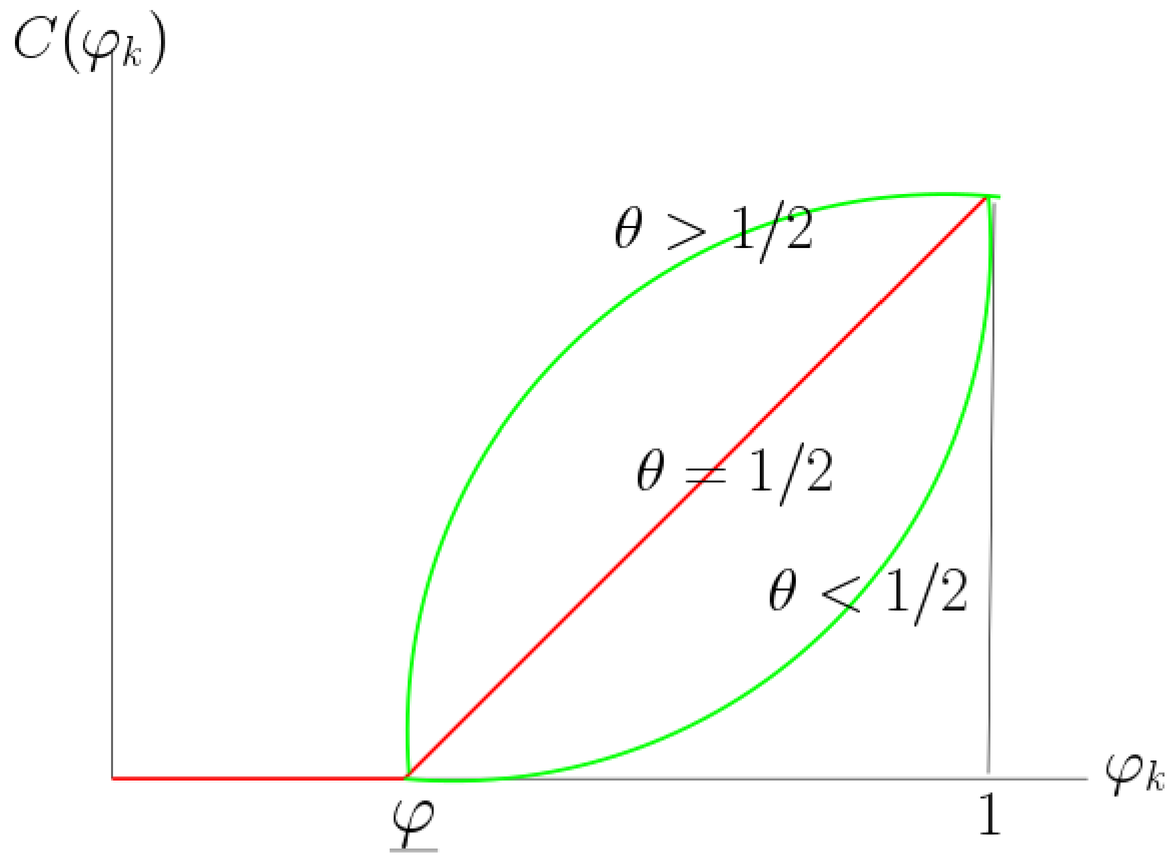
$$\eta_S C(\hat{b}_S/\eta_S) = \hat{b}_S \left[C(\hat{b}_S/\eta_S)/(\hat{b}_S/\eta_S) \right]$$

aggregate total cost to the large party of matching a bid of $\hat{b}_S$ is

$$\eta_L C(\hat{b}_S/\eta_L) = \hat{b}_S \left[C(\hat{b}_S/\eta_L)/(\hat{b}_S/\eta_L) \right]$$

smaller for the large party if $C(\hat{b}_S/\eta_S)/(\hat{b}_S/\eta_S) > C(\hat{b}_S/\eta_L)/(\hat{b}_S/\eta_L)$

- average per capita cost is increasing the large party is advantaged
- when average per capita cost is decreasing the small party is advantaged
- properties of $C(\varphi_k)$ above $\underline{y}$: convex then globally convex, large party advantaged
- concave then can go either way



$C(\varphi_k)$
$\theta > 1/2$
$\theta = 1/2$
$\theta < 1/2$
$\underline{\varphi}$
1
φ_k

Advantage Theorem

If the cost function is either convex or it is concave and the small party is advantaged, then the advantaged party equilibrium bidding function FSD that of the disadvantaged party.

- in particular advantaged party has higher probability of winning and higher expected turnout
- in quadratic case if for high enough θ small party is advantaged then there is a range of θ for which cost is concave, the large party is advantaged, yet it has smaller expected turnout and smaller probability of winning

High Stakes Elections

- specialize to $\theta = 1/2$ intermediate case of constant marginal cost
- probability the small group bids $\underline{y}\eta_S$ and concedes the election is

$$\frac{V - (\eta_S - \eta_L \underline{y})}{V}$$

- probability large group bids η_S and takes the election is

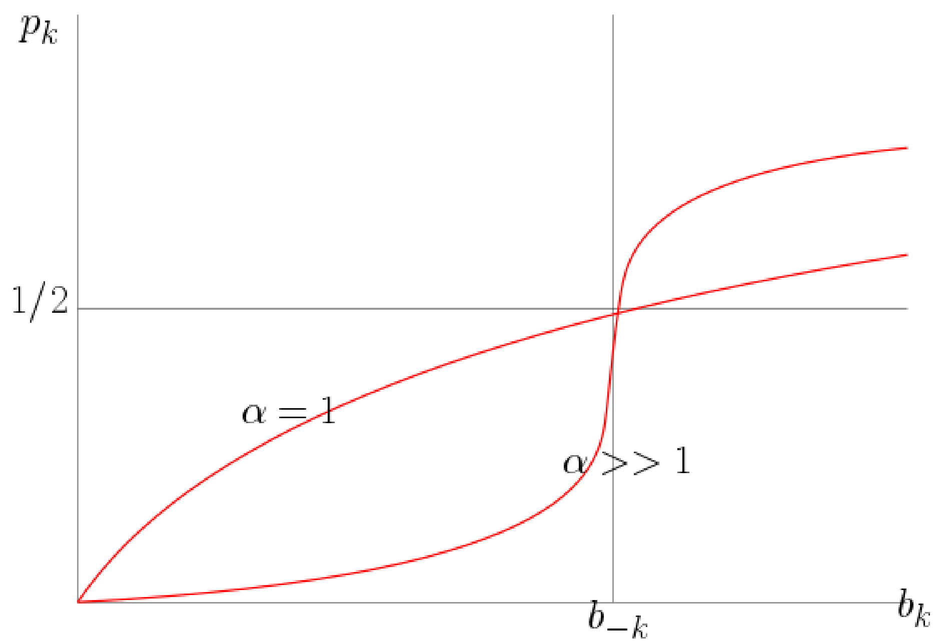
$$\frac{V - \eta_S(1 - \underline{y})}{V}$$

- the key point: as V grows large these probabilities both approach 1
- quite an opposite result from what happens when there is extrinsic uncertainty: in that case the probability for each group of bidding the maximum η_k goes to 1
- we are pretty sure there is some extrinsic uncertainty about the relative size of the parties

The Tullock Model

Tullock conflict resolution function - the probability of winning:

$$p_k(b_k, b_{-k}) = \frac{b_k^\alpha}{b_k^\alpha + b_{-k}^\alpha}$$



The Problem with Tullock

- we only know what happens for low α and in the limit as $\alpha \to \infty$
- an alternative way of introducing extrinsic uncertainty
- some probability decided without noise and some probability decided with noise
- one possibility for “with noise” the Tullock with $\alpha = 1$
- does not work so well

Linear Differential Model

Tullock with $\alpha = 1$ gives rise to the winning differential

$$p_k(b_k, b_{-k}) - p_{-k}(b_{-k}, b_k) = \frac{b_k - b_{-k}}{b_k + b_{-k}}$$

an alternative the linear differential: differential in the probability of winning is equal to the difference in the number of votes divided by the number of possible votes rather than the number of votes cast

since the number of total possible votes has been normalized to 1 this is

$$p_k(b_k, b_{-k}) - p_{-k}(b_{-k}, b_k) = b_k - b_{-k}$$

comes from the linear conflict resolution function

$$p_k(b_k, b_{-k}) = \frac{1 + b_k - b_{-k}}{2}$$

As a Random Turnout Model

a known fixed fraction of voters $0 \leq \iota \leq$ are independents drawn randomly from the two parties

fraction of voters lost to the independents for each party is $1-\iota$

total loss of voters is proportional to the size of the party, actual size of a party is given by $(1-\iota)\eta_k$.

party intends to bid $b_k = \eta_k \varphi_k$ taking account of independents the actual bid is $(1-\iota)b_k = (1-\iota)\eta_k\varphi_k$

fraction of independent voters that support party k uniform on $[0,1]$

$$p_k(b_k, b_{-k}) = \frac{\iota + (1-\iota)(b_k - b_{-k})}{2\iota}$$

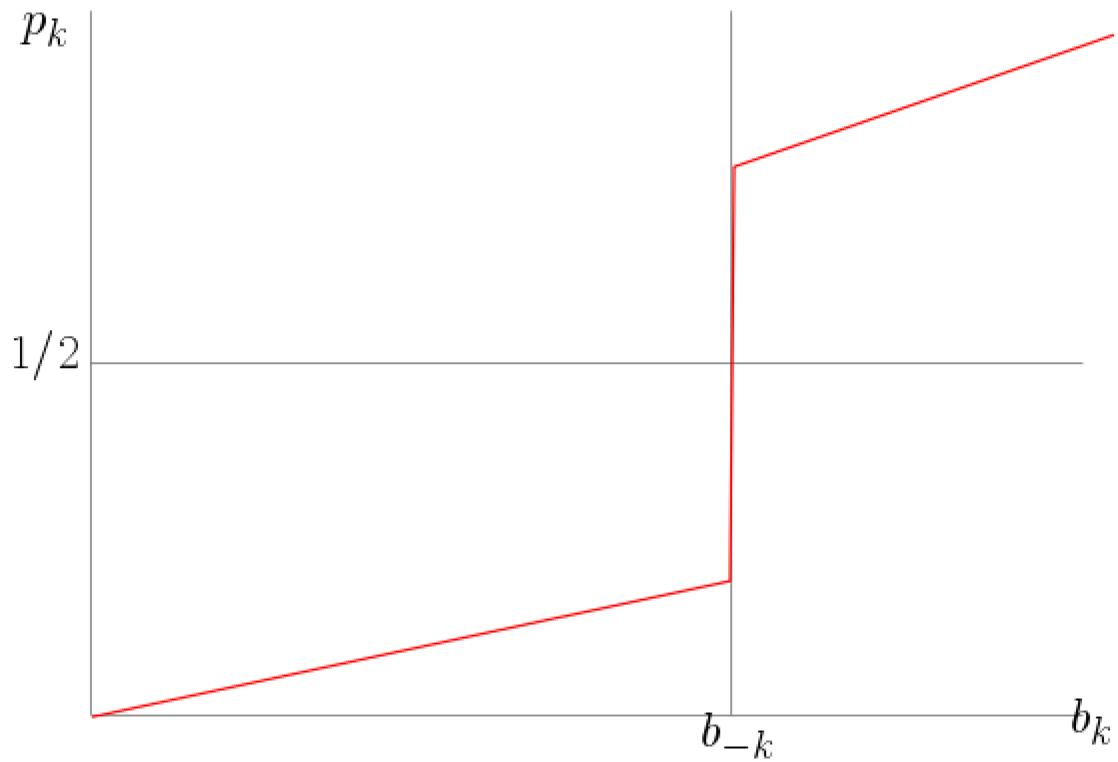
the model above is for $\iota = 1/2$

Mixed Model

probability p_0 that there are independents

- with probability $1 - p_0$ the election is decided by the greatest effort, that is, the all-pay auction model
- with probability p_0 the election is decided by the linear conflict resolution model, that is by the vote differential

let $\tilde{G}_{-k}$ denote the probability of winning schedule derived from opponent bidding schedule and the tie-breaking rule


p_k
$1/2$
b_{-k}
b_k

Group Objective Function

$$\left((1-p_0)\widetilde{G}(b_k|b_{-k}) + p_0 \tfrac{1+b_k-b_{-k}}{2} \right) V - (1-p_0/2)\max\{0, b_k - \eta_k \underline{y}\}$$

in the relevant range $b_k \geq \eta_k \underline{y}$ this may be written as

$$\widetilde{G}(b_k|b_{-k})(1-p_0)V - (b_k - \eta_k \underline{y})(1 - p_0(V+1)/2) + p_0(1+\eta_k \underline{y} - b_{-k})V/2$$

all pay auction with prize $(1-p_0)V$ and marginal cost $(1-p_0(V+1)/2)$

the key point: if V is big enough this is negative, turn out all voters

Bimodality

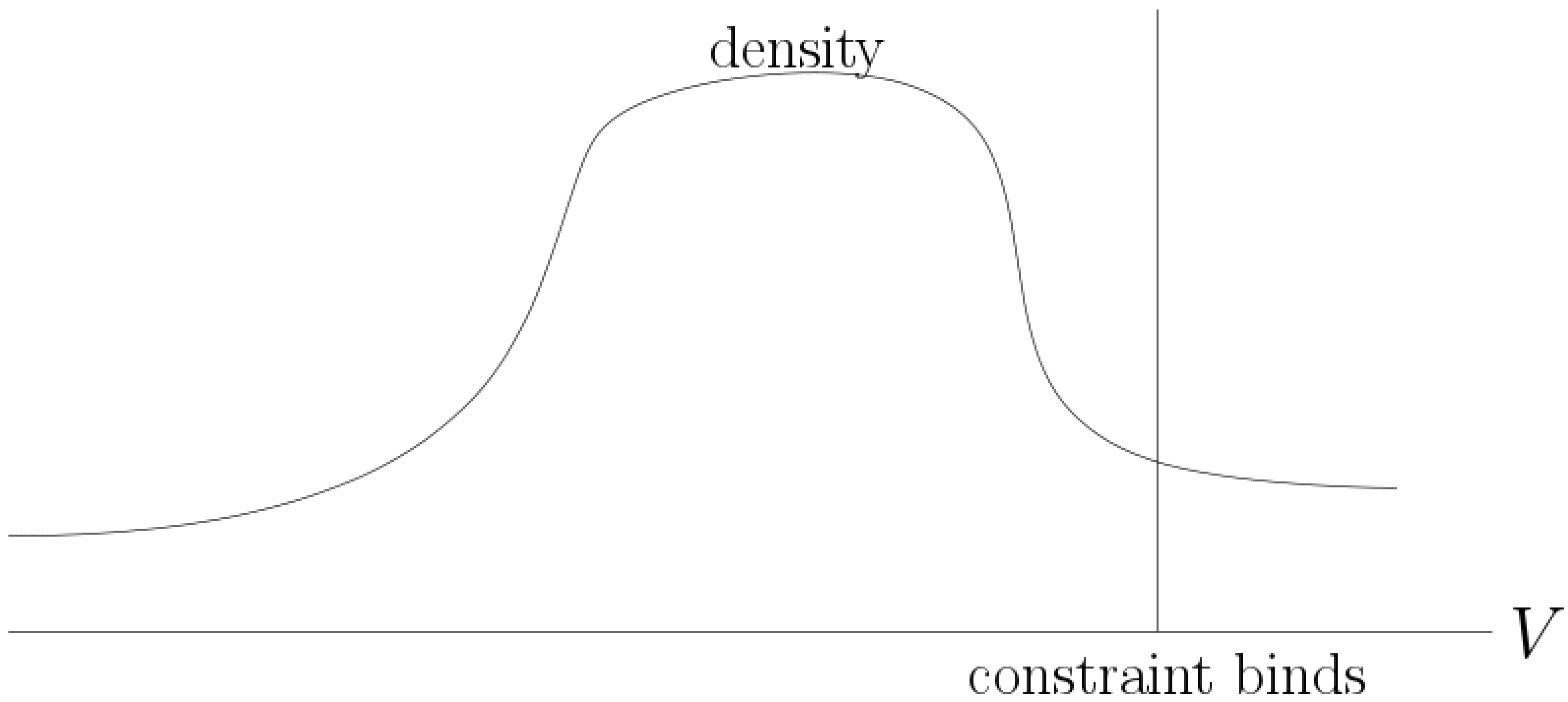
- the key point: if V is big enough this is negative, turn out all voters
- think of comparing elections with different values of V

bimodality

- low V small concedes large takes – low turnout
- high V everyone turns out – high turnout
- not so much in between

Bimodality in Tullock

due to truncation at the top



The Data

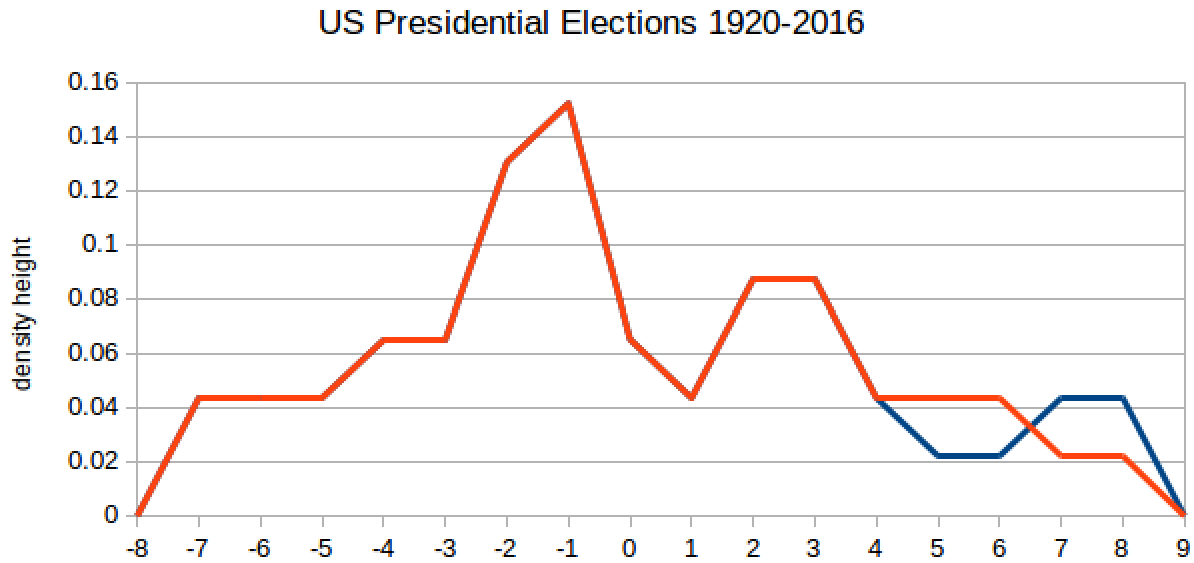
US Presidential and UK General elections since women got the vote in each country

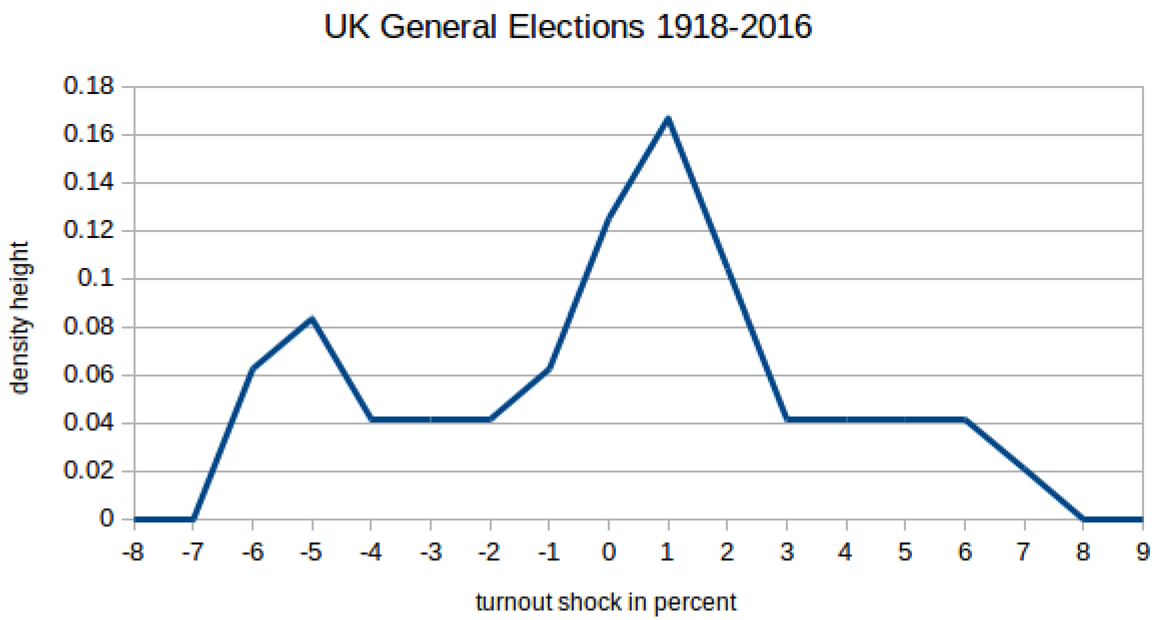
simple Hodrick-Prescott filter to take out slow moving components $z_t = Z_t - (\lambda Z_{t-1} + (1 - \lambda Z_{t-2}))$ chosen to be "round numbers" such that serial correlation is low

Country	λ	lag regression ρ
US	0.5	.06
UK	0.75	.08

Estimation of the Density

- in percent use kernel estimation with uniform and bandwidth equal to 1% to estimate the density
- in practical terms if q is an integer representing a percent take all the observations between $q-1$ and $q+1$ inclusive
- normalize to get a density function


US Presidential Elections 1920-2016
density height
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9


UK General Elections 1918-2016
density height
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
turnout shock in percent

Overall Turnout

US average: 55%

UK average: 73%