Peer Discipline and the Strength of Organizations

David K. Levine and Salvatore Modica

Introduction

- Groups do not act as individuals
- Olson and others have emphasized incentives within groups matter
- Not so much formal research on the subject, especially on the internal working of group discipline
- Group strength depends on including size and cohesion of the group.
- We study self-sustaining discipline through a model of costly peer auditing and punishment in a collusive group
- This punishments should be self-enforcing, so there must be an infinite sequence of audit rounds
- Initial choice of action by group members in a base game followed by an open-ended game of peer punishment

The Discipline Model

$N > 2$ identical players $i = 1, \ldots, N$ in group

initial round - round zero, players choose primitive actions $a^i \in A$

$a^R \in A$ action of representative member, player i gets payoffs $u(a^i, a^R)$

this initial primitive round is followed by an infinite sequence of possible audit rounds where players are assigned to audit other players

auditors receive signals of the auditee's behavior in the previous round only

based on the signal auditors may assign additively separable punishments

there is no discounting, but the game may be (randomly) ended

in effect the discount factor is a design parameter

it is not desirable to let the audits continue with too high a probability, since the punishments cumulate (Hatfields and McCoys)

Signals

behavior in the primitive round generates a binary good/bad signal $z_0^i \in \{0, 1\}$ with probability of a bad signal 0 equal to $\pi_0(a^i, a^R)$

(non-binary signals also considered)

obviously the signal should provide some information about whether a player deviated if it is to be useful – this is called enforceability and the paper gives the relevant criteria

Audit Rounds

$t = 1, 2, \ldots$

players matched in pairs as *auditor* i and *auditee* j*,* matches may be *active* or *inactive*

if $i = j$ match *inactive*

- current auditee an auditor in inactive match in previous round, current match inactive
- remaining matches are active

round $t \geq 1$ in an active match auditor i assigned to audit j observes signal $z_{t-1}^{j} \in \{0, 1\}$ of the behavior of the auditee and has two choices recommend punishment (1) or not to recommend punishment (0),

based on a member i's behavior as auditor in an active match at t signal $z_t^i \in \{0, 1\}$ generated

punish on bad signal or not on good signal, bad signal with probability π else with probability $\pi^p \geq \pi$

Costs and Punishments

Payoffs additively separable between initial primitive utilities and costs incurred or imposed during auditing

No discounting

Following a recommendation of punishment a punishment is imposed.

Auditor i suffers a utility loss of C_t^i auditee suffers a utility loss of $P_t^i \geq 0$ other $N-2$ members of the group share a utility loss of C_t^{iG}

Implementations

procedure for matching and a profile of punishment costs

note that “all matches inactive” means that *de facto* the audit rounds are over

matching is “exogenous” may depend randomly on history of previous matchings and punishment profiles but not on private signals or punishment recommendations

auditor does not need to worry that his future matchings will depend on what he does

Peer Discipline Equilibrium

pure strategy perfect public Nash equilibrium in which all players follow the strategy of punishing on the bad signal and not punishing on the good signal

we are interested in collusive groups, so are interested in the peer discipline equilibrium that supports a particular first period primitive action a^R and minimizes enforcement costs

Two-Stage Implementation

beginning of the first audit round - or equivalently at the end of the initial primitive round - the probability of the game continuing to the first audit round $0 < \delta_0 < 1$

beginning of the second audit round and in all subsequent rounds the continuation probability is $0 < \delta < 1$

matchings are symmetric

punishments are fixed constants P_1, C_1, C_1^G

$C_1 + C_1^G + P_1 \geq 0$ so that there is no net benefit to the group from carrying out a punishment

The Gain Function

$$\tilde{G}(a^i, a^R) = \frac{u(a^i, a^R) - u(a^R, a^R)}{\pi_0(a^i, a^R) - \pi_0(a^R, a^R)}.$$

$$G^R = \max_{a^i} \widetilde{G}(a^i, a^R)$$

Optimal Punishment Plans

Theorem: *Utility of a representative group member is maximized given the non static-Nash enforceable initial action a^R when the incentive constraints hold with equality. Specifically letting G^R denote the maximum gain to deviating from a^R this occurs when*

$\delta_0 = \frac{G^R}{P_1}$, $\delta = \frac{|C_1|}{[\pi^p - \pi] P_1}$. *If $\delta_0 \leq 1, \delta < 1$ the equilibrium utility level is*

$$u^R - \left[\pi_0^R + \frac{|C_1|/P_1}{[\pi^p - \pi] - |C_1|/P_1} \pi \right] \frac{C_1 + C_1^G + P_1}{P_1} G^R.$$

Ratios and Robustness

In ratios equilibrium utility level is

$$u^R - \left[\pi_0^R + \frac{|\theta|}{[\pi^p - \pi] - |\theta|}\pi\right] (1 + \theta + \psi)\, G^R.$$

and condition for existence is $|\theta| \leq \pi^p - \pi$

This theorem is robust to matching and ending procedures and punishment profiles that are linearly scalable

Application: Group Size and the Strength of Groups

- group members provide indivisible effort to purchase a political favor
- willingness to pay: single-peaked in group size
- competition between groups in an auction
- agenda setting

Group Size and the Strength of Groups

- What determines strength of a group?
- Simple measure of group effectiveness: ability to mobilize resources
- Examine willingness to pay
- Group might be attempting to corrupt a politician or could be a consortium bidding on a contract.

Structure of the Model

- maintain the assumption of a linear feasible set and $|\theta| < \Delta\pi$ so peer punishment
- is feasible
- linear cost of effort and prize worth S divided equally among the group, each
- group member getting $s = S/N$
- how much effort is the group willing to provide to get the prize?
- use Becker-DeGroot-Marschak (BDM) elicitation procedure
- basically a second price auction

Bidding

- bid is a commitment to an implementation and basic actions that are incentive compatible with respect to that implementation
- includes also the possibility of not using peer discipline
- effort provided only after the bid is accepted
- (otherwise the situation one of an all-pay auction)
- lobbyist goes to a politician and says “my group will provide so many campaign contributions and provide so many volunteers in your next election if you provide us with S.”

Divisibility of Effort

- strategic difference
- effort divisible, everyone can contribute equally a small amount, and it is relatively easy to monitor whether individuals made the agreed upon contribution
- as practical matter effort is not indivisible: lobbying, protesting, bribing and so forth require overhead cost of thinking about and organizing oneself to participate
- not feasible to spend two minutes a year contributing to a group effort in an effective way
- hence focus on the case where each group member j can provide either 0 or 1 unit of effort: $e^j \in \{0, 1\}$.

Coordination of Effort

- With indivisible effort to bid B group should appoint subset of B members each to provide an effort level of 1
- coordinated through messaging technology
- group sends messages to individuals indicating whether they are expected to contribute:
- each individual j receives an independent signal $\sigma^j = \{0, 1\}$ of whether or not to provide effort
- $\mu \equiv \text{pr}(\sigma^j = 1) = B/N.$
- actual effort level that will be provided is random but bid is evaluated according to the expected value

BDM mechanism

chooses a random number $\tilde{B}$

if $\tilde{B} \leq B$ the bid is accepted

when the bid is accepted $\tilde{B}$ is a *floor* on effort

Monitoring

- auditors can tell whether or not the auditee has contributed effort, but observe whether or not they received a signal with noise
- observable whether or not auditee turned up at the rally, but if he did not, he may say “I never got the phone call” and auditor cannot perfectly determine the truth of this.

Specifically in first audit round auditor i observes auditee j's

- action
- a signal $s^j \in \{0, 1\}$ which is equal to σ^j with probability $1 - \epsilon$ and to the opposite $1 - \sigma^j$ with probability $\epsilon \leq 1/2$

Enforceability and Signal Compression

- auditor observes a pair $(e^j, s^j)$where e^j is the effort provided by the auditee and $s^j$is the auditors garbled version of the signal received by the auditee.
- hence four - rather than two - possible values of the signal
- However: in general if enforceability ialways possible using randomization to reduce a multi-value signal to a binary signal without consequence for the cost of punishment or incentives

The Problem

four possible signal combinations $(e^j, s^j) = (1,1), (0,1), (1,0), (0,0)$

four possible punishments $P_{11}, P_{01}, P_{10}, P_{00}$

$P_1 = \max\{P_{00}, P_{01}, P_{10}, P_{11}\}$ and define

$P_{00} = \beta_{00} P_1, P_{01} = \beta_{01} P_1, P_{10} = \beta_{10} P_1, P_{11} = \beta_{11} P_1$.

β's may be interpreted as probabilities with one corresponding to the highest P is equal to 1

β's chosen to maximize

$$s - \mu - [\pi_0(01, 01) + \Pi](\theta + \psi + 1)G^R$$

where G^R is determined by all the π_0's

Signal Compression Theorem

Theorem: If $\Pi \leq \frac{(1-\mu)(1-2\epsilon)}{\epsilon}$ group utility maximization implies

$$\beta_{00} = \beta_{10} = \beta_{11} = 0, \beta_{01} = 1$$

Otherwise

$$\beta_{10} = \beta_{11} = 0 \text{ and } \beta_{00} = \beta_{01} = 1.$$

Per capita group utility is equal to

$$s - \mu - (\theta + \psi + 1) \cdot \min\{\frac{(1-\mu)\epsilon + \Pi}{1-\epsilon}, (1-\mu) + \Pi\}$$

A Surprise: Groups really are different

- group utility may be increasing in μ - a higher level of effort may be preferred to a lower
- in per capita group utility effort level μ has two effects

  as μ goes up everyone has to contribute a greater amount of expected effort

  as μ goes up the cost of punishing the basic actions is proportional to $1-\mu$, and this goes down

- consider the case $\mu = 1$ - then no cost of punishing the basic action: everyone is asked to contribute and punishment only occurs when there is a failure to contribute - which never happens on the equilibrium path
- when μ is smaller sometimes people are erroneously punished, with a corresponding social cost

Group Utility and the Group Bid

Group utility is increasing in μ if and only if $(\theta + \psi + 2)\epsilon \geq 1$

two cases

- cost of punishment is high and dominates the cost of effort
- effort level is high and noise ϵ is large, so that frequency of false signals is high

Subtle Implication

- Consider public policy to weaken or discourage a criminal gang or rent-seeking lobbying group
- raise the cost of punishment: gang punishes members by murdering them, so vigorously prosecute gang murders
- if the increased cost to the gang of peer discipline causes a shift from the regime in which utility is decreasing in μ to one in which utility is increasing in μ it could actually trigger an increase in gang activity

Willingness to Pay and Group Size

group bids an effort level $B = N\mu$ then group utility is

$$S - N\mu - N(\theta + \psi + 1)\min\left\{\frac{(1-\mu)\epsilon+\Pi}{1-\epsilon}, (1-\mu)+\Pi\right\}$$

alternative not to bid

group should bid the highest value of μ that gives positive utility, or else not bid at all.

The Optimal Bid Function

The bid function is single peaked as a function of N. Define

$$\hat{N} = \frac{S}{1+(\theta+\psi+1)\Pi} \qquad \hat{\hat{N}} = \frac{S}{(\theta+\psi+1)\frac{\epsilon+\Pi}{1-\epsilon}}$$

For $N \leq \hat{N}$ the bid is N. If $\epsilon(\theta + \psi + 2) \leq 1$ and $(\theta + \psi + 1)\frac{\epsilon(1+\Pi)}{1-\epsilon} < 1$

then $\hat{\hat{N}} > \hat{N}$ and for $\hat{N} \leq N \leq \hat{\hat{N}}$ the bid is given by the following function, decreasing in N:

$$\tilde{B} \equiv \frac{(1-\epsilon)S - N\left[(\epsilon+\Pi)(\theta+\psi+1)\right]}{1-\epsilon(\theta+\psi+2)}$$

In all other cases the group does not bid.

Implications

- “optimal” group size $\hat{N}$ increases linearly with the size of the prize S but
- at a rate that is less than 1. Moreover $\hat{N}$ falls as the cost of peer punishment $\Pi(\psi + \theta + 1)$ rises
- Remark on scaling and farm lobbies in countries of different sizes
- compare to voluntary public goods contribution models

Agenda Setting: Endogenous Prizes

- two groups of size $N_1 < N_2$ competing for a prize in a second price auction
- prize a transfer payment between the groups
- who chooses the size of the prize? small group, large, or seller (politician)

- agenda setter will determine two things: which group j will pay for the transfer, and how large the transfer - the prize S - is
- if a group is setting the agenda they obviously pick the other group to pay
- for a given utility all groups lexicographically prefer a smaller prize to a larger one.
- in case of a tie in the bidding, we impose continuity requirement that the group that would win when the prize was slightly higher wins the tie
- low punishment cost case only

Agenda Setting Theorem

A transfer takes place only if the small group sets the agenda, in which case it sets the prize S to $S_2^L \equiv (\theta + \psi + 1)\frac{\epsilon+\Pi}{1-\epsilon}N_2$. The small group bids a positive amount and pays zero; the large group pays $\min\{S_2^L, N_2\}$ to the small group.

If the large group sets the agenda, it sets the prize equal to zero

The winning group pays a positive amount only if the politician sets the agenda, in which case she chooses the large group to pay for the prize, which she sets equal to

$$\hat{S} \equiv \frac{N_1[1-\epsilon(\theta+\psi+2)]+N_2\Big[(\epsilon+\Pi)(\theta+\psi+1)\Big]}{1-\epsilon}$$

Both groups bid N_1 and the large group wins the bidding, so there is no transfer between the groups but simply a payment by the large group of N_1 to the politician.