Peer Discipline and the Strength of Organizations

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Introduction

- Groups do not act as individuals
- Olson and others have emphasized incentives within groups matter
- Not so much formal research on the subject, especially on the internal working of group discipline
- Group strength depends on including size and cohesion of the group.
- Study self-sustaining discipline through a model of costly peer punishment
- Examine schemes a collusive group can use to minimize cost of enforcing particular actions

Overview

- Initial choice of action by group members in a base game followed by an open-ended game of peer punishment
- calculation of equilibria of this game similar to strongly symmetric computation Abreu, Pearce and Stachetti, avoids complications of Fudenberg, Levine and Maskin and Sugaya
- Closest to efficiency wage model and Laffont 1999, but these only have one round and no cost of punishment on the equilibrium path
- Other literature has focused on a reduced form relationship between group characteristics and strength, or a model of voluntary public goods contribution
- We point out that the voluntary contribution model has scaling problems

Results

- Use the model to analyze willingness to pay and competition between two groups in a second price auction
- With non-rival goods strength increases with size
- With a fixed prize strength increases to a maximum then declines
- Homogeneous groups are stronger than heterogeneous
- With competing groups strongest group is closest to “optimal size”
- Various inefficiencies
- Agenda setting: small group and seller can exploit agenda setting power, large group cannot
- Model has quantitative predictions suitable for laboratory study

The Discipline Model

$N > 2$ identical players $i = 1, \ldots, N$ in group

unlimited number of rounds

initial round - round zero

group members choose primitive actions $a^i \in A$

$a^R \in A$ action of representative member

player i gets payoffs $u(a^i, a^R)$

generates a binary good/bad signal $z_0^i \in \{0, 1\}$ with probability of a bad signal 0 equal to $\pi_0(a^i, a^R)$

non-binary signals later

Audit Rounds

$t = 1, 2, \ldots$ commences

players matched in pairs as *auditor* i and *auditee* j

matches may be *active* or *inactive*

if $i = j$ match *inactive*

- current auditee an auditor in inactive match in previous round, current match inactive
- remaining matches are active

round $t \geq 1$ in an active match auditor i assigned to audit j observes signal $z^j_{t-1} \in \{0, 1\}$ of the behavior of the auditee and has two choices recommend punishment (1) or not to recommend punishment (0),

based on a member i's behavior as auditor at t signal $z^i_t \in \{0, 1\}$ generated

punish on bad signal or not on good signal, bad signal with probability π else with probability $\pi^p \geq \pi$

Costs and Punishments

Payoffs additively separable between initial primitive utilities and costs incurred or imposed during auditing

No discounting

Following a recommendation of punishment a punishment is imposed.

Auditor i suffers a utility loss of C_t^i auditee suffers a utility loss of $P_t^i \geq 0$ other $N-2$ members of the group share a utility loss of C_t^{iG}

Implementations

procedure for matching and a profile of punishment costs

matching is “exogenous” may depend randomly on history of previous matchings and punishment profiles but not on private signals or punishment recommendations

auditor does not need to worry that his future matchings will depend on what he does

Equilibrium

pure strategy: an initial action and subsequently choices of signal dependent punishment recommendations, depends in general on public and private history

public strategy: depends only on current signal and history of matchings and punishment profiles

Nash equilibrium as usual

peer discipline equilibrium: Nash equilibrium in which all players follow the strategy of punishing on the bad signal and not punishing on the good signal

a^R *incentive compatible in the implementation* if there is a peer discipline equilibrium with a^R as common initial action

The Gain Function

a^R *enforceable* if there is some punishment scheme based on the signal such that a^R is incentive compatible

$\sigma(a^i, a^R) \equiv \mathrm{sgn}(\pi_0(a^i, a^R) - \pi_0(a^R, a^R))$

The gain function

if $u(a^i, a^R) = u(a^R, a^R)$ then $\tilde{G}(a^i, a^R) = 0$ otherwise:

if $\sigma(a^i, a^R) = 0$ then $\tilde{G}(a^i, a^R) = \mathrm{sgn}(u(a^i, a^R) - u(a^R, a^R)) \cdot \infty$

if $\sigma(a^i, a^R) \neq 0$ then

$$\tilde{G}(a^i, a^R) = \frac{u(a^i, a^R) - u(a^R, a^R)}{\pi_0(a^i, a^R) - \pi_0(a^R, a^R)}.$$

$$G(a^R) \equiv \max_{\sigma(a^i, a^R) \geq 0} \tilde{G}(a^i, a^R)$$

Enforceability

$G^{-}(a^R) \equiv \min_{\sigma(a^i,a^R)<0} \tilde{G}(a^i,a^R)$ or $+\infty$

Lemma: *The group action a^R is enforceable with the punishment $P_1 \geq 0$ if and only if $\max\{0, G(a^R)\} \leq P_1 \leq G^{-}(a^R)$ hence it is enforceable if and only if $\max\{0, G(a^R)\} \leq G^{-}(a^R)$.*

Two-Stage Implementation

beginning of the first audit round - or equivalently at the end of the initial primitive round - the probability of the game continuing to the first audit round $0 < \delta_0 < 1$

beginning of the second audit round and in all subsequent rounds the continuation probability is $0 < \delta < 1$

matchings are symmetric

punishments are fixed constants P_1, C_1, C_1^G

$C_1 + C_1^G + P_1 \geq 0$ so that there is no net benefit to the group from carrying out a punishment

Characterization of Equilibrium in the Two-Stage Implementation

Theorem: *If the action a^R is not static Nash it can be incentive compatible in the two-stage implementation only if a^R is enforceable and $P_1 \geq \max\{G^R, |C_1|/[\pi^p - \pi]\}$. In this case, a^R is incentive compatible if and only if*

$$\delta_0 P_1 \geq G^R\,,\ \delta P_1 \geq |C_1|/[\pi^p - \pi].$$

The resulting equilibrium utility level is

$$u^R - \left[\delta_0 \pi_0^R + (\delta_0 \delta/(1-\delta))\pi\right](C_1 + C_1^G + P_1).$$

Optimal Punishment Plans

Theorem: *Utility of a representative group member is maximized given the non static-Nash enforceable initial action a^R when the incentive constraints hold with equality, that is*

$\delta_0 = \frac{G^R}{P_1}$, $\delta = \frac{|C_1|}{[\pi^p - \pi] P_1}$ *The equilibrium utility level is*

$$u^R - \left[\pi_0^R + \frac{|C_1|/P_1}{[\pi^p - \pi] - |C_1|/P_1} \pi\right] \frac{C_1 + C_1^G + P_1}{P_1} G^R.$$

In ratios

$$u^R - \left[\pi_0^R + \frac{|\theta|}{[\pi^p - \pi] - |\theta|} \pi\right] (1 + \theta + \psi) \, G^R.$$

and condition for existence is $|\theta| \leq \pi^p - \pi$

This theorem is robust to matching and ending procedures and punishment profiles that are linearly scalable

Cost of Peer Punishment

$$\kappa \equiv \left[\frac{|\theta|}{[\pi^{p} - \pi] - |\theta|} \pi \right] (\theta + \psi + 1)$$

$$u^R - \left[\pi_0^R (\theta + \psi + 1) + \kappa \right] G^R.$$

Group Size and the Strength of Groups

- a prize worth S that will be divided equally among the group, each group member getting a benefit of $s = S/N$
- how much effort is the group willing to provide to get the prize?
- use a Becker-DeGroot-Marschak (BDM) elicitation procedure
- bid a commitment to an implementation and basic actions that are incentive compatible with respect to that implementation
- effort is provided only after the bid is accepted
- linear cost of effort

Indivisible Effort

each group member j can provide either 0 or 1 unit of effort: $e^j \in \{0, 1\}$

to provide B group appoints a subset of B members each to provide an effort level of 1

uses a messaging technology

each individual j receives an independent signal $\sigma^j = \{0, 1\}$ of whether or not to provide effort, where $\mu \equiv \mathrm{pr}(\sigma^j = 1) = B/N$

bid evaluated by expected effort level

Imperfect Monitoring

auditors can tell whether or not the auditee has contributed effort, but observe whether or not they received a signal with noise

in the first audit round auditor i observes auditee j's action and a signal $s^j \in \{0,1\}$ which is equal to σ^j with probability $1-\epsilon$ and to the opposite $1-\sigma^j$ with probability $\epsilon \leq 1/2$.

Basic Strategies

four possible strategies.

01: contribute on $\sigma = 1$, do not contribute on 0

00: never contribute

11: always contribute

10: contribute on $\sigma = 0$, do not contribute on 1

interested in the enforceability of 01.

Enforceability and Signal Compression

four possible signal combinations $(e^j, s^j) = (1,1), (0,1), (1,0), (0,0)$ so four possible punishments $P_{11}, P_{01}, P_{10}, P_{00}$

can always use binary signal (random function of underlying signal)

in this case: it is optimal to punish only when $(e^j, s^j) = (0,1)$: when no contribution and a signal indicates that contribution should have taken place

Willingness to Pay and Group Size

willingness to pay by the group is computed from

$$N\mu = S - N\left[(1-\mu)\epsilon(\theta+\psi+1)+\kappa\right]\frac{1}{1-\epsilon}$$

Small cost corollary: *Fix S and assume $\epsilon(\theta+\psi+2)<1$. Then the group's bid is single peaked as a function of N. Precisely, the group bids N for $N \le \hat{N}$ where $\hat{N} \equiv \frac{1-\epsilon}{1-\epsilon+\kappa}S$. For larger N the bid is*

$$\tilde{B} \equiv \frac{(1-\epsilon)S - N\left[\epsilon(\theta+\psi+1)+\kappa\right]}{1-\epsilon(\theta+\psi+2)}$$

which decreases with N and becomes zero for all $N \ge \hat{\hat{N}} \equiv (1-\epsilon)S/[\epsilon(\theta+\psi+1)+\kappa]$.

High Cost Case

$\epsilon(\theta+\psi+2)>1$ then per capita group utility is *increasing* in μ

$s-\mu-[(1-\mu)\epsilon(\theta+\psi+1)+\kappa]\,\frac{1}{1-\epsilon}$

μ has two effects

as μ goes up everyone has to contribute a greater amount of expected effort

as μ goes up the cost of punishing the basic actions $(1-\mu)\epsilon(\theta+\psi+1)$ goes down

consider the case $\mu=1$: no cost of punishing the basic action

why? everyone asked to contribute; punishment only occurs when there is a failure to contribute and a signal indicating that contribution should have taken place. This never happens on the equilibrium path

High Cost Result

High cost corollary: *Fix S and assume $\epsilon(\theta + \psi + 2) > 1$. The group's bid is again single-peaked with same highest-bidding size as in the previous case. In the present case the group bids N for $N \leq \hat{N}$ and zero for larger N.*

Voluntary Contributions With Prizes That Can Be Withdrawn

realized group effort level $\tilde{E}$ is a noisy signal of intended group effort μN

withdraw the prize based on $\tilde{E}$ a voluntary contribution mechanism may be used to provide an incentive for a positive level of contributions

realized effort level $\tilde{E}$ follows a binomial with parameters μ as success probability and N as number of trials

bid includes a threshold $\hat{\mu}$ with the agreement that if effort level $\tilde{E}$ falls below $N\hat{\mu}$ the prize will be withdrawn.

Must have $s \geq 1$ or nobody would provide effort

Full Effort

$\mu = 1$ and $\hat{\mu} = 1$

everyone is decisive so we have incentive compatibility

no noise in the aggregate statistic $\$\tilde{E}$

instead assume an upper bound $\overline{\mu} < 1$, so can't send noise free signals

Theorem: *For all s and $1 > \overline{\mu} > \underline{\mu} > 0$ there exists an $\overline{N}$ such that $N > \overline{N}$ implies that any incentive compatible $\mu < \underline{\mu}$.*

Basically Fudenberg, Levine and Pesendorfer

Note: can always get some donation by picking a single person.

This does not scale properly!

Altruism (that makes sense) doesn't help

U.S. there are about 3 million farmers and 2 million farms

Competing Groups

two groups, second price auction, groups identical except in size, identical value prize, low punishment cost case $\epsilon(\theta + \psi + 2) < 1$

if $N_1 \geq \hat{\hat{N}}$ both groups bid zero, otherwise

if $N_1 \geq \hat{N}$ or $N_2 \geq \hat{\hat{N}}$ the small group wins

if $N_2 \leq \hat{N}$ the large group wins

if $N_1 < \hat{N} < N_2 \leq \hat{\hat{N}}$ there are cases where either group may win

Agenda Setting: Endogenous Prizes

one group or the seller chooses the size of the prize and which group pays for the prize with the constraint that the group must be able to afford it: $S \leq N_j$

for a given utility all groups lexicographically prefer a smaller prize to a larger one

tie in the bidding determined by continuity requirement: group that would win when the prize was slightly higher wins the tie

Agenda Result

Theorem: *A transfer takes place only if the small group sets the agenda, in which case it sets the prize S to*

$$S_2^L \equiv \frac{\epsilon(\theta+\psi+1)+\kappa}{1-\epsilon} N_2,$$

bids a positive amount and pays zero; the large group pays $\min\{S_2^L, N_2\}$ to the small group. If the large group sets the agenda, it sets the prize equal to zero. The winning group pays a positive amount only if the politician sets the agenda, in which case she chooses the large group to pay for the prize, which she sets equal to

$$\hat{S} \equiv \frac{N_1[1-\epsilon(\theta+\psi+2)]+N_2\left[\epsilon(\theta+\psi+1)+\kappa\right]}{1-\epsilon}$$

Both groups bid N_1 and the large group wins the bidding, so there is no transfer between the groups but simply a payment by the large group of N_1 to the politician.