Variable-Lived Short-Run Selves

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The Problem

- Models of long-run planning and short-run impulsive selves provide a quantitative explanation of a wide variety of “behavioral” paradoxes, including the Rabin paradox, the Allais paradox, preferences for commitment in menu choice, hyperbolic discounting, the effect of cognitive load on decision making and reversals due to probabilistic rewards
- These models have a fixed horizon for the short-run self that cannot explain overwhelming evidence that delay impacts decisions continuously
- We introduce a model of short-run selves who live a random length of time as a method of maintaining the underlying strength and simplicity of the long-run/short-run self model while accounting for the continuous effect of delay

The Model

periods are discrete and unbounded, $k = 1, 2, \ldots$.

fixed, period and history invariant set of actions A for the short-run selves

a measure space Y of states

a set R of self-control actions for the long-run self, $0 \in R$ means no self-control is used

A, Y, R closed subsets of Euclidean space

finite history of play $h \in H$ of the past states and actions, $h = (y_1, a_1, r_1, \ldots, y_k, a_k, r_k)$ plus the null history 0

H_t the set of k-length histories H_k, length of the history $k(h)$, final state in h is $y(h)$, initial state y_1

probability distribution over states at $k+1$ depends on period-k state and action y_k, a_k by stochastic kernel $\mu(y, a)$

note that the long-run self's action r has no effect on states game is between long-run self with strategies $\sigma_{LR} : H \times Y \to R$, and sequence of short-run selves

period k short-run self plays in only one period, observes self-control action of long-run self prior to moving; uses strategy $\sigma_t : H_t \times Y \times R \to A$

collection of one for each SR is denoted σ_{SR}

for every measurable subset $R' \subseteq \mathbf{R}, A' \subseteq \mathbf{A}$ the functions $\sigma_{LR}(\cdot,\cdot)[A'], \sigma_k(\cdot,\cdot,\cdot)[R']$ are measurable

strategies together with measure μ give rise to a measure π_k over length k histories

utility of the short-run self is $u(y,r,a)$: long-run player's self-control action influences the short-run player's payoff

the long-run self is completely benevolent

$$U_{LR}(\sigma_{LR}, \sigma_{SR}) = \sum_{k=1}^{\infty} \delta^{k-1} \int u_k(h) d\pi_k(h)$$

Assumption 0 (Upper Bound on Utility Growth): For all initial conditions

$$\sum_{k=1}^{\infty} \delta^{k-1} \int \max\{0, u\,(h)\} d\pi_k(h) < \infty.$$

short-run self optimizes following every history: *SR-perfect*

interested in SR-perfect Nash equilibria

Assumption 1 (Costly Self-Control): If $r \neq 0$ then $u(y,r,a) < u(y,0,a)$.

Assumption 2 (Unlimited Self-Control): For all y,a there exists r such that for all a', $u(y,r,a) \geq u(y,r,a')$.

with these two assumptions we may define the cost of self-control

$$C(y,a) \equiv u(y,0,a) - \sup_{\{r|u(y,r,a)\geq u(y,r,\cdot)\}} u(y,r,a)$$

Assumption 3 (Continuity): $u(y,r,a)$ is continuous in r,a.

the supremum can be replaced with a maximum Assumptions 1 & 3 imply cost continuous and

Property 1: (Strict Cost of Self-Control) If $a \in \arg\max_{a'}(u(y,0,a'))$ then $C(y,a) = 0$, and $C(y,a) > 0$ for $a \notin \arg\max_{a'}(u(y,0,a'))$.

Assumption 4 (Limited Indifference): for all $a' \neq a$, if $u(y,r,a) \geq u(y,r,a')$ then there exists a sequence $r^n \to r$ such that $u(y,r^n,a) > u(y,r^n,a')$.

short-run self is indifferent, long-run self can break tie for negligible cost *reduced-form* optimization problem

$H^{AY} = \{(y_1, a_1, \ldots, y_k, a_k)\}_k$ *reduced* histories

problem of choosing a strategy from reduced histories and states to actions, $\sigma_{RF} : H^{AY} \times Y \to \mathbf{A}$, to maximize the objective function

$$U_{RF}(\sigma_{RF}) = \sum_{k=1}^{\infty} \delta^{k-1} \int [\, u(y(h), 0, a) - C(y(h), a)\,] \, d\sigma_{RF}(h, y(h))[a] \, d\pi_k(h)$$

Theorem 1 (Equivalence of Subgame Perfection to the Reduced Form): Under Assumptions 1-4, every SR-perfect Nash equilibrium profile is equivalent to a solution to the reduced form optimization problem and conversely.

Convex Opportunity Based Cost of Self Control

$$C(y,a) = g(\max_{a'} u(y,0,a') - u(y,0,a))$$

Fixed Short-Run Self Lives

Each short-run player lives for one period – in empirical work, usually 24 hours

This introduces a discontinuity between things that happen in the next 24 hours and everything that happens afterwards

Hyperbolic Discounting Data

from Myerson and Green [1995]

months	Interest rate over $1000 right now
0.23	132
1	93.6
6	49.7
12	46.2
36	32.7
60	22.8
120	16.1
300	10.4

Random Short-Run Self Lives/Period Length

Period k lasts a random length of time τ

For notational simplicity, suppose that τ takes on integer values called "days"

Each day, chance μ current short-run self continues for another day, and chance $1-\mu$ replaced by ifferent short-run self

kth SR self born at the random time $t(k) \geq k$ in state Y he has lifetime utility

$$Q_{kt} = \sum_{\tau=0}^{\infty} (\delta\mu)^{\tau} Ev(Y_{\tau}, r_{\tau}, a_{\tau}).$$

- ♦ death of a SR self is an observable event that LR can condition on
- ♦ LR can commit to plan for lifetime of the current SR self
- ♦ Likewise, SR self can commit to a survival contingent plan for his lifetime

Reformulation in the Original Framework

Define the state $y_k = (Y_{t(k)}, \Delta_k)$

where $\Delta_1 = 1$ and $\Delta_k = \delta^{t(k-1)-k} \Delta_{k-1}$

Define utility $u((Y, \Delta), r, a) = \Delta v(Y, r, a)$

And everything works as before

The Temptation

$$\bar{Q}_{kt} = \max_{\{a_\tau\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} (\delta\mu)^{\tau} Ev(Y_\tau, 0, a_\tau)$$

The Reduced Form

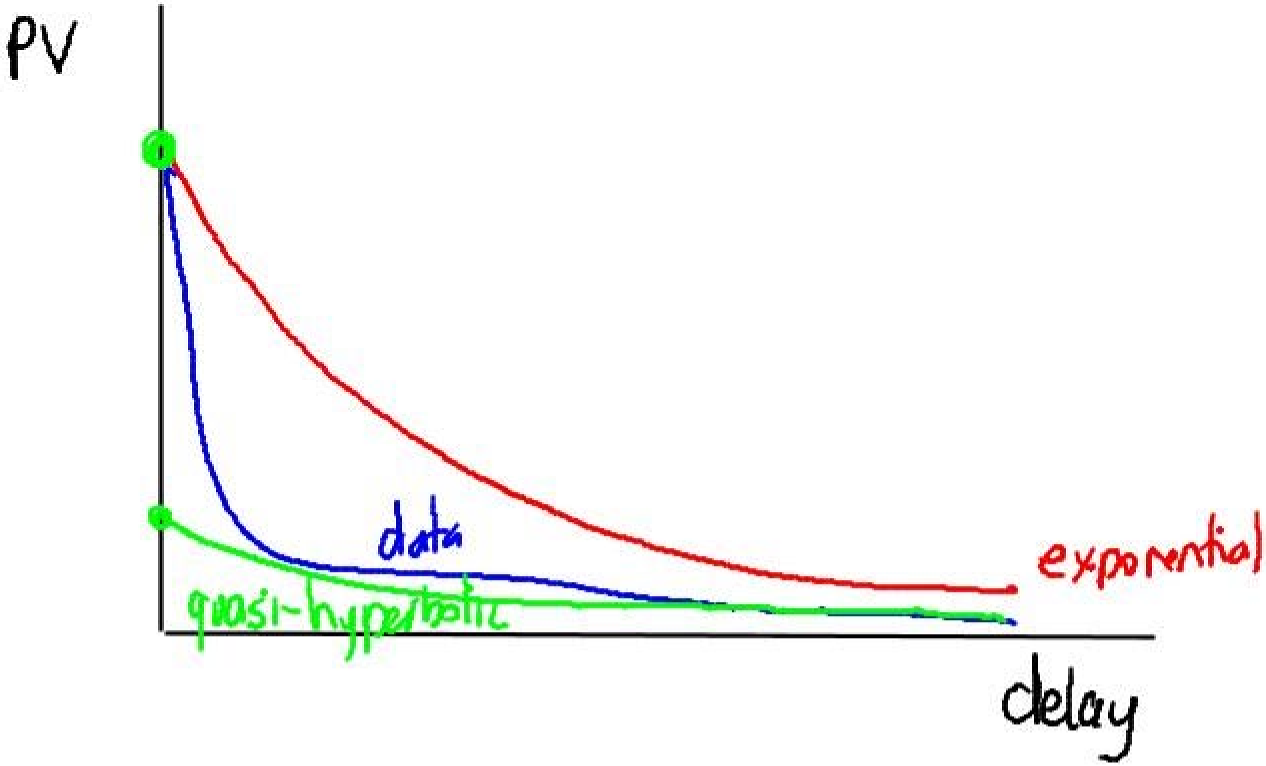
Let m_{kt} be the probability that k is born on day t

we can write the reduced form utility as

$$U_{RF}(\sigma_{RF}) =$$
$$\sum_{k=1}^{\infty}\sum_{t\geq k}\delta^{t-1}\left(Q_{kt} - g(\bar{Q}_{kt} - Q_{kt})\right)m_{kt}$$

Comments

- ♦ The model nests quasi hyperbolic discounting $\mu = 0$
- ♦ and geometric discounting $\lim \mu \to 1$
- ♦ and is well behaved in the continuous time limit



by continuity intermediate values of μ should better fit the data

Commitment versus Self-Control

the state Y is wealth $w \in \Re_+$

the action a is a level of consumption $c \in \Re_+$

utility is $v(y, a) = \log(c)$

$w_{t+1} = R(w_t - c_{kt})$

for simplicity all income discounted into wealth

commitment takes the form of mental accounting allocating a sequence of "pocket cash" limits x_{kt} that constrain the SR self according to

$c_{kt} \leq x_{kt}$

less flexible than self-control, but avoids self-control cost

Perfect Foresight Case

by stationarity the value function when new SR is born depends only on wealth: $V(w_t)$ so the Bellman equation is

$$V(w_t) =$$

$$\max_{\{c_t\}} \sum_{\tau=t}^{\infty} (\delta\mu)^{\tau-t} \log(c_\tau) + \sum_{\tau=t}^{\infty} \delta^{\tau-t+1}(1-\mu)(\mu)^{\tau-t} V(w_{\tau+1})$$

Theorem: *the solution is* $c_t = (1-\delta)w_t$ *independent of which SR self shows up*

this can be implemented by choosing $x_{kt} = (1-\delta)w_t$

$$V(w_t) = \frac{\log(w_t)}{1-\delta} + \left\{ \frac{\log(1-\delta)}{1-\delta} + \frac{\delta \log(R\delta)}{(1-\delta)2} \right\}$$

Unanticipated Initial Period Opportunity

Choose today between z_0 right away and z_t at time t

$$\max_{\{c_\tau\}} C - g\Big(\bar{Q}_0 - \sum\nolimits_{\tau=0}^{\infty} (\delta\mu)^\tau \log(c_\tau)\Big)$$
$$+\sum\nolimits_{\tau=0}^{\infty} (\delta\mu)^\tau \log(c_\tau)$$
$$+\sum\nolimits_{\tau=0}^{\infty} \delta^{\tau+1}(1-\mu)(\mu)^\tau \frac{\log(w_{\tau+1})}{1-\delta}$$

where

$$C = \delta(1-\mu)\frac{B}{1-\delta\mu}$$

$$x_\tau = (1-\delta)(R\delta)^\tau w_0.$$

at time 0 learned that an amount z_t is to be received at time t

after the amount is received, R self may save some of it

a sequence $Z_t \geq 0$ of net increments to pocket cash

$$\begin{aligned} &Z_\tau = 0 \quad \tau < t \\ &\sum_{\tau=t}^{\infty} R^{t-\tau} Z_\tau \leq z_t \end{aligned} \qquad \text{(*)}$$

the temptation of the SR self is

$\bar{Q}_0 = \max_{\{c_\tau, Z_\tau\}} \sum_{\tau=0}^{\infty} (\delta\mu)^\tau \log(c_t)$ subject to $c_\tau \leq x_\tau + Z_\tau$ and (*)

versus

$$\max_{\{c_\tau, Z_\tau\}} C - g\Big(\bar{Q}_0 - \sum_{\tau=0}^{\infty} (\delta\mu)^\tau \log(c_\tau)\Big)$$

$$+\sum_{\tau=0}^{\infty} (\delta\mu)^\tau \log(c_\tau)$$ also subject to (*)

$$+\sum_{\tau=0}^{\infty} \delta^{\tau+1}(1-\mu)(\mu)^\tau \frac{\log(w_{\tau+1})}{1-\delta}$$