Voter Participation with Collusive Parties

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Overview

Woman who ran over husband for not voting pleads guilty *USA Today* April 21, 2015

- Palfrey-Rosenthal setting
- rational voter participation
- two collusive parties: similar to “ethical voters”
- parties can enforce social norms through peer punishment
- results in unique mixed strategy equilibrium of all-pay auction
- enforcement costless and equal prize: large party advantaged
- costly enforcement and equal prize of intermediate size: small party advantaged

Mixing

- ethical voter models of Federson/Sandroni and Coate/Conlin use “sufficiently large” aggregate shocks to avoid mixed equilibria
- we stick to the original Palfrey/Rosenthal model
- we observe that GOTV (get out the vote) effort by parties is a carefully guided secret which makes sense only if the party is engaging in a mixed strategy
- we also look at conditions for pure strategy equilibria and the role of pivotality

Cost of Voting

identical party members privately draw a type y from a uniform

distribution on $[0, 1]$

determines a cost of voting $c(y)$, possibly negative and continuously differentiable, has $c'(y) > 0$ and $c(\underline{y}) = 0$ (committed voters)

participation cost of voting

$C(y) = 0$ for $y < \underline{y}$

$C(y) = \int_{\underline{y}}^{y} c(y)dy$ for $y \geq \underline{y}$

(quadratic in Coate/Conlin)

Peer Monitoring Model

simplified version of Levine/Modica, based on Kandori

social norm a threshold $\hat{y}$ and rule to vote if $y \le \hat{y}$

- each member of the party audited by another party member
- auditor observes whether or not auditee voted
- auditee did not vote and norm not violated probability π that auditor will learn this.

$\pi = 0$ then the auditor learns nothing

$\pi = 1$ the auditor perfectly observes whether y is above or below the threshold $\hat{y}$

(auditing costless so unlike Levine/Modica only one round needed)

Peer Punishment

party can impose punishments $0 \leq P \leq \overline{P}$ on members.

- auditee voted or is discovered not to have violated the policy: not punished
- auditee did not vote and the auditor cannot determine whether or not the auditee violated the policy, the auditee is punished with a loss of utility P

social norm is *incentive compatible*

if and only if $P = c(\hat{y})$

Cost of Monitoring

ϕ participation rate of the party (probability of voting)

total cost of inducing participation $D(\phi) = C(\phi) + M(\phi)$

participation cost: $C(\phi) = \int_{\underline{y}}^{\phi} c(y)dy$ is the total cost

$C'(\phi) = c(\phi)$ so $C(\phi)$ is increasing and convex

monitoring cost: $M(\phi) = \int_{\underline{y}}^{1}(1-\pi)Pdy$

incentive compatibility requires $P = c(\hat{y}) = c(\phi) = C'(\phi)$

so write $M(\phi) = (1-\pi)(1-\phi)C'(\phi)$.

$c(\overline{y}) = \overline{P}$ most possible turnout

Convexity and Concavity

$C(\phi)$ is necessarily convex

$M(\phi)$ is not

and so $D(\phi)$ may or may not be

Theorem: *We have $C(\underline{\phi}) = M(\underline{\phi}) = 0$ so $D(\underline{\phi}) = 0$. The participation cost $C(\phi)$ is twice continuously differentiable strictly increasing and strictly convex. The monitoring cost $M(\phi)$ is continuously differentiable. If $\overline{y} = 1$ (that is $c(1) \leq \overline{P}$ so that full participation is possible) the monitoring cost $M(\phi)$ cannot be concave, must be decreasing over part of its range and $M(1) = 0$ so $D(1) = C(1)$.*

at $\underline{y}$ no punishment cost since punishment is not needed to turn out the committed voters

at $\overline{y} = 1$ everybody votes so nobody is actually punished.

All Pay Auction

population of N voters

two parties $k = S, L$ of size $\eta_k N$ where $\eta_S + \eta_L = 1$.

side that produces the greatest ***expected*** number of votes wins prize worth $v_L > 0$ and $v_S > 0$ per capita

costs of turning out voters $\underline{y}_k < \overline{y}_k$ with cost function $D_k(\varphi_k)$

generic assumption $\eta_S \underline{y}_S \neq \eta_L \underline{y}_L$ and $\eta_S \overline{y}_S \neq \eta_L \overline{y}_L$

large party L can turn out the most voters $\eta_L \overline{y}_L > \eta_S \overline{y}_S$

assume $D_k'(\varphi) > 0$

for $\varphi_k < \underline{y}_k$ cost is $D_k(\varphi_k) = 0$

Strategies

probability measure represented by cumulative distribution function F_k on $[\eta_k \underline{y}_k, \eta_k \overline{y}_k]$

$\eta_k \varphi_k$ is the *bid*

tie-breaking rule a measurable function B_S from $[\max \eta_k \underline{y}_k, \eta_S \overline{y}_S]^2 \to [0, 1]$ with $B_S(\eta_S \varphi_S, \eta_L \varphi_L) = 0$ for $\eta_S \varphi_S < \eta_L \varphi_L$ and $B_s(\eta_S \varphi_S, \eta_L \varphi_L) = 1$ for $\eta_S \varphi_S > \eta_L \varphi_L$ with $B_L = 1 - B_S$

Equilibrium

F_S, F_L are an *equilibrium* if there is a tie-breaking rule B_S such that

$$\int v_k B_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) F_k(d\eta_k\varphi_k) F_{-k}(d\eta_{-k}\varphi_{-k}) - \int D_k(\varphi_k) F_k(d\eta_k\varphi_k) \quad \geq$$
$$\int v_k B_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) \tilde{F}_k(d\eta_k\varphi_k) F_{-k}(d\eta_{-k}\varphi_{-k}) - \int D_k(\varphi_k) \tilde{F}_k(d\eta_k\varphi_k)$$

for all cdfs $\tilde{F}_k$ on $[\eta_k \underline{y}_k, \eta_k \overline{y}_k]$

by the Lesbesgue decomposition theorem the cdf F_k may be decomposed into a density for a continuous random variable f_k and a discrete density ϕ_k along with a singular measure (such as a Cantor measure) that can be ruled out in equilibrium

Advantaged and Disadvantaged Parties

$\hat{\varphi}_k$ defined by $D(\hat{\varphi}_k) = v_k$ or $\hat{\varphi}_k = \overline{y}_k$ if there is no solution

most the part is willing and able to turnout (willingness to pay)

generic assumption $\eta_L \hat{\varphi}_L \neq \eta_S \hat{\varphi}_S$

d (the "disadvantaged" party) for which $\eta_d \hat{\varphi}_d < \eta_{-d} \hat{\varphi}_{-d}$

$-d$ the "advantaged" party

Conceding and Taking Elections

a party *concedes* the election if it makes a bid that has zero probability of winning in equilibrium

a party *takes* the election if it makes a bid that has probability one of winning in equilibrium.

the election is *contested* if neither party either concedes or takes the election.

Main Theorem

There is a unique mixed equilibrium. The disadvantaged party earns zero and the advantaged party earns $v_{-d} - D_{-d}((\eta_d/\eta_{-d})\hat{\varphi}_d) > 0$. If $\hat{\varphi}_k \leq (\eta_{-k}/\eta_k)\underline{y}_{-k}$ then party k is disadvantaged, always concedes the election by bidding $\eta_k \underline{y}_k$ and party $-k$ always takes the election by bidding $\eta_{-k}\underline{y}_{-k}$.

If $\hat{\varphi}_k > (\eta_{-k}/\eta_k)\underline{y}_{-k}$ for $k \in \{S, L\}$ then in $(\max_k \eta_k \underline{y}_k, \eta_d \hat{\varphi}_d)$ the mixed strategies of the players have no atoms, and are given by continuous densities

$$f_k(\eta_k \varphi_k) = (1/\eta_{-k}) D'_{-d}((\eta_k/\eta_{-k})\varphi_k)/v_{-k}.$$

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Only a disadvantaged party concedes the election by bidding $\eta_d \underline{y}_d$ with probability

$$\phi_d(\eta_d \underline{y}_d) = 1 - D_{-d}((\eta_d/\eta_{-d})\hat{\varphi}_d)/v_{-d} + D_{-d}((\eta_L/\eta_{-d})\underline{y}_L)/v_{-d}$$

and it has no other atom.

Only an advantaged party with the most committed voters turns out its committed voters with positive probability equal to

$$\phi(\eta_{-d}\overline{y}_{-d}) = D_d((\eta_{-d}/\eta_d)\underline{y}_{-d})/v_d.$$

When the small party is advantaged it has no other atom. If the large party is advantaged and $\hat{\varphi}_S = \overline{y}_S$, theparty takes the election with probability

$$\phi(\eta_S \overline{y}_S) = 1 - D_{-d}((\eta_S/\eta_L)\overline{y}_S)/v_S$$

by bidding $\eta_S \overline{y}_S$

Comparative Statics

1. only the relative sizes of parties matters

2. value of the prize to the party with the least committed voters is small enough then disadvantaged and concedes the election with very high probability. value of the prize to large party very large with very high probability small party turns out only its committed voters and large party acts preemptively turning out as many voters as the small party is capable of turning out

3. if advantaged party has a higher probability of winning a contested election than the disadvantaged party, it has an overall higher probability of winning the election. Otherwise the disadvantaged party can have a better than 50% chance of winning the election

4. in contested election probability of winning by advantaged party increases with own valuation. surplus of advantaged party (and hence welfare) strictly increasing with its own valuation and reduction in the valuation of the disadvantaged party

Common Prize

$N_S v_S = N_L v_L = V$

$D(\phi)$ strictly increasing and twice differentiable in $[\underline{y}, \bar{y}]$ and $D"(\phi)$ univalent meaning $D(\phi)$ either convex or concave on $[\underline{y}, \bar{y}]$, but not both.

Theorem: If $D(\varphi)$ is convex than the small party is disadvantaged. If $D(\varphi)$ is concave and for some $\underline{y} < \hat{y} < \overline{y}$ we have $D(\hat{y}) < V/\eta_L < D(\overline{y})$ and

$$\frac{2\underline{y}D'(\underline{y})}{\hat{y}^2 - \underline{y}^2} < -\max_{\underline{y} \leq y \leq \hat{y}} D"(y)$$

then for $(\eta_S/\eta_L)\overline{y} > \hat{\varphi}_L$ and in particular for η_S close enough to η_L the small party is advantaged.

Small Party Advantaged

V is neither too large nor too small

- too large loses because of large turnout
- too small issue decided by committed voters

small $\underline{y}$

not too constrained by $\overline{y}$

D must be sufficiently concave for the small party to overcome the size advantage of the large party

- high costs of monitoring (generates high concavity)
- homogeneous costs of participation (generates low convexity)

Efficiency

measured by surplus $v_{-d} - D((\eta_d/\eta_{-d})\widehat{\varphi}_d) > 0$

(not by whether the party with the largest v_j won)

worst case: when parties are very similar and $\overline{y}$ constraint does not bind

note: something very fishy about efficiency here

not clear we have a good theoretical grasp of why voting might be a good idea

(why not select a random subset of voters to vote?)

Interpretation of $\widehat{\varphi}_k$

$\widehat{\varphi}_k$ in general (not just for voting) measures willingness to pay when there is a 0-1 decision

- demonstrate, do not demonstrate
- strike, do not strike
- lobbying effort

Remark: the disadvantaged party gets a surplus of zero, the advantaged party gets the surplus of winning minus of submitting a bid equal to the willingness to pay of the disadvantaged part

exactly the same surpluses as a second price auction in weakly undominated strategies; same true for first price auction if equilibrium exists

- in the case of lobbying φ is not “lost” but may be in part income to politicians

Interpretation of $\underline{y}$

$\underline{y}$ are “committed voters”

may in fact be due to a different social norm: “civic duty to vote” also enforced by monitoring but independent of party

- seems less likely to be a factor in non-voting situations such as lobbying, demonstrations, or striking
- not that there wouldn't be people committed to demonstrating, etc. but just that there are probably few of them compared to committed voters)

in the case of lobbying we expect $\underline{y} < 0$, that is the lowest individual cost is positive

$D(0) = 0$ but $\lim_{y \to 0^+} M(0) > 0$

fixed cost of getting anybody to contribute – studied by Levine/Modica

much more favorable to small group

Vote Suppression (Martinelli)

each party can increase monitoring cost θ of opposing party to an amount $\overline{\theta} > \theta$ by incurring cost $G > 0$.

Theorem: If $\overline{\theta}$ is sufficiently close to θ then only the advantaged party will suppress votes. If G is sufficiently small it will choose to do so and this will be a strict Pareto improvement.

Political Contests

conflict resolution function: probability of winning the election a continuous function of the expected number of voters each party turns out

- outcome of the election decided by the actual number of votes rather than the expected number (binomial)
- correlation in the draws of y by voters
- random errors in the counting of votes
- ballots validation
- court intervention

pivotality in the incentive constraint

going to assume $\overline{y} = 1$, large enough (even if terribly costly) punishments

The Contest Model

probability of the small group winning the prize is given by a conflict resolution function $p_S(\eta_S\varphi_S, \eta_L\varphi_L) \in [0,1]$ with $p_L(\eta_L\varphi_L, \eta_S\varphi_S) = 1 - p_S(\eta_S\varphi_S, \eta_L\varphi_L)$.

strategy a cumulative distribution function F_k on $[0,1]$

per capita costs of turning out voters $D_k(\varphi_k, F_{-k})$ depends on F_{-k} because of pivotality

p_S, D_k continuous (weak convergence for probability measures)

no assumption of monotonicity (makes little sense with pivotality)

Equilibrium

We say that F_S, F_L are an *equilibrium* of the conflict resolution model if

$$\int v_k p_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) F_k(d\eta_k\varphi_k) F_{-k}(d\eta_{-k}\varphi_{-k}) - \int D_k(\varphi_k, F_{-k}) F_k(d\eta_k\varphi_k) \quad \geq$$
$$\int v_k p_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) \tilde{F}_k(d\eta_k\varphi_k) F_{-k}(d\eta_{-k}\varphi_{-k}) - \int D_k(\varphi_k, F_{-k}) \tilde{F}_k(d\eta_k\varphi_k) \qquad .$$

Theorem: An equilibrium of the conflict resolution model exists.

Upper Hemi-Continuity

a sequence of conflict resolution models
$p_S^N(\eta_S \varphi_S, \eta_L \varphi_L), D_S^N(\varphi_S, F_L), D_L^N(\varphi_L, F_S)$

all-pay auction with costs $D_k(\varphi_k)$ differentiable
on $(\underline{y}_k, 1]$ with $(1/\gamma) > D_k{}'(\varphi) > \gamma$ for some $\gamma > 0$ and $\eta_S \hat{\varphi}_S \neq \eta_L \hat{\varphi}_L$.

conflict resolution models *converge* to the all-pay auction if for all $\epsilon > 0$ and $\eta_S \varphi_S > \eta_L \varphi_L + \epsilon$ we have $p_S^N(\eta_S \varphi_S, \eta_L \varphi_L) \to 1$ uniformly, and $\eta_S \varphi_S < \eta_L \varphi_L - \epsilon$ implies $p_S^N(\eta_S \varphi_S, \eta_L \varphi_L) \to 0$ uniformly, and $D_k^N(\varphi_k, F_{-k}) \to D_k(\varphi_k)$ uniformly.

Theorem: If F_k^N are equilibria of the conflict resolution models and F_k is the unique equilibrium of the all-pay auction then $F_k^N \to F_k$.

Population Size

N represents population size and conflict resolution function binomial arising from independent draws of type by the different voters. Chebychev's inequality gives the needed uniform convergence of $p_S^N(\eta_S\varphi_S, \eta_L\varphi_L)$

High Value Elections

Theorem: Suppose $v_L \to \infty$. Then $F_L(1-\epsilon) \to 0$.

- as prize grows large the large group almost certainly turns out all of its voters
- in all-pay auction case it turns out only enough voters to beat the small party

first fix N and make the size of the prize large enough that the large party will turn out most of its voters

now fix the size of the prize and increase the number of voters so that equilibrium converges to all-pay auction equilibrium

so that the turnout of the large party must decline until it matches the number of voters in the small party

declining turnout with population size, but not due to pivotality

Pure Strategy Equilibrium

objective functions

$p_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k})v_k - D_k(\varphi_k, F_{-k})$ single-peaked in $\eta_k\varphi_k$

for example: p_k is concave and D_k convex, at least one strictly

all equilibria are pure strategy equilibria (as in Coate-Conlin)

suppose symmetry $p_L(\eta_L\varphi_L, \eta_S, \varphi_S) = p_S(\eta_L\varphi_L, \eta_S\varphi_S)$, when is $p_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) = p(\eta_k\varphi_k/\eta_{-k}\varphi_{-k})$ concave?

when one party turns out twice as many voters as the other it must none-the-less have at least a 25% chance of losing

concavity means "a lot" of variance in the outcome.

Tullock Contests

types y_k have a particular common and idiosyncratic component where the common component may be correlated between the two groups can get the probability of winning to be the Tullock contest success function

$$\frac{1}{1+(\eta_{-k}\varphi_{-k}/(\eta_k\varphi_k))^\alpha} = \frac{(\eta_k\varphi_k)^\alpha}{(\eta_k\varphi_k)^\alpha + (\eta_{-k}\varphi_{-k})^\alpha}.$$

sufficient condition to be concave is that $\alpha \leq 1$

as $\alpha \to \infty$ approach the case of the all-pay auction

Pivotality

social norm φ_k

two partial conflict resolution functions

$P_k^0(\eta_k\varphi_k, \eta_{-k}\varphi_{-k})$ all voters but one follow the social norm, remaining does not vote

$P_k^1(\eta_k\varphi_k, \eta_{-k}\varphi_{-k})$ all voters but one follow the social norm, remaining does vote

differentiable and non-decreasing in φ_k

conflict resolution function is given by

$p_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) = \varphi_k P_k^1(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) + (1-\varphi_k)P_k^0(\eta_k\varphi_k, \eta_{-k}\varphi_{-k})$

probability of being pivotal

$Q_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) = P_k^1(\eta_k\varphi_k, \eta_{-k}\varphi_{-k}) - P_k^0(\eta_k\varphi_k, \eta_{-k}\varphi_{-k})$.

Incentive Constraints

pivotal cutoff $\gamma_k(\varphi_k, F_{-k})$ solution to $c(\gamma_k) = Q_k(\eta_k\varphi_k, \eta_{-k}F_{-k})v_k$.

unique and continuous.

For $y \geq \gamma_k(\varphi_k, F_{-k})$ incentive constraint for voting accounting for pivotality $c(y) - Q_k(\eta_k\varphi_k, \eta_{-k}F_{-k})v_k \leq P_k$

noting the probability of being pivotal depends on the mixed strategy of the other group

monitoring cost for $\varphi_k \geq \gamma_k(\varphi_k, F_{-k})$ is
$M(\varphi_k, F_{-k}) = \psi(1-\pi)(1-\varphi_k)\left(c(\varphi_k) - Q_k(\eta_k\varphi_k, \eta_{-k}F_{-k})v_k\right)$.

assumption about cost of getting someone not to vote does not matter

recall that ψ multiplies the monitoring cost

Theorem: If $\hat{\varphi}_k < 1$ then as $\psi \to \infty$ we have
$F_k^{\psi}(|\varphi_k - \gamma_k(\varphi_k, F_{-k}^{\psi})| \leq \epsilon) \to 1$.