Time, Interest, and Discounting the Future

interest at an annual rate of $r$
paid annually:
$1$ in the bank, and in one year collect $1+r$

discount factor:
to have $1$ in the bank in one year time, must put
\[ \delta = \frac{1}{1+r} \text{ in the bank today} \]
A Useful Approximation

\[
\frac{1}{1 + r} \approx 1 - r \text{ if } r \ll 1
\]

<table>
<thead>
<tr>
<th></th>
<th>(\frac{1}{1 + r})</th>
<th>1 - r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>.9901</td>
<td>.9900</td>
</tr>
<tr>
<td>10%</td>
<td>.9091</td>
<td>.9000</td>
</tr>
<tr>
<td>50%</td>
<td>.6667</td>
<td>.5000</td>
</tr>
</tbody>
</table>
Present Value

1 dollar at the beginning of every year for $\tau$ years is worth what right now?

what is $z = 1 + \delta + \delta^2 + \ldots + \delta^{\tau-1}$?

$\delta z = z - 1 + \delta^\tau$

$(1 - \delta)z = 1 - \delta^\tau$

\[
1 + \delta + \delta^2 + \ldots + \delta^{\tau-1} = z = \frac{1 - \delta^\tau}{1 - \delta}
\]
Mortgage Interest

You buy a house for $250,000. You make a 20% down payment, and get a 30 year fixed rate mortgage at 8% annual interest. How much are your monthly payments.

- suppose that monthly interest is $8%/12=0.67\%$
- so $\delta = \frac{1}{1 + .0067} \approx .9933$
- mortgage is for $200,000$
- number of payments $\tau = 360$
Find the Monthly Payment

let \( p \) be the monthly payment

then

\[
200000 = (\delta + \delta^2 + \ldots + \delta^t)p = \delta \frac{1 - \delta^t}{1 - \delta} p
\]

or

\[
p = 200000 \frac{1 \ 1 - \delta}{\delta \ 1 - \delta^t}
\]

\[
\approx 200000 \frac{1}{.9933} \cdot .006655 \cdot .9111 \approx 1471
\]
Capital and Investment

Present value is an essential tool for evaluating investments

*Investment* creates *capital*

there are several types of capital

• financial capital (for example, the bank account)
• physical capital (a house, machine, factory)
• human capital (what you get by investing in education)

Next: Repeated Games
The Repeated Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Player 1</th>
<th>don’t confess</th>
<th>confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>don’t confess</td>
<td>32,32</td>
<td>28,35</td>
</tr>
<tr>
<td>confess</td>
<td>35,28</td>
<td>30,30</td>
</tr>
</tbody>
</table>

- This is a simultaneous move game with a unique Nash equilibrium, and a unique strictly dominant strategy solution at 30, 30.
- The unique non-cooperative solution is Pareto dominated by 32, 32.
- With repeated play, incentives are changed by the possibility of punishments and rewards in the future.
More Than One Equilibrium

a basic feature of repeated games: regardless of the discount factors, the repeated static equilibrium is a subgame perfect equilibrium of the repeated game
Grim Trigger Strategies

The **grim trigger strategy** in the repeated game is

- cooperate in the first period
- cooperate in subsequent periods as long as all players have cooperated in every previous period
- cheat in any period in which some player has cheated in any previous period
What to Do?

- payoff to cheating
  \[(35 + 30\delta + 30\delta^2 \ldots) = 5 + 30/(1 - \delta)\]
- payoff to cooperating
  \[32/(1 - \delta)\]
- optimal to cooperate if
  \[32 \geq 35 - 5\delta \text{ or } \delta \geq 3/5\]
- if \(\delta \geq 3/5\) both players playing the grim strategy is a subgame perfect equilibrium
- why is this subgame perfect?
Incentive Compatibility

The condition that cooperation is better than cheating

\[ 32 \geq 35 - 5\delta \]

is called an *incentive constraint*

if it is satisfied then cooperation is said to be *incentive compatible*
All payoffs in the game were in points. At the end of each session, the points earned by each subject were converted into dollars at the exchange rate 200 points=$1 and paid privately in cash. In addition, subjects were paid a 5 dollar show up fee.
Repetition

Infinite horizon

\[ \delta = 0,1/2,3/4 \] expected length 1,2,4 (how did he do this??)

Finite horizon

\[ H = 1,2,4 \]

subjects played all infinite or all finite

done in both orders – increasing length and decreasing length
## Theory

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>PD1</th>
<th>PD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>DD</td>
<td>DD</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>DD, DC, CD</td>
<td>DD, CC</td>
</tr>
<tr>
<td>( \frac{3}{4} )</td>
<td>All</td>
<td>All</td>
</tr>
</tbody>
</table>
# Results on Cooperation

Table 5: Percentage of cooperation by match and treatment* 

<table>
<thead>
<tr>
<th></th>
<th>Match</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>(\delta = 0)</td>
<td>26.26</td>
<td>18.18</td>
<td>10.61</td>
<td>11.62</td>
<td>12.63</td>
<td>12.63</td>
<td>5.56</td>
<td>5.26</td>
<td>5.26</td>
<td>5</td>
</tr>
<tr>
<td>Dice (\delta = \frac{1}{2})</td>
<td>28.36</td>
<td>27.12</td>
<td>34.58</td>
<td>35.53</td>
<td>21.60</td>
<td>19.08</td>
<td>29.84</td>
<td>35.96</td>
<td>28.16</td>
<td>50</td>
</tr>
<tr>
<td>(\delta = \frac{3}{4})</td>
<td>40.44</td>
<td>28.57</td>
<td>27.78</td>
<td>32.92</td>
<td>46.51</td>
<td>33.09</td>
<td>44.05</td>
<td>53.51</td>
<td>42.26</td>
<td>45.83</td>
</tr>
<tr>
<td>(H = 1)</td>
<td>26.56</td>
<td>18.23</td>
<td>16.67</td>
<td>17.19</td>
<td>11.98</td>
<td>8.02</td>
<td>6.79</td>
<td>10.49</td>
<td>6.14</td>
<td>6.67</td>
</tr>
<tr>
<td>Finite (H = 2)</td>
<td>19.79</td>
<td>15.89</td>
<td>14.84</td>
<td>9.64</td>
<td>11.46</td>
<td>10.80</td>
<td>12.04</td>
<td>10.19</td>
<td>6.58</td>
<td>6.67</td>
</tr>
<tr>
<td>(H = 4)</td>
<td>31.64</td>
<td>30.34</td>
<td>30.47</td>
<td>25.52</td>
<td>25.13</td>
<td>23.77</td>
<td>16.36</td>
<td>19.75</td>
<td>14.91</td>
<td>20.83</td>
</tr>
</tbody>
</table>

*All rounds.
Focus on matches 4-10

Table 6: Percentage of cooperation by round and treatment*

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ = 0</td>
<td>9.17</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dice</td>
<td>δ = \frac{1}{2}</td>
<td>30.93</td>
<td>26.10</td>
<td>19.87</td>
<td>12.50</td>
<td>12.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δ = \frac{3}{4}</td>
<td>46.20</td>
<td>40.76</td>
<td>38.76</td>
<td>34.58</td>
<td>33.04</td>
<td>27.27</td>
<td>24.75</td>
<td>26.28</td>
<td>29.17</td>
<td>26.04</td>
<td>32.29</td>
<td>31.25</td>
</tr>
<tr>
<td>H = 1</td>
<td>10.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finite H = 2</td>
<td>13.31</td>
<td>6.90</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H = 4</td>
<td>34.58</td>
<td>21.55</td>
<td>18.97</td>
<td>10.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H = 4</td>
<td>34.58</td>
<td>21.55</td>
<td>18.97</td>
<td>10.63</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tbody>
</table>

*Matches four through ten.
# Joint Outcomes

Table 7: Distribution of outcomes by stage game and treatment*

<table>
<thead>
<tr>
<th></th>
<th>$\delta = 0$</th>
<th>$\delta = \frac{1}{2}$</th>
<th>$\delta = \frac{3}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PD1</td>
<td>PD1</td>
<td>PD1</td>
</tr>
<tr>
<td>CC</td>
<td>2.98</td>
<td>3.17</td>
<td>20.68</td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>18.83</td>
<td>25.64</td>
</tr>
<tr>
<td>CD &amp; DC</td>
<td>20.83</td>
<td>28.57</td>
<td>30.34</td>
</tr>
<tr>
<td></td>
<td>13.98</td>
<td>25.50</td>
<td>26.03</td>
</tr>
<tr>
<td>DD</td>
<td>76.19</td>
<td>68.25</td>
<td>48.98</td>
</tr>
<tr>
<td></td>
<td>85.75</td>
<td>55.67</td>
<td>48.33</td>
</tr>
</tbody>
</table>

*Matches four through ten, and all rounds.
**Concepts**

- interest rate, discount factor
- present value
- investment, capital
- financial capital, physical capital, human capital
- grim trigger strategy
- incentive compatibility
Skill

given investments with different income streams
find and compare the present values
determine when the incentive constraints are satisfied by grim-trigger strategies