Bayes Law

\[ \mu(E|F) = \frac{\mu(F|E)\mu(E)}{\mu(F)} \]

“likelihood” times “prior”
**Ratio Form**

\[
\frac{\mu(E|F)}{\mu(-E|F)} = \frac{\mu(F|E)\mu(E)}{\mu(F|-E)\mu(-E)}
\]

(Kahneman and Tversky)

Steve is very shy and withdrawn, invariably helpful but with very little interest in people or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail. \(\hat{F} = \text{Steve}\)

Is Steve more likely to be a librarian or a farmer? \(E = \text{farmer}\)

Important bit of information: there are about twenty times as many farmers as librarians
Sample Selection Bias and Why We Do RCTs
Using Bayes Law

Pr(shot|return) what we see
Pr(return|shot) what we want

Getting these two confused is a common source of error
Drug Testing

A drug test has a 5% chance of error. A group of parolees is given the test. Of the parolees, 60% are drug users. If the test is positive how likely is it the parolee is using drugs?

E=using drugs
F=positive test

\[ \mu(E|F) = \frac{\mu(F|E)\mu(E)}{\mu(F)} \]

\[ = \frac{.95 \times .6}{.95 \times .6 + .05 \times 4} = .97 \]
Airline Pilots

Now the test is given to a group of airline pilots of whom only 2% are drug users. If the test comes out positive how likely is it the pilot is using drugs?

\[
\mu(E|F) = \frac{.95 \times .02}{.95 \times .02 + .05 \times .98} = .28
\]
The Ann Landers Problem

Ann Landers says that all heroin users once used marijuana, so that if you use marijuana, you will surely end up using heroin.

\[ \mu(E|F) = \frac{\mu(F|E)\mu(E)}{\mu(F)} = \frac{\mu(E)}{\mu(F)} \]

so that if there are 100 times as many marijuana users as heroin users, using marijuana means only a 1% chance of using heroin.
Independence

We say two events $E, F$ are independent if

$$
\mu(E \cap F) = \mu(E)\mu(F)
$$

What is the conditional probability when events are independent?

$$
\mu(E|F) = \frac{\mu(E \cap F)}{\mu(F)} = \frac{(\mu(E)\mu(F))}{\mu(F)} = \mu(E)
$$
**Nature’s Moves**

Add an additional player “Nature” with random moves

Example: Chain Store in declining industry
Decision Analysis

To drill for oil or not to drill for oil? Cost $100,000.

How much will you pay for a geological survey before drilling?

Value of Oil:
$0 (dry) with probability 50%
$300,000 with probability 50%

The survey has a 10% error rate
no risk aversion
Decision Tree
Where to Put Nature’s Move

As late as possible to make it easier to do backwards induction
Filling in the ‘?’s

\[
pr(dry|+) = \frac{pr(+|dry) pr(dry)}{pr(+)} = \frac{.1 \times .5}{.5} = .1
\]
Compute Expected Utilities
drill or survey; survey if $85 - x > 50$ or $x < 35$
First Rule of Decision Analysis

- Do not pay for information that will not change your decision
- widely violated during Covid vaccination pauses
Concepts

- Conditional probability
- Bayes law
- independence
- value of information
- first rule of decision analysis
Skill

given information about conditional probabilities
do a cost/benefit analysis