Matching Pennies

Holmes (row) versus Moriarity (column)

	Canterbury	**Dover**
Canterbury	-1,1	1,-1
Dover	1,-1	-1,1

Holmes got off at Canterbury and Moriarity continued on to Dover

Holmes won

not an equilibrium

how about randomizing 50-50? *mixed strategy equilibrium*

a *mixed strategy* is a probability distribution over pure strategies

Do People Randomize?

- If they are Conan Doyle apparently not
- once in Japan catchers were equipped with mechanical randomization devices to call the pitch

(later ruled unsporting and banned from play)

- good tennis players in important matches do it right
- professional soccer players do it right
- submarine captains and the RAND corporation do it right

The Uncertainty Principle for Social Sciences

Two parties

the party leader can mobilize voters at a cost of 3 per voter

the winning party gets 8, split in case of a tie

	0	**1**	**2**
0	4,4	0,5*	0*,2
1	5*,0	1,1	-3,2*
2	2,0*	2*,-3	-2,-2

No pure strategy equilibrium

The Logic of Strategy

if we know who is going to win election the loser should not turn out

if one party is not going to turn out at all the other party should turn out just one voter

but it can't be that one party turns out no voters and the other just one: better turn out two and win

Why Nate Silver Is Not As Numerate As He Thinks

party leaders tell members how to vote

a poll is taken

then party leaders can change their mind

whatever the poll says someone wants to change their mind (no pure strategy equilibrium)

Rational Expectations

If you have a correct theory of human behavior and if everyone believes your theory then your theory is Nash equilibrium

“rational expectations” if everyone believes you they will behave according to your theory – in game theory, Nash equilibrium = rational expectations

if there is no pure strategy equilibrium then people cannot be certain of what other are going to do

if some expert says that they have a theory that is not rational expectations what they are saying is that if they can convince people their theory is true then it will stop working

Predicting Stock Market Crashes

- For some reason everyone thinks that if economists were any good we could do this
- We are good enough to know we can't
- Suppose I predicted the stock market would crash next week and everyone believed me?

Might as well complain about physicists not being able to simultaneously predict the position and speed of a particle at the same time

Mixed Strategy Equilibrium

- Not easy to give a recipe for finding mixed Nash equilibria
- To mix a player must be indifferent, this is the usual method of solving: find the strategies for player 2 that makes player 1 indifferent and vice versa
- Every finite game has a mixed Nash equilibrium
- Especially important in situations of conflict

Coordination Game

	Station	**Airport**
Station	1*,1*	0,0
Airport	0,0	1*,1*

Two pure equilibria, but also a mixed equilibrium where both players play 50-50.

- Interpretation of mixed equilibrium in terms of uncertainty

Kitty Genovese Problem

March 1964 the *New York Times* published a shocking article:

on March 13 in a poor area of Queens New York Kitty Genovese was raped and murdered on the sidewalk in front of her apartment building while 38 people watched from their windows

none of them intervened or called the police

the crime is shocking but is it shocking that nobody called the police?

Kitty Genovese Model

n people all identical

benefit is someone calls the police is x

cost of calling the police is 1: at the best of times dealing with the police is difficult and unpleasant, the more so in a poor neighborhood

Assumption: $x > 1$: these are not bad people

model of large anonymous apartment building where people do not coordinate

no designated “neighborhood watch person”

model this by assuming a symmetric equilibrium where everyone behaves the same way

Mixed Strategy Equilibrium

symmetry: everyone calls, nobody calls, or mixed

nobody calls, everyone want to

everyone calls, nobody wants to

so look for symmetric mixed strategy equilibrium where p is the common probability of each person calling the police

Indifference

p is the symmetric equilibrium probability for each player to call the police

each player *i* must be indifferent between calling the police or not

if *i* calls the police, gets $x - 1$ for sure.

If *i* doesn't, gets 0 with probability $(1-p)^{n-1}$, gets *x* with probability

$$1 - (1-p)^{n-1}$$

so indifference when

$$x - 1 = x\left(1 - (1-p)^{n-1}\right)$$

Solution

solve for *p*

$$p = 1 - (1/x)^{1/(n-1)}$$

probability police are called

$$1 - (1-p)^n = 1 - \left(\frac{1}{x}\right)^{\frac{n}{n-1}}$$

n increases $n/(n-1)$ which is bigger than 1 decreases towards 1

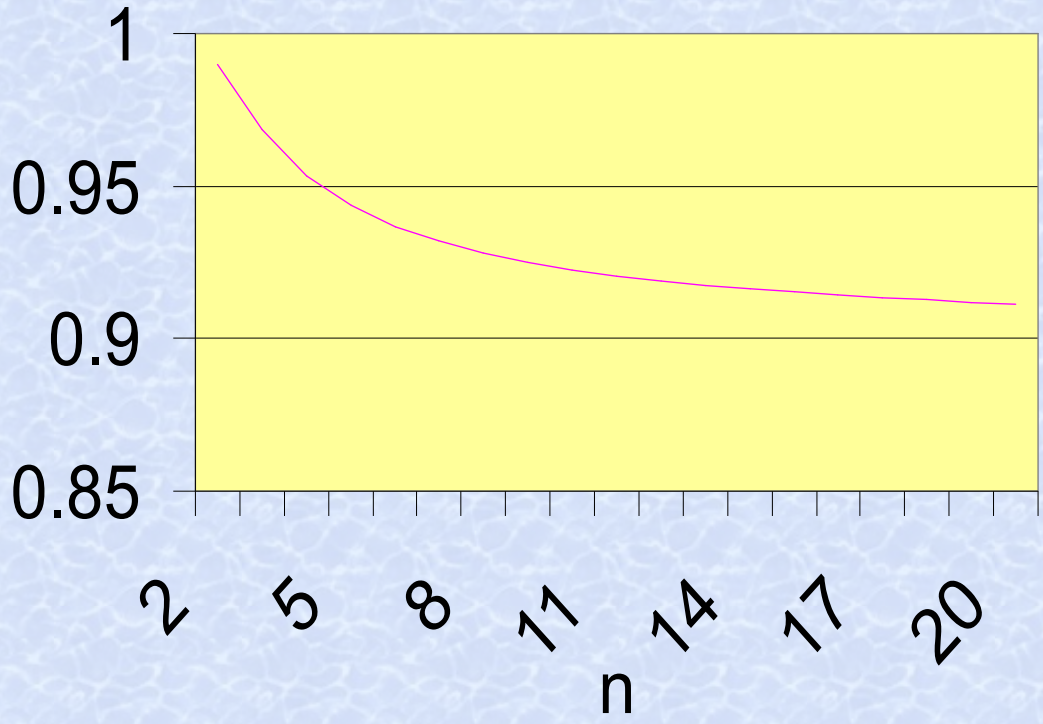
$1/x < 1$ so lower exponent means bigger value

so probability goes down as n goes up

Graph

$$x = 10$$

probability police are called



Many Neighbors

For large n the probability the police is called is approximately

$$1 - 1/x$$

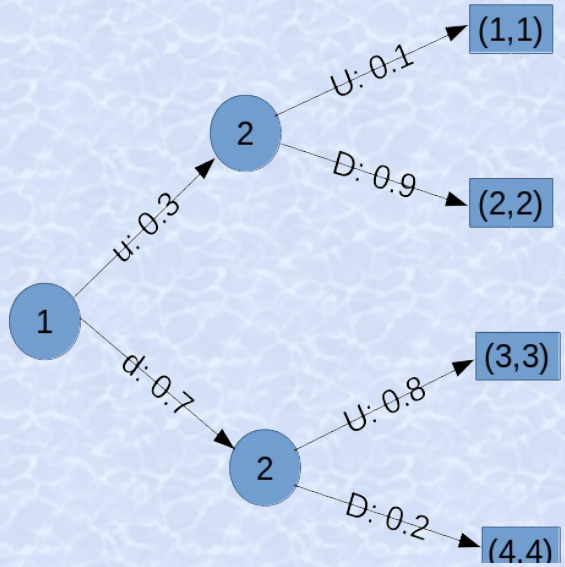
for example: if the cost is half the benefit there is only about a 50% chance the police are called no matter how many people are watching

Mixed vs. Behavior Strategies

A mixed strategy is a probability distribution over strategies in the normal form

A behavior strategy gives conditional probabilities of acting at information sets

Example



	UU(.08)	UD(.02)	DU(.72)	DD(.18)
u(.3)	1,1	1,1	2,2	2,2
d(.7)	3,3	4,4	3,3	4,4
	(.06)	(.04)	(.74)	(.16)

Comparison

Pr(U|u)=Pr(UU)+pr(UD)=0.1

Pr(U|d)=Pr(UU)+pr(DU)=0.8

For practical purposes mixed and behavior strategies are the same

Concepts

- **mixed strategy**, **Nash equilibrium**
- **uncertainty principle for the social sciences**
- conflict, strategic voting
- behavior strategy

Skill

given a 2x2 matrix game

find the mixed strategy equilibrium